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Robust disturbance suppression for nonlinear systems based on H_∞ control

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Abstract

The disturbance suppression problem for nonlinear systems is examined in this paper. We review the so-called nonstandard mixed sensitivity problem, which introduces an integrator to a selected weight, as well as the linear classical disturbance suppression problem and the linear H_∞ disturbance suppression problem. We extend this H_∞ problem to the nonlinear case, and present a method to reduce the order of the state feedback Hamilton-Jacobi PDE (Partial Differential Equation) for this nonlinear H_∞ problem by extending the concept of comprehensive stability [8] [7]. Finally, we investigate the structure of the output feedback H_∞ controller for disturbance suppression, and draw the conclusion that, as in the linear case, there must also be an integrator in the controller.

1 Introduction

This paper is mainly concerned with the constant disturbance rejection problem for nonlinear systems, and it uses H_∞ methods to examine the problem. An important objective of control system design is to minimise the effects of external disturbances. The problem of disturbance rejection (especially constant disturbance rejection) arises in many industrial fields, such as motion-control, active noise control and vibration control. The classical method for constant disturbance rejection is to include an integrator into the controller. However, this technique requires separate consideration of stability, so it does not directly deal with the robust stability issue.

In recent years, H_∞ methods have been employed to handle disturbance suppression problems [10] [7] for linear systems. The main methodological device is to introduce an integrator in a selected weight function and then formulate the disturbance rejection problem as a mixed sensitivity problem. However, these problems are nonstandard H_∞ problems, because they have an un-stabilisable pole at the origin, which violates the pre-requisite conditions of standard H_∞ control theory. There are several indirect ways to get around this problem, such as by using singular perturbation techniques or changing the system block diagram to absorb the integrator weight into the control loop [10].

Paper [7] uses so-called extended H_∞ theory to give a relatively direct alternative solution of this nonstandard H_∞ problem for linear systems. Furthermore, the integrator weighting leads to order reduction of the Riccati equation by using a so-called quasi-stabilising solution. As for a classical control design, the controller arising from either of the two H_∞ approaches normally contains an integrator. Here we extend these ideas to the nonlinear disturbance suppression problem. As in the linear case, for the nonlinear H_∞ problem there are once again standard and nonstandard problems. Not surprisingly, the H_∞ disturbance rejection problem for the nonlinear case inherits the difficulty of the linear case: the existence of un-stabilisable states makes the problem nonstandard.

The main bottleneck of nonlinear H_∞ control, which is similar to the problem encountered in nonlinear optimal control, is the need to solve the Hamilton-Jacobi (HJ) partial differential equation (PDE)[6]. Although explicit globally-defined solutions of most HJ PDEs are hard to obtain, we will present a method which can simplify (by order reduction) the HJ PDE for the nonlinear disturbance rejection problem by using the concept of *comprehensive stability*, which is extended from the linear case (See [7]). Furthermore, we can show that the controller for output feedback control contains an integrator in a sense defined later, in Section 6.

In the next section, we briefly review the classical constant disturbance suppression method. In Section 3, we examine, for linear systems, the mixed sensitivity H_∞ method, and in particular, the so-called extended H_∞ method, which can deal with the robust disturbance suppression problem. In Section 4, we set up the disturbance suppression problem for the nonlinear case. Section 5, the main part, gives an order reduction theorem for the state feedback HJ PDE arising from the nonlinear disturbance suppression problem. Finally in Section 6, we probe the structure of the output feedback H_∞ controller of the system under consideration,

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and show that it normally contains an integrator.

2 Review of the classical constant disturbance rejection technique for linear systems

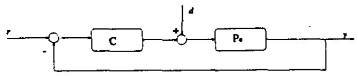

Figure 1: A classical disturbance suppression problem

Let us consider a classical disturbance rejection problem as shown in Figure 1. This depicts a linear time-invariant single-input single output (SISO) system. It consists of the interconnection of a plant $P_0(s)$ and controller $C(s)$ forced by a command signal r, as well as an input disturbance d.

Suppose the command $r(s)$ is identically zero. Then the relationship between the output y and disturbance d is:

$$y(s) = \frac{P_0(s)}{1 + C(s)P_0(s)} d(s). \quad (1)$$

We are normally interested in reducing or eliminating the effect of the disturbance d. When d changes slowly (i.e. the band width of disturbance is low pass), the desired effect can be achieved by adding an integrator into the controller C, provided stability is retained. From (1), we can also see that if $d(s)$ is a step, then an integral controller (i.e. $C(s) = \hat{C}(s)/s$ with $\hat{C}(0)$ nonzero) can totally reject the disturbance (when $t \to \infty$).

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \frac{P_0(s)}{1 + C(s)P_0(s)} d(s) = 0 \quad (2)$$

Unless $P(0) = 0$, it is also necessary that $C(s)$ contains an integrator to secure the rejection property.

3 H_∞ treatment of the classical disturbance suppression problem for linear systems

The classical disturbance suppression technique demands separate theoretical consideration of stability, and certainly does not deal directly with the robust stability issue. To guarantee robust stability, we need to rely on a theory of robust control, such as H_∞ theory. There are at least two ways [10][7] to design an H_∞ controller for the disturbance suppression problem. Apart from redrawing the loop of Fig 1 to correspond to the standard H_∞ formulation, the main methodological device is to introduce an integrator into a selected weight function. The H_∞ problems in [10] and [7] belong to the class of mixed sensitivity problems in H_∞ control. In [10] an integrator is introduced into one of the output weights W_z (see Figure 2), while in [7] there is an integrator in one of the input weights W_d (Figure 3).

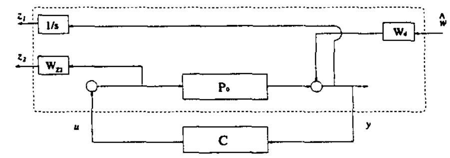

Figure 2: The linear H_∞ framework for the disturbance suppression problem with an integral in the output weight

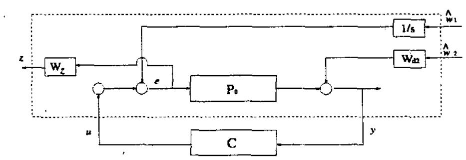

Figure 3: The linear H_∞ framework for the disturbance suppression problem with an integral in the input weight

Evidently, associated with the classical disturbance suppression problem (1), there are at least two kinds of mixed sensitivity H_∞ problems, (Figures 2 and 3). It can be easily checked, for the linear case, that the two H_∞ problems are duals of each other. So, without loss of generality, we can choose the mixed sensitivity problem described in Fig 3 as the basis for our discussions.

In this diagram, P_0 represents the given plant, $1/s$ and W_{d2} are input weights, W_z is an output weight, and C is the controller which needs to be constructed in such a way that it can stabilise the plant P_0, and make the infinity norm of the transfer function from $[\hat{w}_1 \ \hat{w}_2]^T$ to z less than some given bound γ. Note that at zero frequency the integrator ensures that the gain from $\hat{w}_1$ to z will be zero. Given the plant and the weights, the standard approach is to seek to formulate the problem as an H_∞ problem. However, this problem does not satisfy all the pre-requisite conditions of the standard H_∞ control problem (which includes a stability condition [2] [10]), because of an un-stabilisable mode at the origin. Therefore, this problem termed nonstandard. More precisely, consider the state-variable realisation of the "generalised plant" with input $\hat{w}_1, \hat{w}_2$ and u and output z and y in Figure 3. The entire state is not stabilisable from u, because the integrator driven by $\hat{w}_1$ is unaffected by u.

The book [10] gives indirect solutions for such nonstandard problems by using singular perturbation methods (using $\frac{1}{s+\epsilon}$ instead of $\frac{1}{s}$ for small positive ϵ).

The so-called extended H_∞ controller [7] will solve the mixed sensitivity problem described in Figure 3, where a constant disturbance enters at the plant input. By using disturbance-observer-based integral control, the robust stability requirement is satisfied directly. The synthesis of the extended H_∞ controller requires a "quasi-stabilising" solution [7] of the "X"-Riccati equation (the Riccati equation arising in the state feedback problem, which also arises in the output feedback problem.). The original $(n+1)$-th order Riccati equation can be constructed from the solution of a reduced order n-th order equation, n being the degree of P_0.

Similarly we can use extended H_∞ controller design to solve the mixed sensitivity problem of Figure 2, where the constant disturbance enters at the plant output. Not surprisingly, for this dual formulation, it is possible to simplify the controller synthesis by constructing the solution to the original $(n+1)$-th order "Y" Riccati equation (the Riccati equation arising in the output feedback routines which is termed the filter or observer Riccati equation) from the solution of a reduced n-th order equation.

4 Setting up the disturbance suppression problem formulation in the nonlinear case

The problems discussed above are all linear ones. In this section, we give a description of the nonlinear problem.

To begin, we consider the classical disturbance problem shown as in Figure 1, except that the plant may be nonlinear. In order to give a more explicit description, we suppose that the SISO nonlinear plant, P_0, is modelled as follows.

$$P_0: \begin{cases} \dot{x}_0 = A(x_0) + B_1(x_0)w_1 + B_2(x_0)u \\ y = C_2(x_0) + w_2. \end{cases} \quad (3)$$

We assume that the functions appearing in systems of this paper are smooth with bounded first and second order partial derivatives. Here, $w = [w_1\ w_2]^T$, and $w_1 \in R$ corresponds to a plant input disturbance, while $w_2 \in R$ corresponds to a plant output disturbance. The introduction of the disturbance w_2 can be interpreted as a way of capturing modelling uncertainty for output feedback H_∞ control. It should be noted that if w_2 is zero, then the problem becomes singular. In order to simplify our discussion, we shall suppose that $B_1(x_0) = B_2(x_0)$. (For the input disturbance rejection problem, this condition is always satisfied.)

We need to extend this nonlinear disturbance rejection problem as depicted in Figure 1 to an H_∞ style problem. As mentioned in the previous section, for the linear case [10] [7], there are two ways to perform this step. The first one is depicted in Figure 2, and the second one in Figure 3. The first way, as stated in the last section, leads to a solution allowing order reduction of the "Y"-Riccati equation (or observer Riccati equation) in the linear case. However, for the nonlinear case, there is no simple and explicit "filter" HJ PDE which is equivalent to the "Y" Riccati equation of linear H_∞ system theory. If we choose the second formulation, it turns out that we can reduce the order of the control HJ PDE for the state feedback problem (which is equivalent to the "X" Riccati equation in the linear case). Therfore, we elect to extend this nonlinear disturbance suppression problem to an H_∞ problem along the lines of [7].

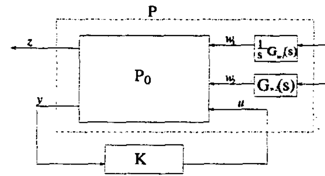

Figure 4: A nonlinear disturbance suppression problem

The framework of this nonlinear H_∞ problem is shown in Figure 4. In order to slightly extend the application scope of our method, we choose $\frac{1}{s}G_{w1}(s)$ instead of only $\frac{1}{s}$ as the weight of $\hat{w}_1$. Here $G_{w1}(s)$ is a stable and proper transfer function. Based on linear classical control theory, $\frac{1}{s}G_{w1}(s)$ can be written as $\frac{\alpha}{s} + G_{w12}(s)$, where α is real and $G_{w12}(s)$ is a stable and strictly proper transfer function (See Figure 5.)

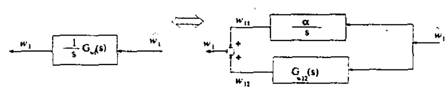

Figure 5: The change of weight transfer function

The state equation of the weight transfer function becomes:

$$\begin{cases} \dot{x}_{w_{11}} = \alpha \hat{w}_1 \\ \dot{x}_{w_{12}} = A_{w_{12}} x_{w_{12}} + B_{w_{12}} \hat{w}_1 \\ w_1 = w_{11} + w_{12} = x_{w_{11}} + C_{w_{12}} x_{w_{12}} \end{cases} \quad (4)$$

We assume that $G_{w_2}(s)$ in Figure 4 is a rational stable proper transfer function with no finite and infinite $j\omega$-axis zeros [1], which can be expressed as:

$$\begin{cases} \dot{x}_{w_2} = A_{w_2} x_{w_2} + B_{w_2} \hat{w}_2 \\ w_2 = C_{w_2} x_{w_2} + D_{w_2} \hat{w}_2 \end{cases} \quad (5)$$

The equations of the generalised system are now:

[1] If $G_{w_2}(s)$ has finite or infinite $j\omega$-axis zeros, then there will be $j\omega$-axis zeros from the disturbance input $\hat{w}$ to the measurement output y. This also leads to a nonstandard problem. In order to concentrate our attention on main problem, we assume that $G_{w_2}(s)$ has no finite and infinite $j\omega$-axis zeros.

$$\begin{cases} \dot{x}_{w_{11}} &= \alpha\hat{w}_1 \\ \dot{x}_{w_{12}} &= A_{w_{12}}x_{w_{12}} + B_{w_{12}}\hat{w}_1 \\ \dot{x}_{w_2} &= A_{w_2}x_{w_2} + B_{w_2}\hat{w}_2 \\ \dot{x}_0 &= A(x_0) + B_1(x_0)(x_{w_{11}} + C_{w_{12}}x_{w_{12}} + u) \\ y &= C_2(x_0) + C_{w_2}x_{w_2} + D_{w_2}\hat{w}_2 \end{cases} \tag{6}$$

The choice of z is important. We elect to set $z = e'$ (See Figure 6), a little difference from the linear case. [2] That is, we split the disturbance w_1 into two components, $w_{11} = x_{w11}$ and $w_{12} = C_{w2}x_{w2}$.

$$z = e' = x_{w_{11}} + u. \tag{6*}$$

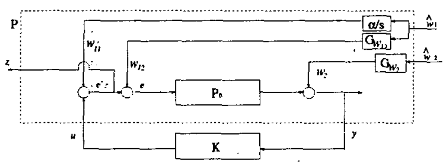

Figure 6: The explanation for e'

5 Simplification of the state feedback HJ PDE for nonlinear disturbance suppression problem under comprehensive stability

Here, we extend the concept of so-called comprehensive stability [7] to the nonlinear H_∞ problem. This includes the nonlinear disturbance rejection problem, which contains un-stabilisable states. The disturbance rejection problem is a nonstandard H_∞ problem, because $x_{w_{11}}$ is not stabilisable from u.

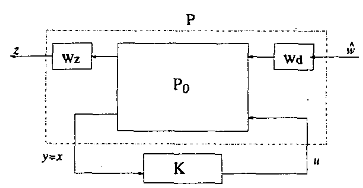

Figure 7: The structure of mixed sensitivity problem

First we introduce the standard nonlinear state feedback H_∞ control problem: See Fig 7, let the state space model for plant P be:

$$\begin{cases} \dot{x} &= A(x) + B_1(x)\hat{w} + B_2(x)u \\ z &= C_1(x) + D_{12}(x)u \\ y &= x. \end{cases} \tag{7}$$

The standard state feedback H_∞ control problem is to find a controller $u = K(x)$ which makes the closed loop (P,K) γ-dissipative and internally stable, see [3]. Internal stability is the condition that $x(t) \to 0$ as $t \to 0$ for all $x(0)$ and $\hat{w} \in L_2[0,\infty)$.

[2] For the linear case, $z = e = y_{w_1} + u$. We also could choose $z = e$ here, but it is more convenient to provide another choice of z.

Theorem 1 *Consider the system defined by equation (7), and suppose that $\exists\alpha,\beta : \alpha I \geq E_1 = D_{12}^T D_{12} \geq \beta I > 0$ for all x. Suppose one can find a strictly positive proper smooth function V of x, such that $V(x) > 0$ for $x \neq 0$, $V(0) = 0$ and which (a) satisfies the state feedback Hamilton Jacobi PDE (HJ PDE)*

$$\begin{aligned} &\nabla_x V\left(A - B_2E_1^{-1}D_{12}^TC_1\right) \\ +\ &\frac{1}{2}\nabla_x V\left(\gamma^{-2}B_1B_1^T - B_2E_1^{-1}B_2^T\right)\nabla_x V^T \\ +\ &\frac{1}{2}C_1^T\left(I - D_{12}E_1^{-1}D_{12}^T\right)C_1 = 0. \end{aligned} \tag{8}$$

and (b) makes the vector field

$$A - B_2E_1^{-1}D_{12}^TC_1 + \left(\gamma^{-2}B_1B_1^T - B_2E_1^{-1}B_2^T\right)\nabla_x V^T$$

asymptotically stable. Then the central controller for the state feedback problem, which guarantees γ-dissipativity and internal stability, is defined as:

$$K^*(x) = -E_1(x)^{-1}[D_{12}(x)^TC_1(x) + B_2(x)^T\nabla_x V(x)^T].$$

Furthermore, even if (b) is not fulfilled, the closed-loop satisfies the dissipation inequality

$$V(x(t)) + \tfrac{1}{2}\int_0^T |z(t)|^2dt \leq \gamma^2\tfrac{1}{2}\int_0^T |\hat{w}(t)|^2dt + V(x(0)).$$

Proof: See [3]. Other proofs are omitted to save space. ∎

For some mixed sensitivity problems, such as (6), there exist some un-stabilisable states, so it is obvious that no stabilising solution for the HJ PDE exists. In order to get around this obstacle, we extend the concepts of comprehensive stability and essential stability [8] to nonlinear systems.

Definition 2 *The closed-loop system (P,K) in Fig 7 is essentially stable if the interconnection of the physical plant P_0 and controller K is internally stable, or equivalently, if the only non-internally-stable modes of (P,K) are those associated with the weighting.*

The motivation is that the weighting is not present in any physical sense, while P_0 and K are physically present.

Definition 3 *The closed-loop system (P,K) in Fig 7 is said to be comprehensively stable if it is essentially stable, and the closed-loop from $\hat{w}$ to z is γ-dissipative. When this is the case, K is called a comprehensively stabilising controller.*

As a first step towards adjusting Theorem 1 to cope with un-stabilisable states, we present Theorem 7 below. This theorem uses the concept of zero detectability; we now define this concept for the system.

Definition 4 *The system (of Figure 7) with input $\hat{w}$ and output z is said to be zero-detectable if the conditions that $\hat{w}(t) = 0$ and $z(t)=0$ for all $t \geq 0$, are sufficient to imply that $\lim_{t\to\infty} x(t) = 0$.*

We present a lemma as follows, which will be needed for the main stability theorem. It comes from a simple extension of La Salle's invariance principle [9].

Definition 5 *We define Π as the projection from $\Re^{\dim(x)}$ to $\Re^{\dim(x_s)}$ in the obvious way by $\Pi([\ x_w^T\ x_s^T\ ]^T) = x_s$.*

Lemma 6 *Let $V(x)$ be a scalar function with continuous partial derivatives. Let $\mathcal{B}_r$ be the set defined as $\{x : V(x) < r\}$. Assume that for all $r \in \Re$, the state x_s is bounded within $\mathcal{B}_r$ and that also within $\mathcal{B}_r$ the following conditions hold*

- $\dot{V}(x) \leq 0$
- *$V(x) > 0$ for $x_s \neq 0$ and $V(x) = 0$ for $x_s = 0$,*
- *for every trajectory of x starting from $x(0)$ within $\mathcal{B}_r$, there is a bound for $x(t)$ (which may possibly depend on $x(0)$).*

Let $\mathcal{N}$ be the set of all points within $\mathcal{B}_r$ where $\dot{V}(x) \equiv 0$ and let $\mathcal{M}$ be the largest invariant set within $\mathcal{N}$. Then for every possible $x(0)$ in $\mathcal{B}_r$, as $t \to \infty$, $x(t) \to \mathcal{M}$ and consequently every associated solution $x_s = \Pi(x)$ tends to $\mathcal{M}_s = \Pi(\mathcal{M})$.

Theorem 7 *Consider the system defined by equation (7), and suppose that $\exists \alpha, \beta : \alpha I \geq E_1 = D_{12}^T D_{12} \geq \beta I > 0$ for all x. Suppose also that $u = K(x)$ for some K such that $K(0) = 0$. Suppose that the state vector of P is of the form $[x_w^T\ x_s^T]^T$, in which the components x_s are stabilisable from u and the components x_w are associated only with weights and are not necessarily stabilisable. Then the closed-loop system (P, K) will be comprehensively stable, given the following conditions are satisfied:*

- *There exists a storage function V, such that $V(x) > 0$ if $x_s \neq 0$ and $V(x) = 0$ if $x_s = 0$, which satisfies the dissipative inequality:*

$$\dot{V}(x) \leq \frac{1}{2}[\gamma^2 ||\hat{w}||^2 - ||z||^2]. \tag{9}$$

- *The states x_s are zero-detectable.*

Next, let us go back to the nonlinear disturbance suppression problem. We present a theorem which gives a sufficient condition for system (6) to be comprehensively stabilised. This condition is relatively moderate, and easier to check than that in Theorem 1.

Theorem 8 *Consider the system described by equation (6). Suppose that the state vector of the plant P is of the form $[x_{w_{11}}\ x_s^T]^T = [x_{w_{11}}\ x_{w12}^T\ x_{w_2}^T\ x_0^T]^T$, where the sub-state x_s is zero-detectable. If there exists a function $\bar{V}(x)$, such that $\bar{V}(x) > 0$ if $x_s \neq 0$ and $\bar{V}(x) = 0$ if $x_s = 0$, which satisfies the following HJ PDE:*

$$\nabla_{x_s}\bar{V}\bar{A} + \frac{1}{2}\nabla_{x_s}\bar{V}\left(\gamma^{-2}\bar{B}_1\bar{B}_1^T - \bar{B}_2\bar{E}_1^{-1}\bar{B}_2^T\right)\nabla_{x_s}\bar{V}^T = 0, \tag{10}$$

then the system (6) can be comprehensively stabilised by the central controller $K^\star(x_s)$, defined as:

$$K^\star(x_s) = -\bar{E}_1(x_s)^{-1}[\bar{D}_{12}(x_s)^T C_1(x) + \bar{B}_2(x_s)^T \nabla_{x_s}\bar{V}(x)^T] \tag{11}$$

In the above equations the terms are given by

$$\bar{A} = \bar{A}(x_s) = \begin{bmatrix} A_{w_{12}} x_{w_{12}} \\ A_{w_2} x_{w_2} \\ A(x_0) + B_1(x_0) C_{w_{12}} x_{w_{12}} \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} B_{w_{12}} & 0 \\ 0 & B_{w_2} \\ 0 & 0 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} 0 \\ 0 \\ B_1(x_0) \end{bmatrix},$$

$C_1(x) = x_{w_{11}}, \bar{D}_{12} = D_{12}, and \bar{E}_1 = \bar{D}_{12}^T \bar{D}_{12}.$

Remarks:

- By applying Theorem 8, we can achieve a $(n + n_{w_{12}} + n_{w_2})$-th order HJ PDE (10) instead of a $(n + n_{w_{12}} + n_{w_2} + 1)$-th order HJ PDE (8) to construct the comprehensively stabilising controller for the disturbance suppression problem.
- It is obvious that $\bar{V} = 0$ is one solution of the HJ PDE (10); since when $\bar{V} = 0$, then $z = 0$, and the γ-dissipativity condition is satisfied. However for such a $\bar{V}$, stability is not necessarily guaranteed. On the other hand, if P_0 is a stable plant, the stability requirement is automatically satisfied with $\bar{V} = 0$. This means that the state feedback controller is just $-x_{w_{11}}$, and therefore output feedback just needs to feed back the estimate of the state $x_{w_{11}}$. This greatly simplifies the design of the output feedback H_∞ controller for this problem, because we only need to observe the state $x_{w_{11}}$.

6 The structure of the disturbance suppression output feedback controller for the nonlinear plants

In this section, we shall discuss the use of output feedback rather than state feedback to achieve constant disturbance suppression. We first show how one might use a nonlinear observer in conjunction with a state-feedback $\mathcal{H}_\infty$ controller in order to develop an output feedback controller. The form of solution suggests that an output feedback controller which rejects constant disturbances may contain an integrator. We aim to confirm that the controller normally acquires an integrator, a phenomenon well known in the linear case, and the basis of classical constant disturbance suppression ideas.

Nonlinear H_∞ output feedback control is particularly difficult. The standard solution of the linear H_∞ output feedback control problem normally depends upon solving two Riccati equations [10]. One of these, which arises in the state-feedback control problem, is replaced by an HJ PDE in the nonlinear case. The other, however, is replaced by a still more complicated equation (involving an information state), see [3]. Practical approaches to solution of this latter equation are so far lacking.

As an alternative, one can draw on ideas of nonlinear observer theory [4]-[5], and substitute a state estimate $\hat{x}$ instead of the state x in a state feedback controller, retrospectively checking the γ-dissipativity and stability of the closed-loop system. In this case, the controller remains finite-dimensional, which is not normally the case when information state methods are used.

In the linear case, the disturbance rejection output feedback controller necessarily includes an integrator unless the plant has a zero at the origin. We now investigate the output feedback controller structure for the nonlinear case. We first define a notion of internal stability.

Definition 9 *A closed loop system is internally stable around all constant operating points if when subjected to inputs composed of the sum of a constant signal plus signal in $\mathcal{L}_2$, all internal states $x(t)$ of the closed loop system become the sums of constant signals plus a signals in $\mathcal{L}_2$. Output stability is defined similarly.*

This definition reduces to the standard notion of stability in the linear case. By analogy with the linear case we shall adopt the following definition.

Definition 10 *A nonlinear system contains an integrator iff there exist some initial conditions for the state, and some input signal in $\mathcal{L}_2$ which results in the output being the sum of a non-zero constant signal plus a signal in $\mathcal{L}_2$.*

Theorem 11 *Consider the constant disturbance suppression problem described by equations (6) and (6*) and depicted in Figure 6. Suppose that an output feedback $\mathcal{H}_\infty$ controller exists, such that the resultant closed loop is both internally stable and output stable around all constant operating points (Definition 9). Then either the plant P_0 or the controller must contain an integrator (Definition 10).*

7 Conclusion

This paper presents a modest extension of nonlinear H_∞ theory in order to solve the constant disturbance rejection problem. We have suggested a nonlinear extension of a concept introduced for the corresponding linear problem, that of the *"comprehensively stabilising"* controller, and have achieved an order reduced HJ PDE for the state feedback problem. Furthermore, we draw the conclusion that the output feedback controller normally must contain an integrator for constant disturbance suppression. This method improves our intuitive understanding of the linear problem.

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