Proceedings of the American Control Conference
Chicago, Illinois • June 2000

Differentially coprime kernel representations and closed-loop identification of nonlinear systems

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Abstract

In this paper, we utilize the differential kernel representation concept to convert a nonlinear closed-loop identification problem into one of open-loop identification utilizing nonlinear version of the Youla parametrization. The main advantage of our approach using kernel representations over fractional descriptions is that we address a larger class of nonlinear systems.

1 Introduction

The development of algorithms for the identification of linear plants operating in a linear closed-loop has been a topic of research in the last decade. There are essentially two reasons that explain the interest in closed-loop identification. The first one is that, most often, the data are collected in closed-loop, i.e. running the plant in open-loop is not possible (unstable plant) or would disturb operating conditions. The second reason is that, as shown in [1, 2], models identified using closed-loop techniques capture the essential dynamical characteristics that are important for control. We refer to [1, 2] for a sample of the available linear closed-loop techniques.

Nonlinear identification has essentially been tackled in the literature utilizing the ability to parametrize the unknown plant in terms of a (known) nominal model and the (known) nonlinear controller along with a so called Youla-Kucera parameter associated with the plant. The main advantage of this method is that rather than identifying the plant one identifies the Youla-Kucera parameter; in the linear case this results (and in the nonlinear case this may result) in the closed-loop identification problem being transformed into one that is open-loop in nature. This problem was initially investigated in [3] and further investigated in a nonlinear framework in [4, 5] based on the left coprime factorizations as developed in e.g. [6]. In [4], using coprime factorizations of the plant and controller, the identification of nonlinear time-varying plants operating under linear, possibly time-varying feedback was investigated. The authors show that there are left and right coprime based fractional descriptions for the set of all plants stabilized by the linear controller given that the nominal plant model is linear. They use models of the plant that can be based either on the left or right coprime factors of the nominal plant and controller. The authors of [5] extend the theory to enable one to find a left coprime factorization based description of the set of all plants stabilized by a given controller, given a nominal plant model and a controller that are not necessarily linear. A notion of differential coprimeness was introduced to help characterize the model set of the plant.

The objective of this paper is to develop these ideas by replacing the use of left factorizations with kernel representations. Results on the parametrization of stabilizing controllers (or its dual) based on kernel representations are available in [7, 8, 9, 10]. However, the parametrization based on kernel representations adopts a different definition of the stability of feedback systems than the usual one used and we cannot use it directly to derive a closed-loop identification procedure. In [11, 12], to overcome such a problem we have employed special kernel representations, i.e. differentially coprime kernel representations, to derive a parametrization based on the usual sense of stability of feedback systems, and they are therefore ideally suited for a closed-loop identification problem. This paper shows how to utilize the parametrization based on differentially coprime kernel representations to convert a closed-loop identification of nonlinear systems to one of open-loop identification.

2 Preliminary background

Please refer to Section 2 of [11], which will be found in this proceeding, for the definitions and notations used in this paper. See also [12] for more detail.

In order to convert a closed-loop identification problem into a open-loop one, we need a Youla parametrization for nonlinear systems, namely, a

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parametrization of all plants that are stabilized by a given controller. In [7, 9], parametrizations of all plants that are *null* internally stabilized by a given controller were derived. However, these parametrizations are not applicable to closed-loop identification problems, because we cannot check whether the given controller *null* internally stabilizes the plant or not. (Because we cannot inject the external signal z_G to the plant R_G.) In [11, 12] the authors have derived the parametrization of all plants that are internally stabilized by a given controller by utilizing differentially coprime kernel representations.

Theorem 1 [11, 12] *Consider a null internally stable feedback system $\{G, K\}$ with bounded kernel representations $R_G : L_{2e}^{m+p} \to L_{2e}^{p} : w \mapsto z_G$ and $R_K : L_{2e}^{m+p} \to L_{2e}^{m} : w \mapsto z_K$ where $w := (u, y)$. Suppose R_K is uniformly differentially coprime, completely controllable and has a construction*

$$R_K(w) = R_K^{\mathrm{smth}}(w) + R_K^{\mathrm{cnst}} \cdot w \qquad (1)$$

with a smoothing operator R_K^{smth} and a constant matrix R_K^{cnst}. Then the parametrization of all weakly Lipschitz plants which are internally stabilized by K is given by G_S with kernel representation

$$R_{G_S} := R_S \; R_{\{G,K\}} : L_{2e}^{m+p} \to L_{2e}^{p} \qquad (2)$$

with a weakly Lipschitz bounded well-defined kernel representation $R_S : L_{2e}^{m+p} \to L_{2e}^{p}$ of any weakly Lipschitz bounded operator S such that $R_S^{\#}$ is weakly Lipschitz and bounded.

Differential coprimeness plays an important role to connect the different definitions of internal stability. Some related properties of differentially coprime kernel representations are clarified in [11, 12] as well. The parametrization stated in Theorem 1 is also a generalized version of that based on left factorizations given in [5], but the assumptions made here are much weaker than the former result. The configuration of the parametrization is shown in Figure 1. K is a stabilizing controller for G, and S is a bounded free parameter.

We also state a lemma borrowed from [11, 12] which will be used in the next section.

Lemma 1 [11, 12] *Consider two weakly Lipschitz operators $\Sigma : L_{2e}^{m} \times L_{2e}^{p} \to L_{2e}^{p}$ and $\Gamma : L_{2e}^{m} \times L_{2e}^{p} \to L_{2e}^{p}$. Suppose Σ is smoothing. Then Γ has a weakly Lipschitz pseudo-inverse operator $\Gamma^{\#}$ such that*

$$z = \Gamma(v, y) \;\; \Leftrightarrow \;\; y = \Gamma^{\#}(v, z) \qquad (3)$$

holds if and only if this also holds for $(\Sigma + \Gamma)$.

3 Closed-loop identification of nonlinear systems

This section demonstrates how the parametrization in Theorem 1 is utilized for closed-loop identification.

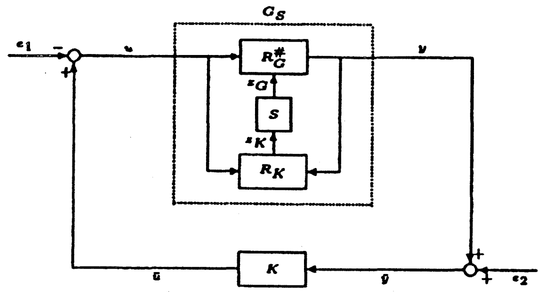

Figure 1: Configuration of the parametrization $\{G_S, K\}$

The configuration as shown in Figure 2 is considered. G is the plant to be identified and we use the (shorthand) notations $r_{12} := (r_1, r_2)$ and r_3 for the known reference signals, $d_{12} := (d_1, d_2)$ for the unknown disturbance signals and $w := (u, y)$ for the measurable loop signals. G_0 is a nominal model and K is a stabilizing controller for both G and G_0 with R_{G_0} and R_K their kernel representations. Conventionally, r_3 would be taken to be zero, and $R_K^{\#}$ would be replaced by K. It suits us to consider the more general arrangement.

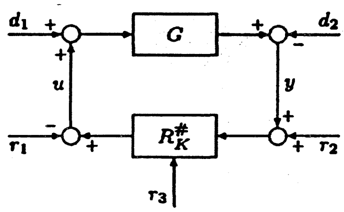

Figure 2: Configuration of closed-loop identification

We make some assumptions pertinent to the application of Theorem 1.

Assumptions

(A1) R_K is completely controllable.
(A2) R_K is differentially coprime.
(A3) R_K has a construction as in (1).
(A4) The feedback system $\{G_0, K\}$ with the kernel representations R_{G_0} and R_K is null internally stable.
(A5) The feedback system $\{G, K\}$ is internally stable.

We assume different internal stabilities of the feedback system with the nominal plant in (A4) and the feedback system with the actual plant to be identified in (A5). This is because *null* internal stability in (A4) is necessary to utilize the parametrization in Theorem 1, whereas internal stability in (A5) is suitable for identification.

From Lemma 1 and Lemma 6 in [11] we can use another set of assumptions instead of (A1)-(A4).

(A1') K is completely controllable.
(A2') K is globally Lipschitz.
(A3') K has a construction as in (1).
(A4') The feedback system $\{G_0, K\}$ with the kernel representations R_{G_0} and $R_K = (-K, \mathrm{Id})$ is null internally stable.

Suppose the assumptions (A1)-(A5) (or (A1')-(A4'),(A5)) are satisfied. Then G can be depicted as Figure 3 with a bounded weakly Lipschitz operator S by Theorem 1, where R_K and $R_K^{\#}$ have the same initial conditions. The objective of this section is to identify the operator S using the reference input signal r_1, r_2 and r_3 and the measurable loop signal w. It is shown in [11, 12] that the mapping $(r_1, r_2, r_3) \mapsto (u, y)$ is bounded and we can use all signals r_1, r_2 and r_3 as reference signals for identification.

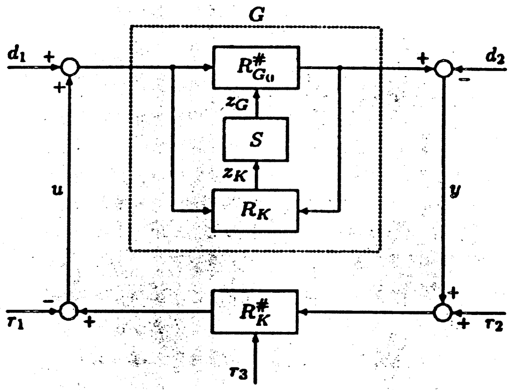

Figure 3: Identification of the Youla parameter S

3.1 Noise free case

This subsection considers the case where the noise $d_{12} = (d_1, d_2) = 0$. It can be easily seen that the input and output signals z_K and z_G of the operator S can be calculated from the measurable loop signal w as follows.

$$
\begin{aligned}
z_G &= R_{G_0}(w) && (4)\\
z_K &= R_K(w) = r_3 - \partial R_{K(w)}(r_{12}) && (5)
\end{aligned}
$$

If we can choose the open-loop input z_K arbitrarily by choosing appropriate r_1, r_2 and r_3, the objective of this subsection will be achieved.

First, consider the case that we can choose r_3 arbitrarily. Then it is obvious that we can choose an arbitrary $z_K \in L_{2e}$ by choosing an appropriate $r_3 \in L_{2e}$ for any fixed $r_1, r_2 \in L_{2e}$, and furthermore we can choose an arbitrary $z_K \in L_2$ by choosing an appropriate $r_3 \in L_2$ for any fixed $r_1, r_2 \in L_2$. The easiest way of choosing a reference signal is to set $r_{12} = 0$ and to set r_3 as the desired open-loop input z_K.

Next, consider the case that we cannot choose r_3 arbitrarily, e.g. a fixed controller is implemented and we cannot inject any nonzero external signal r_3 into the operator K. This situation is the typical setting used in the preliminary results, e.g. [4, 5].

In the linear case, equation (5) reduces to

$$
z_K = r_3 - \partial R_{K(w)}(r_{12}) = r_3 - R_K(r_{12}) = r_3 + \tilde{U} r_2 - \tilde{V} r_1 \qquad (6)
$$

by setting $R_K(y, u) = -\tilde{U} y + \tilde{V} u$ with a left coprime factorization $K = \tilde{V}^{-1}\tilde{U}$. Hence we can choose an arbitrary open-loop input $z_K \in L_{2e}$ by choosing an appropriate $r_1 \in L_{2e}$ when $r_2, r_3 \in L_{2e}$ is fixed. Furthermore the coprimeness of R_K implies the existence of a bounded operator X such that the Bezout identity

$$
R_K X = \mathrm{Id} \qquad (7)
$$

holds. The Bezout identity implies if we choose the reference signal $r_{12} = X(-\alpha + r_3)$ then we can make $z_K = \alpha$ from (6), i.e. we can choose an arbitrary open-loop input $z_K \in L_2$ by choosing appropriate $r_1, r_2 \in L_2$ for any fixed $r_3 \in L_2$.

In our setting, it seems that we can choose an arbitrary $z_K \in L_{2e}$ by choosing an appropriate $r_1 \in L_{2e}$, since $\partial R_{K(w)}$ is pseudo-invertible. But this is not true in general because the loop signal w depends on the reference signals r_1, r_2 and r_3. We need another lemma to prove such a relation.

Lemma 2 *Consider the closed-loop identification as in Figure 3 with $d_{12} = 0$, and suppose the assumptions (A1)-(A5) (or (A1')-(A4'),(A5)) hold. Then,*
(i) we can choose an arbitrary open-loop input $z_K \in L_{2e}$ by choosing an appropriate $r_1 \in L_{2e}$ when $r_2, r_3 \in L_{2e}$ are prescribed,
(ii) we can choose an arbitrary open-loop input $z_K \in L_2$ by choosing appropriate $r_1, r_2 \in L_2$ when $r_3 \in L_2$ is prescribed.

Proof. (i) is proved first. Suppose R_K has a construction as in (1). Then the mapping of $y \mapsto u$ under $z_K = 0$ has to be causal from the definition of kernel representations, and this property implies that R_K^{cnst} has a pseudo-inverse by using Lemma 1. Then again from Lemma 1, R_K has a weakly Lipschitz pseudo-inverse.

On the other hand, the equation (5) reduces to

$$
\begin{aligned}
r_3 - z_K &= \partial R_{K(y,u)}(r_2, r_1)\\
&= \partial R^{\mathrm{smth}}_{K(y,u)}(r_2, r_1) + R_K^{\mathrm{cnst}} \cdot (r_2, r_1)\\
&= \underbrace{\partial R^{\mathrm{smth}}_{K(E_{\{G_S,K\}}(r_{12}))}(r_2, r_1)}_{\text{smoothing}} + \underbrace{R_K^{\mathrm{cnst}} \cdot (r_2, r_1)}_{\text{non-smoothing}}.
\end{aligned} \qquad (8)
$$

Thus the operator in (8) has the construction as in (1). It follows from Lemma 1 that this operator also has a weakly Lipschitz pseudo-inverse, i.e. there exists a weakly Lipschitz operator

$$r_1 = \left(\partial R_{K(E_{\{G_S,K\}}(r_{12}))}\right)^{\#}(r_2, r_3 - z_K). \tag{9}$$

Hence we can choose an arbitrary open-loop input $z_K \in L_{2e}$ by choosing $r_1 \in L_{2e}$ as (9).

(ii) can be proved in a similar way as in the proof of Theorem 6 in [12]. Set the reference signal as

$$r_{12} = X_{(w)}(r_3 - v). \tag{10}$$

Then the mapping of $v \mapsto w$ is weakly Lipschitz and we obtain

$$z_K = v \tag{11}$$

which holds for all $v, z_K \in L_{2e}$. (See [12] for more detail.) Therefore by (10), we can choose an arbitrary open-loop input signal $z_K \in L_2$ by choosing $r_1, r_2 \in L_2$. This completes the proof. □

Lemma 2 describes important properties for identification: we can choose an arbitrary open-loop input signal z_K even when r_3 cannot be used for a reference signal. Of course, when r_1 and r_3 are prescribed, we cannot choose an arbitrary z_K by choosing only r_2 because, as in the linear case, the operator $\tilde{U}$ in (6) is not invertible generally.

3.2 Incorporation of noise

In this subsection, the case of unknown noise $d_{12} = (d_1, d_2) \neq 0$ is discussed. Similar equations as in the noise free case hold.

$$\begin{aligned} z_G &= R_{G_0}(w + d_{12}) = R_{G_0}(w) + \partial R_{G_0(w)}(d_{12}) \\ z_K &= R_K(w + d_{12}) = R_K(w) + \partial R_{K(w)}(d_{12}) \\ &= r_3 - \partial R_{K(w)}(r_{12}) + \partial R_{K(w)}(d_{12}) \end{aligned}$$

Define the known signals $\bar{z}_K := R_K(w)$ and $\bar{z}_G := R_{G_0}(w)$, we can rewrite the configuration of identification as in Figure 3 into an equivalent configuration as shown in Figure 4. The signals z_K and z_G are input and output signals of the operator S but they are not known because of the unknown noise d_{12}. Similar to the arguments in the noise free case in Lemma 2, we can obtain the following theorem.

Theorem 2 *Consider the closed-loop identification as in Figure 3, and suppose the assumptions (A1)-(A5) (or (A1')-(A4'),(A5)) hold. Then,*
(i) we can choose an arbitrary open-loop input $\bar{z}_K \in L_{2e}$ by choosing an appropriate $r_1 \in L_{2e}$ (using the knowledge of r_2, r_3, u and y) when $r_2, r_3, d_1, d_2 \in L_{2e}$ are prescribed,
(ii) we can choose an arbitrary open-loop input $\bar{z}_K \in L_2$ by choosing appropriate $r_1, r_2 \in L_2$ (using the knowledge of r_3, u and y) when $r_3, d_1, d_2 \in L_2$ are prescribed.

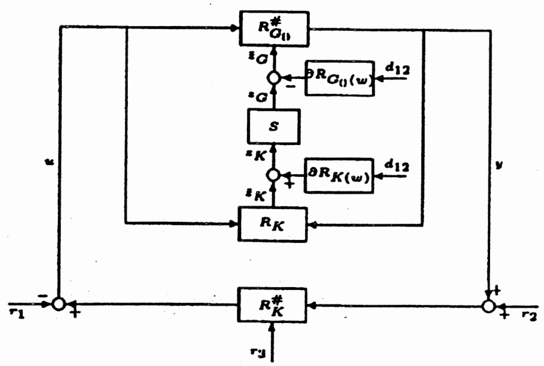

Figure 4: Conversion to nonstandard open-loop identification problem

Proof. (i) is straightforward from (i) in Lemma 2. Choosing the reference signal r_{12} as in (10), (ii) is also straightforward from (ii) in Lemma 2 provided the mapping of $(v, d_{12}) \mapsto w$ is weakly Lipschitz. Hence we only show this fact here. From Theorem 6 (and its proof) in [12], the weakly Lipschitz plant G can always be expressed by the parametrized from G_{0S} with a weakly Lipschitz kernel representation $R_{G_{0S}}$. Then the signal z_{G_0SK} can be calculated as

$$\begin{aligned} z_{G_0SK} &= \begin{pmatrix} R_K^{\rm smth}(w + X_{(w)}(v)) + R_K^{\rm cnst}w + R_K^{\rm cnst}X_{(w)}(v) \\ R_{G_{0S}}(w + d_{12}) \end{pmatrix} \\ &= \begin{pmatrix} R_K^{\rm smth}(w + X_{(w)}(v)) + R_K^{\rm cnst}w + (v - \partial R_{K(w)}^{\rm smth}X_{(w)}(v)) \\ R_{G_{0S}}(w + d_{12}) \end{pmatrix} \\ &= \underbrace{\begin{pmatrix} R_K^{\rm smth}(w + X_{(w)}(v)) - \partial R_{K(w)}^{\rm smth}X_{(w)}(v) \\ 0 \end{pmatrix}}_{\text{smoothing}} \\ &\quad \underbrace{+ \begin{pmatrix} R_K^{\rm cnst} \cdot w \\ R_{G_{0S}}(w + d_{12}) \end{pmatrix} + \begin{pmatrix} v \\ 0 \end{pmatrix}}_{\text{non-smoothing}}. \end{aligned}$$

Hence the mapping $(v, d_{12}) \mapsto w$ is weakly Lipschitz under $z_{G_0SK} = 0$ because of the strong well-posedness of $\{G_{0S}, K\}$ with $R_{\{G_{0S},K\}}$. Therefore, if we choose the reference signal as in (10), we can choose an arbitrary open-loop input signal $\bar{z}_K \in L_2$ by

$$\bar{z}_K = v \tag{12}$$

This completes the proof. □

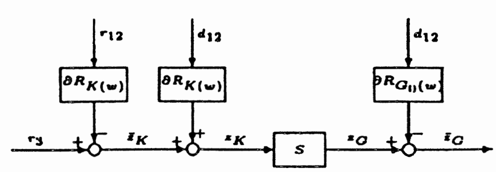

Figure 5: Non-standard open-loop identification problem

Theorem 2 implies that even if the disturbances d_1 and d_2 are not zero, we can choose the open-loop input signal $\bar{z}_K$ arbitrarily. We now obtain the open-loop identification problem as shown in Figure 5. However this configuration is not a standard open-loop identification problem because both the input and output signals are contaminated by noise and the unknown additive signals have correlation with the reference signal $\bar{z}_K$. To overcome such problems, we assume a high SNR (signal-to-noise ratio) as in [4, 5]. More precisely, we make the following assumptions.

Assumptions
(A6) $\|d_{12}\| \ll \|w\|$.
(A7) $\|d_{12}\| \ll \frac{1}{\|R_K\|_L}\|\bar{z}_K\|$.

Assumption (A6) allows us to regard the operators $\partial R_{G_0(w)}(\cdot)$ and $\partial R_{K(w)}(\cdot)$ as being modellable by time-varying linear systems when they are acting on d_{12}. These time-varying systems result from the linearization around the trajectory produced by the input w. Furthermore, employing the differential operator of S we obtain

$$\bar{z}_G = S(\bar{z}_K) + \left(\partial S_{(\bar{z}_K)}\, \partial R_{K(w)} - \partial R_{G_0(w)}\right)(d_{12}). \tag{13}$$

The closed-loop identification problem has been transformed into another open-loop identification problem as shown in Figure 6. Assumption (A7) also allows us to regard the operator $\partial S_{(\bar{z}_K)}$ as a time-varying linear system around the trajectory produced by the input $\bar{z}_K$. Therefore we now obtain a standard open-loop identification problem. Here the operator $(\partial S_{(\bar{z}_K)} \partial R_{K(w)} - \partial R_{G_0(w)})$ is a time-varying linear system and the unknown additive signal has no correlation with the reference signal $\bar{z}_K$. Details about the nonlinear identification procedure can be found in [4].

If the reference signal r_3 is available, then the easiest way for closed-loop identification is to set $r_{12} = 0$ and to set r_3 as the desired open-loop input $\bar{z}_K$, provided the assumption (A6) holds. If (A6) fails, then we should enlarge the loop signal w by choosing appropriate $r_{12} \neq 0$.

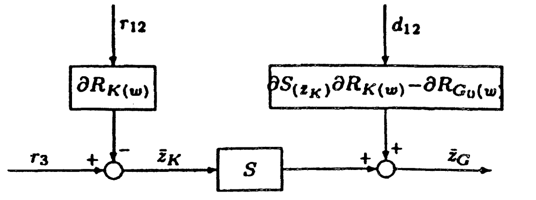

Figure 6: Conversion to standard open-loop identification problem

4 Conclusion

This paper has discussed how to apply the nonlinear Youla parametrization to closed-loop identification of nonlinear systems. We have utilized the differentially coprime kernel representations to derive this procedure. The authors believe that the results in the paper extend the applicability of closed-loop identification to a wider class of nonlinear systems.

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