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Parametrization of all internally stabilizing plant and controller pairs via differentially coprime kernel representations

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Abstract

In this paper, we use the notion of a differentially coprime kernel representation to parametrize the set of all internally stabilizing nonlinear plant and controller pairs using a so called Youla parameter and to unify understanding of some stability concepts for nonlinear systems. The idea of a differential kernel representation allows us to clarify the relationship between three different notions of internal stability available in the literature. Furthermore the parametrization derived here is applicable to nonlinear closed-loop identification problems.

1 Introduction

In this paper, we present a parametrization of all internally stabilizing plant and controller pairs using kernel representations. It is also applicable to the identification of nonlinear systems operating under possibly nonlinear feedback [1, 2]. The parametrization of nonlinear stabilizing controllers were tackled by many researchers and left coprime factorizations approach have derived satisfactory results so far [3]. Furthermore some applications have been obtained based on such parametrizations [4, 5]. Recently kernel representations were introduced as a generalization of left factorizations in [6]. Although there is no essential difference for linear systems, for nonlinear systems it was shown in [7] that state-space realizations of kernel representations are often computable under mild assumptions whereas those of left factorizations are hard or impossible to obtain.

Results on the parametrization of stabilizing controllers (or its dual) based on kernel representations are available in [8, 9, 10, 11]. However, the parametrization based on kernel representations adopts a different definition of the stability of feedback systems than the usual one and we have problems in utilizing it to some applications, e.g. a closed-loop identification [12, 5]. To overcome such problems, we employ a special kernel representation, i.e. we assume that 1. the direct feedthrough part is a constant, and that 2. it has a differential coprimeness property. The property 1 is also employed in [10, 5] to render the parametrized feedback system well-posed. As for the differential coprimeness assumption in 2, it has been introduced in [5] in the context of coprime factorizations and amounts usually to requiring not just boundedness of the closed-loop operator but Lipschitz continuity as well. Kernel representations which obey these two assumptions also connect the stability of feedback systems as employed in the kernel representation approach to the one used in the usual sense. All proofs are omitted for the reason of space. See [10] for the proofs and more detail.

2 Preliminary background

This section gives definitions and preliminary results borrowed from [5, 8, 10, 13].

2.1 Signal spaces

τ_T is the truncation operator on the vector space of functions mapping from $\mathbb{R}$ to $\mathbb{R}^m$. It is defined by

$$\tau_T u(t) := \begin{cases} u(t) & t \le T \\ 0 & t > T. \end{cases}$$

$L_2^m[0,\infty)$ denotes the vector space of $\mathbb{R}^m$ valued square integrable functions with norm defined by $\|u\|^2 := \int_0^\infty |u(t)|^2 dt$. L_2^m and L_2 are used as shorthand for $L_2^m[0,\infty)$.

$L_{2e}^m[0,\infty)$ denotes the vector space of functions u satisfying $\tau_T u(t) \in L_2^m[0,\infty)$ for all $T > 0$. L_{2e}^m and L_{2e} are used as shorthand for $L_{2e}^m[0,\infty)$.

2.2 Operator stability

$\Sigma^{x^\circ} : L_{2e}^m \to L_{2e}^p$ denotes an operator with an initial state $x(0) = x^\circ \in \mathcal{X}^\circ \subset \mathbb{R}^n$ which is a mapping from L_{2e}^m to L_{2e}^p, where $\mathcal{X}^\circ \ni 0$ is a connected subset of $\mathbb{R}^n$. Σ is used as shorthand for Σ^{x°.

For such operators, the following properties are defined. To state the Lipschitz continuity, we employ the differential operator $\partial(\cdot)$ borrowed from [5].

$$\partial\Sigma_{(u)}^{x^\circ}(v) := \Sigma^{x^\circ}(u+v) - \Sigma^{x^\circ}(u) \qquad (1)$$

The operator acts on v, and is parametrized by u; in case Σ^{x° is a linear operator, the differential operator is independent of u and identical with Σ^{x°.

Definition Consider an operator $\Sigma^{x^\circ}: L_{2e}^m \to L_{2e}^p$.
• The operator is said to be *causal* if $\Sigma^{x^\circ}(u) \in L_{2e}^p$ is uniquely determined for $\forall u \in L_{2e}^m$ and $\forall x^\circ \in \mathcal{X}^\circ$, and $\tau_T \Sigma^{x^\circ} \tau_T = \tau_T \Sigma^{x^\circ}$ holds for $\forall T > 0$ and $\forall x^\circ \in \mathcal{X}^\circ$.
• The operator is said to be *bounded* if it is causal and there exists a finite constant γ and a scalar function ϕ satisfying $\phi(0) = 0$ such that the following inequality holds for $\forall u \in L_{2e}^m$ and $\forall x^\circ \in \mathcal{X}^\circ$,

$$\|\Sigma^{x^\circ}(u)\| \le \gamma \|u\| + \phi(x^\circ) \tag{2}$$

• The operator is said to be *unit* if it is causally invertible, and both Σ^{x° and $(\Sigma^{-1})^{x^\circ}$ are bounded.
• The operator is said to be *weakly Lipschitz* (or *weakly Lipschitz continuous*) if it is causal and its Lipschitz semi-norm $\|\tau_T \Sigma^{x^\circ}\|_L$ is finite for every $T > 0$ and $x^\circ \in \mathcal{X}^\circ$, where the Lipschitz semi-norm $\|\cdot\|_L$ is defined as follows.

$$\|\tau_T \Sigma^{x^\circ}\|_L := \sup_{u,v \in L_{2e}^m,\ \tau_T v \ne 0} \frac{\|\tau_T \partial \Sigma_{(u)}^{x^\circ}(v)\|}{\|\tau_T v\|} \tag{3}$$

• The operator $\Sigma^{x^\circ}: L_{2e}^m \to L_{2e}^p$ is said to be *smoothing* if it is weakly Lipschitz and for every $T > 0, \gamma > 0$ and $x^\circ \in \mathcal{X}^\circ$ there exists $t_1 = t_1(T, \gamma, x^\circ) \in (0, T)$ such that

$$\|\tau_{t+t_1}(\Sigma^{x^\circ} \tau_{t+t_1} - \Sigma^{x^\circ} \tau_t)\|_L \le \gamma \tag{4}$$

holds for $\forall t \in [0, T - t_1]$.
• The operator is said to be *globally Lipschitz* (or *globally Lipschitz continuous*) if there exists a finite constant γ such that the following inequality holds for $\forall u, v \in L_2^m$ and $\forall x^\circ \in \mathcal{X}^\circ$.

$$\|\partial \Sigma_{(u)}^{x^\circ}(v)\| \le \gamma \|v\| \tag{5}$$

The smoothing concept is a powerful tool for establishing the well-posedness property of interconnected systems [13] and it will be used to prove the well-posedness of feedback systems.

In this paper, it is also assumed that any operator has the state-space realization

$$\Sigma^{x^\circ}: \left\{ \begin{array}{rcl} \dot{x} &=& f(x,u) \quad x(0) = x^\circ \in \mathcal{X}^\circ \\ y &=& h(x,u) \end{array} \right. \tag{6}$$

where f and h are smooth functions with $f(0,0) = 0$ and $h(0,0) = 0$ (consequently $\Sigma^0(0) = 0$ holds). Systems that have the state-space realizations as given in (6) are causal.

We now define the complete controllability concept which is an intrinsic property of state-space realizations of operators. A causal operator $\Sigma^{x^\circ}: L_{2e}^m \to L_{2e}^p$ with the state-space realization as in (6) is said to be *completely controllable* if for $\forall x^1, x^2 \in \mathcal{X}^\circ$ there exist $T > 0$ and an input $u \in L_2^m[0, T]$ such that the corresponding trajectory $x(t)$ $(0 \le t \le T)$ of the system (6) satisfies $x(0) = x^1$ and $x(T) = x^2$.

2.3 Kernel representations

This subsection introduces kernel representations [8] as generalization of left factorizations. A *kernel representation* of a causal operator $\Sigma^{x^\circ}: L_{2e}^m \to L_{2e}^p$ is a causal operator $R_\Sigma^{x^\circ}: L_{2e}^m \times L_{2e}^p \to L_{2e}^p$ such that

$$y = \Sigma^{x^\circ}(u) \iff R_\Sigma^{x^\circ}(u, y) = 0 \tag{7}$$

holds for $\forall x^\circ \in \mathcal{X}^\circ$ and $\forall u \in L_{2e}^m$ and $y \in L_{2e}^p$.

A kernel representation $R_\Sigma^{x^\circ}: L_{2e}^m \times L_{2e}^p \to L_{2e}^p$ is said to be *well-defined* if there exists the causal pseudo-inverse operator $(R_\Sigma^{x^\circ})^\#: L_{2e}^m \times L_{2e}^p \to L_{2e}^p$ such that

$$y = (R_\Sigma^{x^\circ})^\#(u, z_\Sigma) \iff R_\Sigma^{x^\circ}(u, y) = z_\Sigma$$

holds for $\forall x^\circ \in \mathcal{X}^\circ$ and $\forall u \in L_{2e}^m$ and $y, z_\Sigma \in L_{2e}^p$.

Kernel representations are natural generalizations of left factorizations, because if an operator $\Sigma^{x^\circ}: L_{2e}^m \to L_{2e}^p$ has a left factorization $\Sigma = \tilde{M}^{-1}\tilde{N}$ with $\tilde{N}^{x^\circ}: L_{2e}^m \to L_{2e}^p$ and $\tilde{M}^{x^\circ}: L_{2e}^p \to L_{2e}^p$ then a well-defined kernel representation of Σ is given by $R_\Sigma^{x^\circ}(u, y) = -\tilde{N}^{x^\circ}(u) + \tilde{M}^{x^\circ}(y)$.

Kernel representations are not equivalent to left factorizations because in general any given kernel representation R_Σ is not "separable", namely, it cannot be divided into two operators $\tilde{N}$ and $\tilde{M}$ as above.

Definition A bounded kernel representation $R_\Sigma^{x^\circ}: L_{2e}^{m+p} \to L_{2e}^p$ is said to be *coprime* if there exists a bounded operator $X^{x^\circ}: L_{2e}^p \to L_{2e}^{m+p}$ such that

$$R_\Sigma^{x^\circ} X^{x^\circ} = \mathrm{Id} \tag{8}$$

holds for $\forall x^\circ \in \mathcal{X}^\circ$.

Equation (8) reduces to $-\tilde{N}X_1 + \tilde{M}X_2 = \mathrm{Id}$ when R_Σ specializes to the left factorization. Therefore equation (8) is a natural generalization of the Bezout identity in linear systems theory.

Definition A globally Lipschitz kernel representation $R_\Sigma^{x^\circ}: L_{2e}^{m+p} \to L_{2e}^p$ is said to be *uniformly differentially coprime* if there exists a set of bounded operators $X_{(w)}^{x^\circ}: L_{2e}^p \to L_{2e}^{m+p}$ which are parametrized by the signal w and have a finite gain uniformly over $w \in L_{2e}^{m+p}$ such that

$$\partial R_{\Sigma(w)}^{x^\circ} X_{(w)}^{x^\circ} = \mathrm{Id} \tag{9}$$

holds for $\forall w \in L_{2e}^{m+p}$. Here $X_{(w)}(v)$ is causally dependent on w and v.

If a kernel representation R_Σ is differentially coprime, i.e. there exists a set of bounded operators $X_{(w)}$ such that (9) holds, then the Bezout identity also holds by setting $w = 0$, i.e. R_Σ is coprime in the usual sense. The definition of differential coprimeness is motivated by the fact that it is implied by a small signal closed-loop stability which is necessary for closed-loop identification [1, 2].

Remark 1 A trivial sufficient condition for differential coprimeness of R_Σ is the global Lipschitz continuity of Σ. Consider a globally Lipschitz operator $\Sigma : L_{2e}^m \to L_{2e}^p$ $(u \mapsto y)$. Then its trivial bounded kernel representation given by

$$R_\Sigma(u, y) = -\Sigma(u) + y. \qquad (10)$$

This is differentially coprime with $X = (0, \mathrm{Id})$ and $\partial R_{\Sigma(u,y)}(\tilde{u}, \tilde{y}) = -\partial\Sigma_{(u)}(\tilde{u}) + \tilde{y}$.

2.4 Internal stability

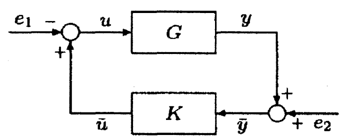

Figure 1: The feedback system $\{G, K\}$ with additive disturbances

We now consider the feedback system depicted in Figure 1. Such a feedback system that interconnects $G^{x_G^\circ} : L_{2e}^m \to L_{2e}^p$ and $K^{x_G^\circ} : L_{2e}^p \to L_{2e}^m$ is denoted by $\{G^{x_G^\circ}, K^{x_K^\circ}\}$ or just $\{G, K\}$. We use the following condensed notations if no confusion arises: $w := (u, y)$, $z_{GK} := (z_K, z_G)$ and $e_{12} := (e_1, e_2)$.

The stability of the feedback system $\{G, K\}$ with additive disturbances as in Figure 1 is considered. Such a configuration is often treated in the literature on right coprime factorizations. Let us define a new operator $E_{\{G,K\}}^{(x_G^\circ, x_K^\circ)} : L_{2e}^{m+p} \to L_{2e}^{m+p}$ which is a mapping from the external additive signal (e_1, e_2) to (u, y) in Figure 1.

Definition A feedback system $\{G, K\}$ is said to be *well-posed* if the operator $E_{\{G,K\}}^{(x_G^\circ, x_K^\circ)}$ exists and is weakly Lipschitz.

If one of G or K is smoothing and the other is weakly Lipschitz, then the feedback system $\{G, K\}$ is well-posed [13].

Definition A well-posed feedback system $\{G, K\}$ is said to be *internally stable* if $E_{\{G,K\}}^{(x_G^\circ, x_K^\circ)}$ is bounded.

2.5 Null and strong internal stability

We state two other stability concepts of feedback systems based on kernel representations in this subsection. Section 3 will connect these concepts. The stability of feedback systems as shown in Figure 2 is discussed here, where $R_G : (u, y) \mapsto z_G$ and $R_K : (y, u) \mapsto z_K$ denote the kernel representations of the components G and K respectively.

By employing R_G and R_K a kernel representation of the operator $E_{\{G,K\}}$ can be defined by

$$R_{E_{\{G,K\}}}^{(x_G^\circ, x_K^\circ)}(e_{12}, w) := \begin{pmatrix} R_K^{x_K^\circ}(w + e_{12}) \\ R_G^{x_G^\circ}(w) \end{pmatrix} = z_{GK}.$$

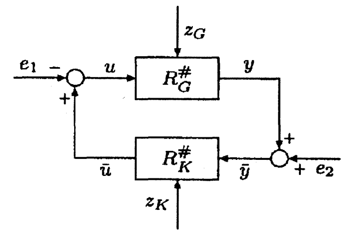

Figure 2: Null and strong internal stability of $\{G, K\}$

It is easy to see that

$$w = E_{\{G,K\}}(e_{12}) \quad \Leftrightarrow \quad R_{E_{\{G,K\}}}(e_{12}, w) = 0$$

which is the definition of the kernel representation (7). Further the kernel representation of the feedback system in the case $e_{12} = 0$ can also be defined by

$$R_{\{G,K\}}^{(x_G^\circ, x_K^\circ)}(w) := R_{E_{\{G,K\}}}^{(x_G^\circ, x_K^\circ)}(0, w) \qquad (11)$$

Then null/strong well-posedness and internal stability defined as follows.

Definition A feedback system $\{G, K\}$ with a weakly Lipschitz kernel representation $R_{\{G,K\}}$ is said to be *null well-posed* if the operator $R_{\{G,K\}}^{-1}$ exists and is weakly Lipschitz. It is said to be *strongly well-posed* if the operator $R_{E_{\{G,K\}}}$ is well-defined and $R_{E_{\{G,K\}}}^\#$ is weakly Lipschitz.

Definition A null well-posed feedback system $\{G, K\}$ with a bounded kernel representation $R_{\{G,K\}}$ is said to be *null internally stable* if $R_{\{G,K\}}^{-1}$ is bounded.

Definition A strongly well-posed feedback system $\{G, K\}$ with a bounded kernel representation $R_{\{G,K\}}$ is said to be *strongly internally stable* if $R_{E_{\{G,K\}}}^\#$ is bounded.

2.6 Parametrization of all strongly internally stabilizing plant and controller pairs

This subsection introduces the concept of strong detectability. Roughly speaking, a kernel representation is strongly detectable if its state-space realization is an asymptotic state-observer of the original system [10, 11].

Definition A kernel representation $R_\Sigma^{x^\circ} : L_{2e}^{m+p} \to L_{2e}^p$ is said to be *strongly detectable* if there exists a finite constant γ and a scalar function ϕ satisfying $\phi(0, 0) = 0$ such that

$$\|R_\Sigma^{x^1}(w + v) - R_\Sigma^{x^2}(w)\| \leq \gamma\|v\| + \phi(x^1, x^2) \quad (12)$$

holds for $\forall w \in L_{2e}^{m+p}$, $\forall v \in L_2^{m+p}$ and $\forall x^1, x^2 \in \mathcal{X}^\circ$.

By assuming the strong detectability of kernel representations, we can derive the parametrization of all plants which are *strongly* (and *null*) internally stabilized by a given controller K. We do not repeat the results here for the reason of space. See [10, 11] for detail. For closed-loop identification, we need the parametrization of all plants which are internally stabilized by a given controller, because we cannot check whether the given controller *strongly* (or *null*) internally stabilizes the plant or not. Therefore the results in [10, 11] cannot be used for closed-loop identification, and we will investigate the parametrization based on internal stability in what follows.

3 Equivalence of internal stabilities

In this section, the relationship between the three different well-posedness and stability definitions for feedback systems are discussed. First we give a basic property about the pseudo-invertibility of weakly Lipschitz operators.

Lemma 1 *Consider two weakly Lipschitz operators* $\Sigma : L_{2e}^m \times L_{2e}^p \to L_{2e}^p$ *and* $\Gamma : L_{2e}^m \times L_{2e}^p \to L_{2e}^p$. *Suppose* Σ *is smoothing. Then* Γ *has a weakly Lipschitz pseudo-inverse operator* $\Gamma^{\#}$ *such that*

$$z = \Gamma(v, y) \quad \Leftrightarrow \quad y = \Gamma^{\#}(v, z) \tag{13}$$

holds if and only if this also holds for $(\Sigma + \Gamma)$. *Furthermore, suppose* Γ *is a constant matrix and* $\Gamma^{\#}$ *exists*[1]. *Then* $(\Sigma + \Gamma)^{\#} - \Gamma^{\#}$ *is smoothing.*

We can now prove the equivalence between null well-posedness and strong well-posedness using the assumption that an operator $\Sigma : u \mapsto y$ has a construction

$$\Sigma(u) = \Sigma^{\text{smth}}(u) + \Sigma^{\text{cnst}} \cdot u. \tag{14}$$

Lemma 2 *Consider a well-posed feedback system* $\{G, K\}$ *with kernel representations* $R_G : L_{2e}^{m+p} \to L_{2e}^p$ *and* $R_K : L_{2e}^{m+p} \to L_{2e}^m$. *Suppose either* R_G *or* R_K *has a construction as in (14). Then the feedback system* $\{G, K\}$ *with the kernel representation* $R_{\{G,K\}}$ *is strongly well-posed if and only if it is null well-posed.*

Furthermore, we can show the equivalence between null internal stability and strong internal stability when one of the kernel representations is globally Lipschitz.

Theorem 1 *Consider a well-posed feedback system* $\{G, K\}$ *with kernel representations* $R_G : L_{2e}^{m+p} \to L_{2e}^p$ *of* G *and* $R_K : L_{2e}^{m+p} \to L_{2e}^m$ *of* K. *Suppose either* R_G *or* R_K *is globally Lipschitz and has a construction as in (14). Then the feedback system* $\{G, K\}$ *with the kernel representation* $R_{\{G,K\}}$ *is strongly internally stable if and only if it is null internally stable.*

[1] If Γ has a construction $z = \Gamma(v, y) = Av + By$ with constant matrices A and B, then $\Gamma^{\#}$ exists iff B is nonsingular and it is represented by $y = \Gamma^{\#}(v, z) = -B^{-1}Av + B^{-1}z$.

If both R_G and R_K have constructions as in (14), then all three well-posedness notions coincide. The equivalence between well-posedness and null well-posedness does not hold in general. We can show the relation between these three well-posedness notions by employing a specific kernel representation.

Lemma 3 *Consider a well-posed feedback system* $\{G, K\}$ *with a kernel representation* $R_K : L_{2e}^{m+p} \to L_{2e}^m$ *of* K *which has a construction as in (14). Then there exists a kernel representation* $R_G : L_{2e}^{m+p} \to L_{2e}^p$ *of* G *such that the system* $\{G, K\}$ *with* $R_{\{G,K\}}$ *is null well-posed.*

Lemma 3 only proves that the null well-posedness of the feedback system with one kernel representation R_G of G. The class of all kernel representations of G such that the feedback system is null well-posed is given by $R_0 R_{\{G,K\}}$ with any well-defined kernel representation R_0 of a zero operator [8] (where $R_{\{G,K\}}$ is invertible). Lemma 2 and Lemma 3 imply that if either of the kernel representations R_G or R_K has a construction as in (14) then the three well-posedness notions are equivalent in some sense. (Indeed null well-posedness and strong well-posedness coincide completely.)

Furthermore, we investigate the property of differential coprimeness of the kernel representation $R_{\{G,K\}}$.

Lemma 4 *Consider an invertible globally Lipschitz kernel representation* $R_\Sigma : L_{2e}^m \to L_{2e}^m$. *Then* R_Σ^{-1} *is globally Lipschitz if and only if* R_Σ *is uniformly differentially coprime.*

Using Lemma 4, we can establish the property of a differential coprime kernel representation $R_{\{G,K\}}$.

Theorem 2 *Consider a feedback system* $\{G, K\}$ *with kernel representations* $R_G : L_{2e}^{m+p} \to L_{2e}^p$ *of* G *and* $R_K : L_{2e}^{m+p} \to L_{2e}^m$ *of* K. *Suppose* $R_{\{G,K\}}$ *is uniformly differentially coprime, and either* R_G *or* R_K *has a construction as in (14). Then the feedback system* $\{G, K\}$ *with the kernel representation* $R_{\{G,K\}}$ *is strongly internally stable.*

Proof. The proof of the theorem is straightforward from Lemma 2, Lemma 4 and Theorem 1. □

Uniform differential coprimeness of $R_{\{G,K\}}$ implies strong internal stability of the feedback system $\{G, K\}$ provided either R_G or R_K has a construction as in (14). This property will be used in the parametrization of *all* internally stabilizing plant and controller pairs in the next section.

4 Parametrization of all internally stabilizing plant and controller pairs

This section discusses the parametrization of all internally stabilizing plant and controller pairs, in contrast to the results in [10, 11] which provide the parametrization of all *strongly* (and *null*) internally stabilizing pairs. Before stating the results, we give an important remark on strong detectability. While a strongly detectable kernel representation R_Σ is globally Lipschitz by definition, the converse also holds if it is completely controllable.

Lemma 5 *Consider a kernel representation $R_\Sigma^{x^\circ} : L_{2e}^{m+p} \to L_{2e}^p$. Suppose it is completely controllable. Then the following properties are equivalent.*
(i) $R_\Sigma^{x^\circ}$ is strongly detectable.
(ii) $R_\Sigma^{x^\circ}$ is globally Lipschitz.
(iii) The following inequality holds for $\forall u, v \in L_2^{m+p}$.

$$\|R_\Sigma^0(u+v) - R_\Sigma^0(u)\| \le \gamma \|v\| \qquad (15)$$

Lemma 5 suggests that the global Lipschitz continuity of a kernel representation is a crucial property when it is utilized for the parametrization. The parametrization of *all* internally stabilizing controllers is now given.

Theorem 3 *Consider a null internally stable feedback system $\{G, K\}$ with bounded kernel representations $R_G : L_{2e}^{m+p} \to L_{2e}^p$ and $R_K : L_{2e}^{m+p} \to L_{2e}^m$. Suppose R_K is uniformly differentially coprime with a weakly Lipschitz operator $X_{(w)}$ in (9), completely controllable and has a construction as in (14). Then the parametrization of all weakly Lipschitz plants which are internally stabilized by K is given by G_S defined by*

$$R_{G_S} := R_S \; R_{\{G,K\}} \; : \; L_{2e}^{m+p} \to L_{2e}^p \qquad (16)$$

with any weakly Lipschitz bounded operator S.

Differential coprimeness plays an important role to connect the different definitions of internal stability. The parametrization stated in Theorem 3 is a generalized version of that based on left factorizations given in [5], but the assumptions made here are much weaker than those in the former result.

The complete controllability of the kernel representation is assumed in Theorem 3. A sufficient condition for this property is given in the following obvious lemma.

Lemma 6 *Consider a causal operator $\Sigma^{x^\circ} : L_{2e}^m \to L_{2e}^p$ which has a state-space realization as in (6) with a kernel representation $R_\Sigma^{x^\circ} : L_{2e}^m \times L_{2e}^p \to L_{2e}^p$. Suppose $(R_\Sigma^\#)^{x^\circ}(u, 0)$ has the same state-space realization as that of Σ^{x°. Then R_Σ is completely controllable if Σ is completely controllable.*

Remark 1, Lemma 1 and Lemma 6 suggest that if the controller K is globally Lipschitz and completely controllable then the trivial kernel representation of $K : y \mapsto u$ as $R_K(y, u) = -K(y) + u$ will be a good candidate kernel representation which satisfies the assumptions in Theorem 3, because it is uniformly differentially coprime, completely controllable and has a construction as in (14).

Next we now give the parametrization of *all* internally stabilizing plant and controller pairs which allows both plant and controller vary.

Theorem 4 *Consider a feedback system $\{G, K\}$ with kernel representations $R_G : L_{2e}^{m+p} \to L_{2e}^p$ and $R_K : L_{2e}^{m+p} \to L_{2e}^m$. Suppose $R_{\{G,K\}}$ is uniformly differentially coprime and completely controllable, and it has a construction as in (14). Then the parametrization of all internally stabilizing plant and controller pairs G_S and K_Q is given by*

$$\begin{array}{rcl} R_{K_Q} & := & R_Q \; R_{\{G,K\}} \; : \; L_{2e}^{m+p} \to L_{2e}^m \\ R_{G_S} & := & R_S \; R_{\{G,K\}} \; : \; L_{2e}^{m+p} \to L_{2e}^p \end{array} \qquad (17)$$

with all internally stable feedback systems $\{S, Q\}$ with the kernel representation $R_{\{S,Q\}}$.

Theorems 3 and 4 will be useful in its application to closed-loop identification [1, 2]. Notice that Theorem 3 can be easily made applicable by choosing the stabilizing controller K to satisfy the differential coprimeness assumption.

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