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RECURSIVE ITERATIVE FEEDBACK TUNING [1]

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Abstract

In this paper, we propose a methodology for iterative recursive feedback tuning of a given controller structure. The algorithm and its convergence (stability) properties are analyzed. The algorithm can be viewed as the dual of the recursive closed-loop output-error identification algorithm as studied in [8].

Keywords: adaptive control, estimation, duality

1 Introduction

In the model-based world, the controller is computed on the basis of a model of the process in order to achieve level of performance on the actual system. The emergence of model-based control has been accompanied by the development of system identification, i.e. a modeling methodology for estimating models on the basis of data collected on the actual process. It is now widely accepted in the identification-for-control community that the identified model should reflect the intended use of the model, i.e. control design. In other words, the control performance criterion should dictate what the identification criterion should be. As shown in [4], it very often turns out that the performance criterion calls for closed-loop identification techniques. A recent revival of these techniques has occurred in the context of the several schemes for iterative closed-loop identification and control design; see [4, 10] for further details. One such method, initially proposed as an exercise in [9], and whose properties have been studied intensively recently, is a method based on a "tailor-made parametrization". The method uses knowledge of the controller; it minimizes an error between the closed-loop transfer functions of the true closed-loop and the model closed-loop, using a parametric model of the open-loop model only; see Figure 1.1.

Model Reference Adaptive Control (MRAC), classical PID tuning and fuzzy logic control are only several of the multitude of non-model-based control methodologies. Recently, a new data-driven model-free iterative control design method has been proposed in [5]. That scheme is based on an iterative tuning of the controller parameter vector along the gradient direction of a control performance criterion that minimizes the closed-loop output error as shown in Figure 2.2. The key contribution of [5] was to show that an unbiased estimate of this gradient can be constructed from filtered versions of the signals measured on the closed-loop system.

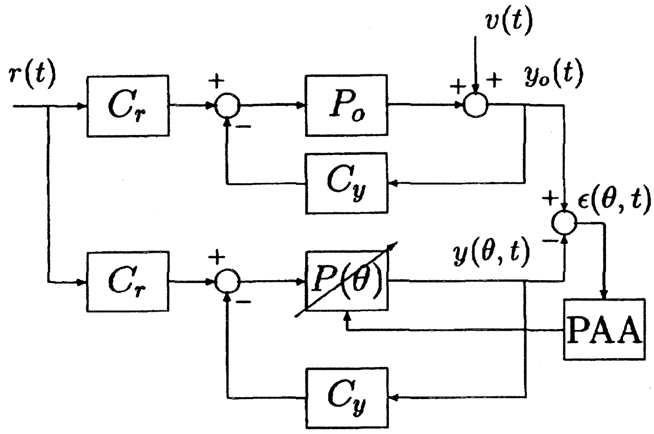

Figure 1.1: closed-loop tailor made or output error identification scheme

As noted in [1] and as can be observed from Figures 1.1 and 2.2, the problem of Iterative Feedback Tuning (IFT) is the dual of the (off-line) closed-loop tailor-made identification problem. In this paper, we study the dual of the (recursive) closed-loop output-error identification problem as studied in [8], i.e. the problem of adaptive or recursive iterative feedback tuning. It turns that this dual problem has close connections with MRAC.

The organization of the paper is as follows. In Section 2, we describe the problem at hand and establish the model reference type algorithm. In Section 3, we provide a stability analysis in the deterministic case. Convergence results in a stochastic environment are given in Section 4 with generalizations in Section 5. In Section 6, we present some numerical simulations. We conclude in Section 7.

2 Recursive iterative feedback tuning

In this section, we establish the equation that will lead to the parameter adaptation algorithm.

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General problem setting

Let us consider a scalar linear time-invariant and discrete-time system described by

$$S : y(t) = P_o\, u(t) + v(t) \qquad (2.1)$$

where P_o is an unknown rational transfer function. Here $u(t)$ is the control input signal, $y(t)$ is the achieved output signal and $v(t)$ is a zero-mean stochastic process with finite moments. The input signal $u(t)$ is determined according to

$$C : u(t) = r(t) - C(\rho)\, y(t) = r(t) - \frac{N(\rho)}{D(\rho)} y(t) \qquad (2.2)$$

where $r(t)$ is an external reference which we assume to be quasi-stationary and uncorrelated with $v(t)$. Also,

$$\begin{aligned} D(\rho) &= 1 + d_1 q^{-1} + \cdots + d_{n_d} q^{-n_d}, & (2.3)\\ &= 1 + q^{-1} D^*(\rho) & (2.4)\\ N(\rho) &= n_1 q^{-1} + \cdots + n_{n_n} q^{-n_n} & (2.5)\\ &= q^{-1} N^*(\rho) & (2.6) \end{aligned}$$

and ρ is a vector containing the adjustable controller parameters, i.e.

$$\rho = [d_1 \ \cdots \ d_{n_d} \ n_1 \ \cdots \ n_{n_n}]. \qquad (2.7)$$

The output of (2.1) in feedback with (2.2) is denoted by $y(\rho, t)$ to emphasize its dependence on the controller parameters; see Figure 2.2.

Let $y_o(t)$ be the desired closed-loop response to the reference signal r: $y_o(t) = T_d\, r(t)$, with T_d some stable reference model which is assumed to be constructable from the stable feedback interconnection of the unknown plant P_o with some linear controller C_o of the form (2.2). *The existence of such controller is only assumed to derive the recursive algorithm and to study its stability. However, the nonexistence of such a controller does not preclude the applicability of our scheme.*

Then the error between the achieved and the desired closed-loop response is given by $\epsilon(\rho, t) = y(\rho, t) - y_o(t)$.

The basic equations

We have the following relations for the **reference loop**

$$\begin{aligned} \bar{u}_o(t) &= C_o\, y_o(t) = \frac{N_o}{D_o} y_o(t) & (2.8)\\ \bar{u}_o(t+1) &= -D_o^*\, \bar{u}_o(t) + N_o^*\, y_o(t) = \rho_o^T \psi(t) & (2.9) \end{aligned}$$

with

$$\begin{aligned} \psi(t) &= [-\bar{u}_o(t) \cdots -\bar{u}_o(t-n_d+1)\ y_o(t) \cdots y_o(t-n_n+1)],\\ \rho_o &= [d_1^o \ \cdots \ d_{n_d}^o \ n_1^o \ \cdots \ n_{n_n}^o]. \end{aligned}$$

Note that in practice, the signals $\bar{u}_o(t)$ and $y_o(t)$ are not accessible and ρ_o is unknown. Similarly, for the **achieved loop** one has

$$\bar{u}(\rho, t+1) = \rho^T \varphi(\rho, t) \qquad (2.10)$$

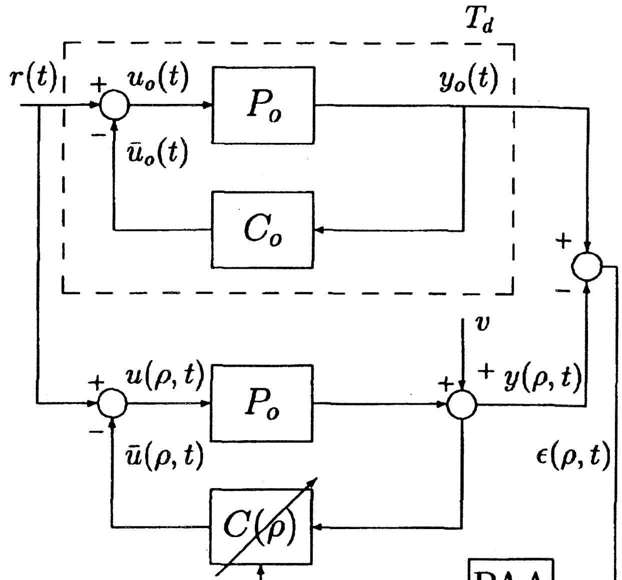

Figure 2.2: Recursive iterative feedback tuning scheme

with

$$\begin{aligned} &\varphi(\rho, t)\\ &= [-\bar{u}(\rho,t) \cdots -\bar{u}(\rho, t-n_d+1)\ y(\rho,t) \cdots y(\rho, t-n_n+1)] \end{aligned}$$

and ρ defined as in (2.7). Let us define the closed-loop input prediction error

$$\bar{\epsilon}(\rho, t) = \bar{u}_o(t) - \bar{u}(\rho, t) = u(\rho, t) - u_o(t). \qquad (2.11)$$

The key observation is that

$$\begin{aligned} \bar{u}_o(t+1) &= \rho_o^T \varphi(\rho, t) + \rho_o^T \psi(t) - \rho_o^T \varphi(\rho, t),\\ &= \rho_o^T \varphi(\rho, t) - D_o^*\,(\bar{u}_o(t) - \bar{u}(\rho, t)),\\ &\qquad + N_o^*\,(y_o(t) - y(\rho, t))\\ &= \rho_o^T \varphi(\rho, t) - (D_o^* + N_o^* P_o)\, \bar{\epsilon}(\rho, t). \qquad (2.12) \end{aligned}$$

Using (2.10), one obtains

$$\bar{\epsilon}(\rho, t+1) = [\rho_o - \rho]^T \varphi(\rho, t) - (D_o^* + N_o^* P_o)\, \bar{\epsilon}(\rho, t)$$

It now follows that

$$\bar{\epsilon}(\rho, t) = \frac{[\rho_o - \rho]^T \varphi(t-1)}{1 + q^{-1}(D_o^* + N_o^* P_o)}. \qquad (2.13)$$

The closed-loop output error is defined as

$$\epsilon(\rho, t) = y(\rho, t) - y_o(t) = P_o\, \bar{\epsilon}(\rho, t) + v(t). \qquad (2.14)$$

It is now straightforward to see that

$$\begin{aligned} \epsilon(\rho, t) &= \frac{P_o\, [\rho_o - \rho]^T \varphi(t-1)}{1 + q^{-1}(D_o^* + N_o^* P_o)} + v(t),\\ &= \frac{T_d}{D_o} [\rho_o - \rho]^T \varphi(t-1) + v(t) \cdot \qquad (2.15) \end{aligned}$$

Equation (2.15) follows from the definitions of N_o^* and D_o^*.

The algorithm
Now replacing the fixed predictor (2.10) by an adjustable predictor, i.e. replacing ρ by $\rho(t)$, one obtains the following a posteriori prediction error in the deterministic case (i.e. $v(t) = 0$),

$$\epsilon(\rho, t) = \frac{T_d}{D_o} [\rho_o - \rho(t)]^T \varphi(t-1). \qquad (2.16)$$

Similarly, the a priori prediction error in the deterministic case is given by

$$\epsilon^o(\rho, t) = \frac{T_d}{D_o} [\rho_o - \rho(t-1)]^T \varphi(t-1). \qquad (2.17)$$

The previous equation has the typical form encountered in discrete-time MRAC and PLR (pseudo-linear regression) recursive identification methods including the output error method for closed-loop identification studied in [8]. Therefore it is reasonable to consider a Parameter Adaptation Algorithm (PAA) of the form used in this case. A general form of such an algorithm and the related stability analysis in connection with an equation of the form (3.9) can be found in [6]. Such a PAA has the form

$$\begin{aligned} \rho(t+1) &= \rho(t) + F(t)\,\varphi(\rho,t)\,\epsilon(\rho,t+1), & (2.18)\\ F^{-1}(t+1) &= \lambda_1(t)\,F^{-1}(t) + \lambda_2(t)\,\varphi(\rho,t)\varphi(\rho,t)^T & (2.19) \end{aligned}$$

where

$$0 < \lambda_1(t) \le 1, \quad 0 \le \lambda_2(t) < 2,$$
$$F(0) > 0, \quad F^{-1}(t) > \alpha F^{-1}(0), \quad 0 < \alpha < \infty.$$

Here, the two sequences $\lambda_1(t)$ and $\lambda_2(t)$ allow one to have different laws of evolution of the adaptation gain.

Remark: One can also set up a less "conventional" modified adaptive algorithm using the following error signal

$$\begin{aligned} \tilde{\epsilon}(\rho,t) &= \gamma_1\,\epsilon(\rho,t) + \gamma_2\,\bar{\epsilon}(\rho,t) & (2.20)\\ &= \frac{T_d}{D_o}(\gamma_1 + \gamma_2 P_o^{-1})\,[\rho_o - \rho(t)]^T \varphi(t-1) & (2.21) \end{aligned}$$

and replacing $\epsilon(\rho,t)$ by $\tilde{\epsilon}(\rho,t)$ in (2.18) and using

$$u_o(t) = S_d\, r(t) \qquad (2.22)$$

where S_d is some stable input reference model. The case $\gamma_1 = 0$ and $\gamma_2 = 1$ corresponds to an input-error type criterion.

3 Stability analysis

We will make the following assumptions

1. The unknown plant P_o is assumed to have (only) stable zeroes.

2. The reference model T_d can be constructed from the stable feedback interconnection of P_o and some stabilizing linear controller C_o.

3. There exists a parameter vector ρ_o such that

$$C_o = C(\rho_o).$$

The results of the stability analysis are presented in the following theorem.

Proposition 3.1 *The recursive parameter estimation algorithm given by (2.18)-(2.19) assures*

$$\lim_{t\to\infty} \epsilon(\rho,t) = 0 \qquad (3.1)$$
$$\lim_{t\to\infty} \epsilon^o(\rho,t) = 0 \qquad (3.2)$$
$$\|\varphi(\rho,t)\| < C, \quad 0 < C < \infty, \quad \forall t \qquad (3.3)$$

for all initial conditions $\rho(0)$, $\epsilon^o(\rho(0),0)$ *and* $\varphi(\rho(0),0)$ *if*

$$\frac{T_d}{D_o} - \frac{\lambda}{2}, \quad \sup_t \lambda_2 \le \lambda < 2, \qquad (3.4)$$

is a strictly positive real transfer function.

Proof: The form of equation (2.16) for the posteriori error, and the equations (2.18) and (2.19) of the PAA, allow one to use the results of [6], and it follows immediately that the condition (3.4) implies (3.1).

It now remains to show that (3.2) and (3.3) hold. To prove (3.2), one has to prove that $\varphi(\rho,t)$ is bounded. However, the components of $\varphi(\rho,t)$ are delayed versions of $y(\rho,t) = y_o(t) + \epsilon(\rho,t)$ and $\bar{u}(\rho,t) = \bar{u}_o(t) - \bar{\epsilon}(\rho,t)$. We have assumed the stability of T_d and therefore $y_o(t)$ and $\bar{u}_o(t)$ are bounded (provided $r(t)$ is bounded). We have already shown that $\epsilon(\rho,t)$ is bounded so it remains to show that $\bar{\epsilon}(\rho,t)$ is bounded. This follows from (2.14) and the fact that P_o has only stable zeroes.

Remark 1: For the modified algorithm obtained using (2.21), it is trivial to see that the positive real condition (3.4) is replaced by

$$D_o^{-1}(\gamma_1\,T_d + \gamma_2\,S_d) - \frac{\lambda}{2} > 0, \quad \sup_t \lambda_2 \le \lambda < 2, \qquad (3.5)$$

being positive real. Notice that the design parameters γ_1 and γ_2 in (2.20) can be used to implement a stable algorithm using (3.5). For γ_1 and γ_2 non-zero, Assumption 1 above has to be replaced by the assumption that

$$\gamma_1\,P_o + \gamma_2 \qquad (3.6)$$

has only stable zeroes. The stability of $\epsilon(\rho,t)$ and $\bar{\epsilon}(\rho,t)$ follows from the stability of $\tilde{\epsilon}(\rho,t)$ in a proof very similar to the one proposed for Proposition 3.1.

Remark 2: One can derive a "filtered" algorithm. Indeed, define

$$\varphi_f(t) = \frac{T_d}{\hat{D}}\,\varphi(t) \qquad (3.7)$$

where $\hat{D} = D(\rho(t))$ is the current estimate of D_o. If $\hat{D}$ contains unstable roots, one has to replace it by the stable spectral factor of $|\hat{D}|^2$ or by its last stable estimate.

Neglecting the non-commutativity of the time-varying operators, (or equivalently, assuming $\rho(t)$ varies slowly enough) (2.15) can be rewritten

$$\begin{aligned} \epsilon(\rho,t) &= \frac{\hat{D}}{D_o}[\rho_o-\rho(t)]^T\frac{T_d}{\hat{D}}\varphi(t-1) \quad (3.8)\\ &= \frac{\hat{D}}{D_o}[\rho_o-\rho(t)]^T\varphi_f(t-1), \quad (3.9)\end{aligned}$$

which allows one to derive a recursive scheme with the filtered observation vector $\varphi_f(t)$. For the filtered algorithm, it is straightforward to see that the positive real condition (3.4) is replaced by

$$\frac{\hat{D}}{D_o}-\frac{\lambda}{2}>0, \quad \sup_t \lambda_2 \le \lambda < 2, \quad (3.10)$$

Clearly in the vicinity of $\rho=\rho_o$, this condition is much more likely to be satisfied than (3.4).

Remark 3: It follows from [5] that the gradients of

$$\begin{aligned} y(\rho,t) &= P_o\,u(\rho,t)+v(t)\\ u(\rho,t) &= r(t)-C(\rho)\,y(\rho,t)\end{aligned}$$

with respect to the j-th parameter ρ_j are given by

$$\begin{aligned} y'_{\rho_j}(\rho,t) &= P_o\,u'_{\rho_j}(\rho,t)\\ u'_{\rho_j}(\rho,t) &= -C(\rho)\,y'_{\rho_j}(\rho,t)-C'_{\rho_j}(\rho)\,y(\rho,t)\end{aligned}$$

for $j=1,\cdots,n_d+n_n$. We have that

$$\begin{aligned} y'_{\rho_j}(\rho,t) &= -\frac{P_o}{1+P_oC(\rho)}C'_{\rho_j}(\rho)\,y(\rho,t)\\ u'_{\rho_j}(\rho,t) &= -\frac{1}{1+P_oC(\rho)}C'_{\rho_j}(\rho)\,y(\rho,t)\end{aligned}$$

for $j=1,\cdots,n_d+n_n$, i.e. the calculation of these gradients involves the unknown plant P_o or more precisely the unknown complementary sensitivity and sensitivity function. In [5], this problem is overcome by using the closed-loop system itself in a recycling experiment. In [2], a model of the closed-loop transfer function obtained using open-loop identification techniques is used for the "recycling" experiment.

In the "filtered" algorithm proposed in this paper,

$$\varphi_f(t)=\frac{T_d}{\hat{D}}\varphi(t)$$

can be viewed as an approximation of the gradient of a quadratic criterion in $\epsilon(\rho,t)$. Indeed, $\varphi(t)$ measured on the actual system can be interpreted as "recycling" (in a non adaptive or batch adaptive context) using an approximation of the complementary sensitivity function, i.e. the reference model T_d.

Remark 4: Conditions (3.4) and (3.5) both involve quantities which (at least until the algorithm has converged) are unknown. Thus one cannot say easily in advance if these (sufficient) conditions for convergence are fulfilled or not.

4 Convergence analysis in a stochastic environment

One of the objectives is to obtain asymptotic unbiased estimates in the presence of noise on the plant output under the assumptions of Section 3. For this analysis, we will use the same ODE approach that has been used in [8] for the dual problem of closed-loop output error identification. The equation for the a posteriori prediction error in the presence of noise is (2.15).

Proposition 4.1 *Consider the PAA (2.18)-(2.19) with* $\lambda_1(t)=1$.

1. *Assume that the stationary processes* $\varphi(\rho,t)$ *and* $\epsilon(\rho,t)$ *can be defined for* $\rho(t)=\rho$ *(i.e.* ρ *is assumed to be constant).*
2. *Assume that* $\rho(t)$ *generated by (2.18)-(2.19) belongs infinitely often to the domain* $\mathcal{D}_S$ *for which the stationary processes* $\varphi(\rho,t)$ *and* $\epsilon(\rho,t)$ *can be defined.*
3. *Define the convergence domain* $\mathcal{D}_C$ *as*

$$\mathcal{D}_C=\{\rho:\varphi^T(\rho,t)[\rho-\rho_o]=0\}\cdot \quad (4.1)$$

Then

$$\text{Prob}\left\{\lim_{t\to\infty}\rho(t)\in\mathcal{D}_C\right\}=1. \quad (4.2)$$

if

$$\frac{T_d}{D_o}-\frac{\lambda}{2}, \quad \sup_t \lambda_2\le\lambda<2, \quad (4.3)$$

is a strictly positive real transfer function. If, furthermore, $\mathcal{D}_C$ *is the singleton* $\{\rho_o\}$ *(richness condition) then (4.3) implies*

$$\text{Prob}\left\{\lim_{t\to\infty}\rho(t)\to\rho_o\right\}=1. \quad (4.4)$$

The proof of the proposition is similar to the proof of Theorem 4.1 in [8] and is therefore omitted.

Remark: For the filtered algorithm, it follows again that (4.3) has to be replaced by (3.10).

5 More general settings

In this section, we consider two degree of freedom controllers and nonlinear controllers.

Two degree of freedom controllers

Suppose (2.2) is replaced by

$$C:u(t)=\frac{1}{D(\rho)}[N_r(\rho)\,r(t)-N_y(\rho)\,y(t)]. \quad (5.1)$$

One can easily show that

$$\epsilon(\rho,t)=\frac{T_d}{N_{ro}}[\rho_o-\rho(t)]^T\varphi(t-1) \quad (5.2)$$

where we suppose (for the sole purpose of establishing the algorithm) that there exists a controller of the form (5.1) with D_o, N_{yo} and N_{ro} such that

$$T_d = \frac{P_o N_{ro}}{S_o + P_o N_{yo}}. \qquad (5.3)$$

Here

$$\rho = [d_1 \ \cdots \ d_{n_d} \ n_{r_1} \ \cdots \ n_{r_{n_r}} \ n_{y_1} \ \cdots \ n_{y_{n_y}}].$$

and

$$\begin{aligned}\varphi(\rho,t) &= [-u(\rho,t) \cdots -u(\rho,t-n_d+1)\; r(\rho,t) \cdots \\ &\quad r(\rho,t-n_r+1) \; -y(\rho,t) \cdots -y(\rho,t-n_y+1)].\end{aligned}$$

Nonlinear plant and/or controller

One can also consider a nonlinear setting as studied in the dual case in [7]. Indeed, suppose the plant is described by

$$S : y(t) = P_o(u(t), v(t)) \qquad (5.4)$$

and that (2.2) is replaced by the stabilizing controller

$$C : u(t) = -C(\rho, y(t), r(t)). \qquad (5.5)$$

Consider the desired input and output signals

$$y_o = T_d(r), \quad u_o = S_d(r), \qquad (5.6)$$

respectively, obtained from output and input reference models that result from the stable feedback interconnection of

$$y_o(t) = P_o(u_o(t), 0) \ \text{ and } \ u_o(t) = -C(\rho_o, y_o(t), r(t)).$$

As previously, the existence of $C(\rho_o)$ is only needed for setting up the algorithm. In the sequel, we assume a high signal-to-noise ratio and we require that the plant and the controller and all closed-loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. This means that if the closed-loop operators are linearized around any (stable) trajectory, the resulting linear (time-varying) systems are BIBO stable; see e.g. [3]. Dropping time arguments for convenience,

$$u_o = -[C(\rho_o, y_o, r) - C(\rho_o, y(\rho), r)] - C(\rho_o, y(\rho), r).$$

Now,

$$\begin{aligned}&C(\rho_o, y_o, r) - C(\rho_o, y(\rho), r) \\ &= C(\rho_o, y_o, r) - [C(\rho_o, y_o, r) + \partial C_y(\rho_o, y_o, r)(y(\rho) - y_o)] \\ &= \partial C_y(\rho_o, y_o, r)(y_o - y(\rho)). \qquad (5.7)\end{aligned}$$

Here $\partial C_y(\rho, y, r)$ denotes the linearization of $C(y, r)$ in response to a perturbation in y around the trajectory y and r. Also,

$$\begin{aligned}y_o - y(\rho) &= P_o(u_o, 0) - P_o(u(\rho), v) \qquad (5.8)\\ &= P_o(u_o, 0) - [P_o(u_o, 0) \\ &\quad + \partial P_{o_u}(u_o, 0)(u(\rho) - u_o) - \partial P_{o_v}(u_o, 0)\, v] \\ &= \partial P_{o_u}(u_o, 0)(u_o - u(\rho)) + \partial P_{o_v}(u_o, 0)\, v.\end{aligned}$$

Here $\partial P_{o_u}(u_o, 0)$ and $\partial P_{o_v}(u_o, 0)$ denote the linearizations of $P_o(u, v)$ in response to, respectively, u and v around the trajectory $u = u_o$ and $v = 0$. It now follows from (5.7) and (5.8) that

$$\begin{aligned}u_o &= -C(\rho_o, y(\rho), r) \\ &\quad -\partial C_y(\rho_o, y_o, r)\partial P_{o_u}(u_o, 0)(u_o - u(\rho)) \\ &\quad + \partial C_y(\rho_o, y_o, r)\partial P_{o_v}(u_o, 0)\, v.\end{aligned}$$

Define the signal $\bar{\epsilon}_{CL}(\rho) = u_o - u(\rho)$. It now follows that

$$\begin{aligned}\bar{\epsilon}_{CL}(\rho) &= -C(\rho_o, y(\rho), r) + C(\rho, y(\rho), r) \\ &\quad - \partial C_y(\rho_o, y_o, r)\partial P_{o_u}(u_o, 0)\, \bar{\epsilon}_{CL} \\ &\quad + \partial C_y(\rho_o, y_o, r)\partial P_{o_v}(u_o, 0)\, v.\end{aligned}$$

Define

$$C'(\rho, y(\rho), r) = [C'_{\rho_1}(\rho, y(\rho), r) \ \cdots \ C'_{\rho_n}(\rho, y(\rho), r)]$$

where $C'_{\rho_1}(\rho, y(\rho), r)$ denotes the partial derivative of $C(\rho, y(\rho), r)$ with respect to ρ_j for $j = 1, \cdots, n$; n being the dimension of ρ. Since

$$\begin{aligned}&-C(\rho_o, y(\rho), r) + C(\rho, y(\rho), r) \\ &\qquad = -C(\rho_o, y(\rho), r) + [C(\rho_o, y(\rho), r) \\ &\qquad\quad + C'(\rho, y(\rho), r)(\rho - \rho_o)] \\ &\qquad = C'(\rho, y(\rho), r)(\rho - \rho_o),\end{aligned}$$

it follows that

$$\begin{aligned}\bar{\epsilon}_{CL}(\rho) &= -[1 + \partial C_y(\rho_o, y_o, r)\partial P_{o_u}(u_o, 0)]^{-1} \times \\ &\qquad C'(\rho, y(\rho), r)(\rho_o - \rho) \\ &\quad +[1 + \partial C_y(\rho_o, y_o, r)\partial P_{o_u}(u_o, 0)]^{-1} \times \\ &\qquad \partial C_y(\rho_o, y_o, r)\partial P_{o_v}(u_o, 0)\, v. \qquad (5.9)\end{aligned}$$

Define $\epsilon_{CL}(\rho) = y_o - y(\rho)$. It now follows from (5.8) that

$$\epsilon_{CL}(\rho) = \partial P_{o_u}(u_o, 0)\, \bar{\epsilon}_{CL}(\rho) + \partial P_{o_v}(u_o, 0)\, v. \qquad (5.10)$$

Equations (5.9) and (5.10) can be used to implement an algorithm of the type (2.18)-(2.19) with

$$\varphi(t) = [C'(\rho, y(\rho), r)]^T. \qquad (5.11)$$

We refer to [7] for an extension of the stability theorems of Sections 3 and 4.

6 A numerical illustration

In this section, we apply the "non-conventional" algorithm (2.20)-(2.21) to a seventh order plant of the form

$$y(t) = k(t)\frac{B(q)}{A(q)} u(t) + v(t) \qquad (6.1)$$

where the coefficients of the polynomials $A(q)$ and $B(q)$, represented in vector form as a and b, are respectively

$$\begin{aligned}a &= [1\ 0.0049\ -0.0848\ -0.1953\ 0.1450 \\ &\qquad -0.0159\ -0.0505\ 0.0145], \\ b &= [0\ 1\ 3.0119\ 1.7150\ 0.1795\ 1.0926\ 0.1477\ 0.0004].\end{aligned}$$

The plant is stable but has unstable zeroes. The noise sequence is white and $v(t)$ is uniformly randomly distributed over the interval $[-0.01\ \ 0.01]$. The gain $k(t)$ is varying as a sinusoid over the interval 0.2 ± 0.0125 with period 0.004 rad/sec. In this simulation example, the underlying sampling time is $T = 1$ second. The output and input reference model are chosen to be second order with transfer function, respectively,

$$T_d(q) = \frac{0.13\,q^2 + 0.26\,q + 0.13}{q^2 - 0.75\,q + 0.27}, \quad (6.2)$$

$$S_d(q) = \frac{0.06\,q^2 + 0.09\,q + 0.024}{q^2 + 0.09\,q + 0.145}. \quad (6.3)$$

and the reference signal $r(t)$ is a randomly (uniformly over $[-2\ \ 2]$) generated signal filtered using a Butterworth filter with cut-off frequency 0.01.

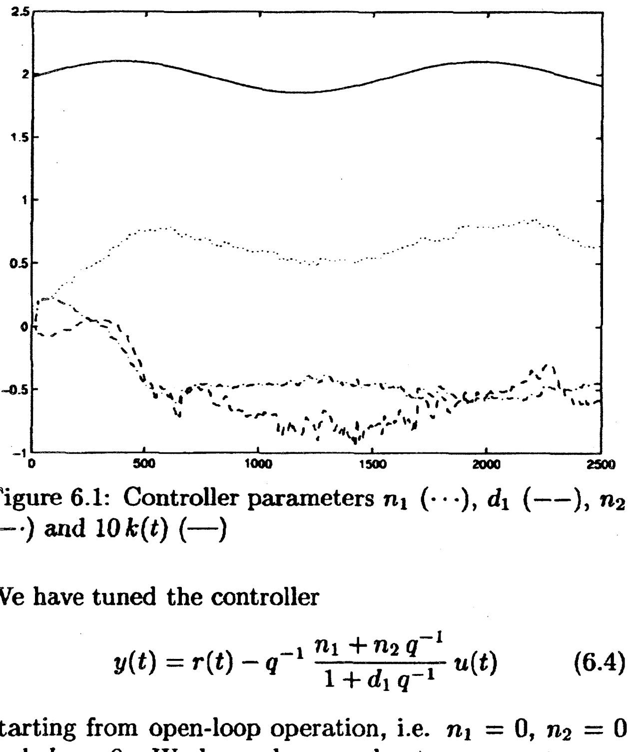

Figure 6.1: Controller parameters n_1 ($\cdots$), d_1 ($--$), n_2 ($-\cdot$) and $10\,k(t)$ (—)

We have tuned the controller

$$y(t) = r(t) - q^{-1}\,\frac{n_1 + n_2\,q^{-1}}{1 + d_1\,q^{-1}}\,u(t) \quad (6.4)$$

starting from open-loop operation, i.e. $n_1 = 0$, $n_2 = 0$ and $d_1 = 0$. We have chosen a least squares strategy with forgetting factor and

$$\gamma_1 = 0.1, \quad \gamma_2 = 0.9, \quad \lambda_1 = 0.99, \quad \lambda_2 = 1. \quad (6.5)$$

Note that (3.6) is satisfied, i.e. $\gamma_1\,b + \gamma_2\,a$ has only stable roots. The gain $k(t)$ and controller parameters n_1, n_2 and d_1 are shown in Figure 6.1. The desired output $y_o = T_d\,r(t)$ and the achieved output are shown at different time intervals in Figure 6.2.

7 Conclusions

In this paper, we have considered iterative feedback tuning as a dual of closed output-error identification. This has allowed us to present a series of adaptive algorithms that show some similarities with some direct adaptive

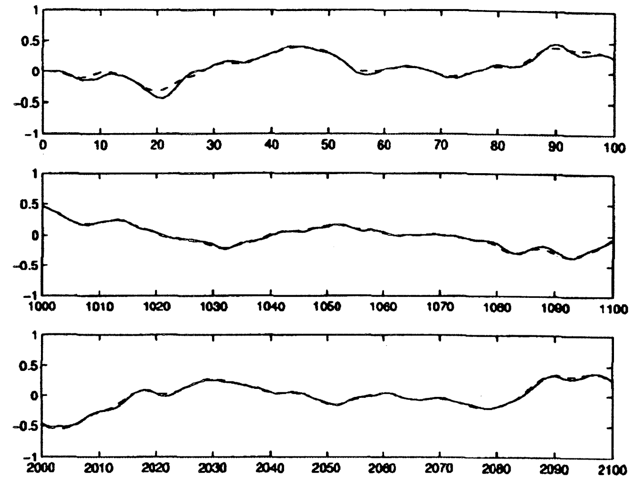

Figure 6.2: The desired output $y_o = T_d\,r(t)$ ($--$) and the achieved output (—).

control schemes. Some of the algorithms derived in this paper show promise for systems with unstable zeroes and can be easily extended to a nonlinear framework.

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