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Variable Constraint Control of Underactuated Free Flying Robots – Mechanical Design and Convergence –

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Abstract

A new control method for underactuated nonlinear systems called a variable constraint control (VCC) will be proposed. In this control method, the first integral obtained by adding control constraints is used as an invariant manifold. The results will be applied to the posture control problems of free flying robots and relations between convergence of the control and structural parameters of robots are discussed.

1 Introduction

We will propose a new control method for nonholonomic systems called a variable constraint control (VCC) and apply the results to the posture control of underactuated free flying robots such as hopping robots and space robots subject to the conservation law of angular momentum. Relations between convergence of the control and parameters of robots will be discussed.

As to the posture control of the free flying robots, Murray et al. [1] introduced feedforward control from the point of view of a path planning. Mukherjee [4], Papadopulas [3] and Reyhanoglu et. al. [2] proposed feedforward controls based on the idea of the holonomy whereas Sugita et al. [5] and Mukherjee et al. [6] proposed a feedback control.

On the other hand, since it is known that such systems are transformed to chained forms with two inputs if $n = 3$ [7], many control methods developed based on the chained form [7], [8], [9], [10] may be applicable to solve the posture control problems. However, transformations to chained forms have singular manifolds and they interrupt convergence of the control. There seems no research results avoiding this problem as far as the authors know.

In this paper, we will propose a two stage control method called *variable constraint control (VCC)* for nonholonomic systems that is applicable to the posture control of free flying robots. In the first stage of the control, we impose a holonomic constraint referred to as a *control constraint* to the nonholonomic system by the control to change the existing nonholonomic constraint to holonomic, that is, integrable. Since the time derivative of its first integral is zero, the integral will serve as an invariant manifold as long as the control constraints are preserved. In the second control stage, we will control a group of variables that represents the zero dynamics in the first stage control while keeping the control constraints in order to maintain the invariant manifold.

By this control, the transformation to the chained form is not necessary. However, since the first stage control will brings the system into some singular circumstances, we have to investigate the convergent condition. We will discuss the problem by applying VCC to a hopping robot and space robot.

2 Variable constraint control (VCC)

2.1 First integral and control constraint

We will deal with a mechanical system which has the generalized coordinate $q \in R^n$, the control input $u \in R^m$ and the $n-m$ dimensional velocity constraints described by the following Pfaffian form [1]

$$A(q)\dot{q} = 0 \ : \quad A \in R^{(n-m)\times n} \tag{1}$$

We assume that $A(q)$ is of full row rank around the reference point q_r, $m < n$ and (1) can be decomposed as

$$A_1(q)\dot{q}_1 + A_2(q)\dot{q}_2 = 0 : \quad \dot{q}_1 \in R^m,\ \dot{q}_2 \in R^{n-m} \tag{2}$$

with $|A_2| \neq 0$. Also, we assume that $\dot{q}_1$ can be treated as the control input u so that (1) can be described by

$$\dot{q} = B(q)u : \quad q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad B(q) = \begin{bmatrix} I_m \\ -A_2^{-1}A_1 \end{bmatrix}, \tag{3}$$

where B satisfies $AB = 0$ which means that Range(B) spans the null space of A. We suppose that the m column vectors in Range(B) are not involutive, i.e., (1) is not integrable, but satisfies the controllability Lie algebra condition [1]. This means that (3) is a controllable (small-time locally controllable) nonholonomic system which cannot be stabilized by any smooth feedback control.

Throughout this paper, the control purpose is to bring $q \in R^n$ to a given $q_r \in R^n$ and maintain it there. To this end, we will use a two stage controlling method called *variable constraint control (VCC)* explained below.

In the first stage, we will introduce the following l dimensional holonomic control constraint

$$H(q) = 0 : \quad H(q) \in R^l \tag{4}$$

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which is regarded as fixing some of the coordinates or providing them some algebraic relations. Then from the time derivatives of (4), $(\partial H/\partial q)\dot{q} = 0$ is added to (1) as a velocity constraint. Furthermore, (1) itself will be changed to some $\hat{A}(q)\dot{q} = 0$ since some of the coordinates will be fixed by (4). Therefore, new velocity constraints will be described by

$$G(q)\dot{q} := \begin{bmatrix} \frac{\partial H}{\partial q} \\ \hat{A}(q) \end{bmatrix} \dot{q} = 0 \tag{5}$$

If the null space of $G(q)$ can be made involutive by appropriately selecting $H(q)$, (5) becomes holonomic, i.e., integrable, and we can find its first integral. The first integral implies apparently $H(q) = 0$ and yields a function $F(q)$ of dimension $n - m$ which satisfies

$$\frac{\partial F}{\partial q} = \hat{A} \tag{6}$$

and

$$F(q) = F_0, \quad \dot{F}(q) = \hat{A}(q)\dot{q} = 0. \tag{7}$$

In (7), F_0 is the integral constant which will be specified later so as to satisfy the terminal condition. Since the time derivative of $F(q)$ is zero, $F(q) = F_0$ becomes an invariant manifold (uncontrollable manifold), that is: if $F(q) = F_0$ is satisfied for some t_1, it will hold for all $t \geq t_1$, as long as $H(q) = 0$ remains in force.

We will introduce such a circumstance asymptotically using a nonlinear decoupling control which will provide the decay models

$$\dot{H} = -\lambda_1 H, \quad \dot{F} = -\lambda_2(F - F_0), \tag{8}$$

to (3), where λ_1 and λ_2 are positive constants. Since the number of the inputs is m, we need the following condition

$$l + (n - m) \leq m \tag{9}$$

By this control, we can control $l + (n - m)$ coordinates. However, the remaining $n - (l + n - m) = m - l$ coordinates cannot be controlled as a zero dynamics. Let's the coordinate expression of the zero dynamics be $N(q) \in R^{m-l}$. Then, in the second stage of control, we will control $N(q)$ while keeping $H(q) = 0$ to preserve $F(q) = F_0$ as the invariant manifold. As the control law achieving this objectives, we also introduce a decoupling control which yields

$$\dot{H} = -\lambda_1 H, \quad \dot{N} = -\lambda_3(N - N_0), \tag{10}$$

where $\lambda_3 > 0$. Throughout the control, F_0 and N_0 must be chosen to satisfy

$$H(q_r) = 0, \quad F(q_r) = F_0, \quad N(q_r) = N_0 \tag{11}$$

to bring q to q_r.

By the way, as will be seen in below, $H(q)$ must make R_1 nonsingular in addition to make the null space of $G(q)$ involutive. Therefore, to make the design procedure simple, we will restrict the dimension of the null space to one because it is always involutive. This requires

$$n - (l + n - m) = m - l = 1 \tag{12}$$

which together with (9) leads to $n - 1 \leq m$. Since n link planar free flying robots have $m = n - 1$ inputs, the choice of

$$l = n - 2 \tag{13}$$

satisfies (9) as well as (12). All the examples below will satisfy these conditions as $n = 3$, $m = 2$ and $l = 1$.

2.2 Control law and singularity

We will show the form of the controls explicitly and raise some problems assuming that $m = n - 1$ and $l = n - 2$ hold.

In the first stage of control, we will choose u to ensure satisfaction of (8) as

$$\begin{bmatrix} \dot{H} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial F}{\partial q} \end{bmatrix} B(q)u = \begin{bmatrix} \frac{\partial H}{\partial q} \\ \hat{A} \end{bmatrix} B(q)u = -\begin{bmatrix} \lambda_1 H \\ \lambda_2(F - F_0) \end{bmatrix} \tag{14}$$

If $H(q)$ can be chosen to make

$$R_1(q) := \begin{bmatrix} \frac{\partial H}{\partial q} \\ \hat{A}(q) \end{bmatrix} B(q) \tag{15}$$

nonsingular as a function of q, the control is given by

$$u = -R_1^{-1} \begin{bmatrix} \lambda_1 H \\ \lambda_2(F - F_0) \end{bmatrix} \tag{16}$$

In the second stage of control, the condition

$$\begin{bmatrix} \dot{H} \\ \dot{N} \end{bmatrix} = \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial N}{\partial q} \end{bmatrix} B(q)u = -\begin{bmatrix} \lambda_1 H \\ \lambda_3(N - N_0) \end{bmatrix} \tag{17}$$

leads to

$$u = -R_2^{-1} \begin{bmatrix} \lambda_1 H \\ \lambda_3(N - N_0) \end{bmatrix}, \tag{18}$$

if

$$R_2(q) = \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial N}{\partial q} \end{bmatrix} B(q) \tag{19}$$

is nonsingular. The zero dynamics in this second stage is shown to be $F(q) = F_0$.

The problems of VCC are the following.

P1. Let's $\hat{B}$ denotes B when $H = 0$. Then, when $H = 0$, $AB = 0$ becomes $\hat{A}\hat{B} = 0$ which means that the last low of R_1 will be zero, and we cannot form its inverse. However, it will be turn out that so long as (3) is controllable and one has $\lambda_2 > \lambda_1$, even though R_1 is asymptotically singular, the control u converges as t tends to infinity. This will be shown in examples.

P2. Since the zero dynamics $N(q)$ in the first stage cannot be controlled, it may drive $R_1(q)$ to singularities before convergence. In some cases such as a hopping robot, we can avoid this problem by choosing parameters of the robot. However, in general, we cannot completely get around the problem. The initial posture q_0 restricts the choice of q_r.

P3 The zero dynamics $N(q)$ is essentially of the form of an integrator (as illustrated in the examples). The integration may accumulate to become excessively large. We can get around this problem by switching the first stage control to the second at an appropriate time T_s, which may cause some settling error.

2.3 Test in a chained form

We will check VCC by applying it to a system described by a chained form. In the chained form, we don't encounter the problems P2 and P3.

We treat a chained form with $n = 3$ and $m = 2$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ q_2 & 0 \end{bmatrix} u \tag{20}$$

which is derived from the following nonholonomic velocity constraint

$$A(q)\dot{q} = \dot{q}_3 - q_2\dot{q}_1 = 0 \tag{21}$$

Let's introduce $n - 2 = 1$ control constraint as

$$H(q) = q_2 - q_{r2} = 0 \tag{22}$$

Then assuming that (22) hold, (21) can be changed to a holonomic constraint:

$$\hat{A}(q)\dot{q} = \dot{q}_3 - q_{r2}\dot{q}_1 = 0 \tag{23}$$

The first integral of (23) is given by

$$F(q) = F_0 : \quad F(q) = q_3 - q_{r2}q_1 \tag{24}$$

where F_0 is chosen as

$$F_0 = q_{r3} - q_{r2}q_{r1} \tag{25}$$

in order to guarantee $q_3 = q_{r3}$ when $q_1 = q_{r1}$ is obtained at the end of the second stage control. Note that because the problem is so simple, we have no need to introduce $G(q)$ and exploit the involutive nature of its nullspace.

Since

$$R_1 = \begin{bmatrix} 0 & 1 & 0 \\ -q_{r2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ q_2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ H & 0 \end{bmatrix} \tag{26}$$

the control in the first stage is given by

$$u = \begin{bmatrix} -\lambda_2 \frac{F - F_0}{H} \\ -\lambda_1 H \end{bmatrix} \tag{27}$$

It follows from (8) that H converges to zero exponentially fast. Therefore u_2 converges. In addition, $H(q)$ does not become 0 before convergence and u_1 is expressed by

$$u_1 = -\lambda_2 e^{-(\lambda_2 - \lambda_1)t} \frac{F(q(0)) - F_0}{H(q(0))} \tag{28}$$

Therefore, if we choose

$$\lambda_2 > \lambda_1 \tag{29}$$

then u_1 also converges to 0 as long as $H(q(0)) \neq 0$. When $H(q(0)) = 0$ at the beginning, we have to introduce an additional control before VCC so as to leave q_2 from q_{r2} just like [8].

Since the Jacobian matrix between q and $(q_1, H, F - F_0)^T$ can be shown to be nonsingular, the zero dynamics of the closed loop system is given by q_1 which leads to

$$\dot{N}(q) = u_1 \tag{30}$$

Since, u_1 approaches 0 exponentially fast, the zero dynamics becomes constant and does not diverge. Therefore, we can choose the switching time T_s arbitrarily in the chained form to keep the closed system stable.

In the second stage of the control, we aim for

$$N(q) = N_0 : \quad N_0 = q_{r1} \tag{31}$$

in addition to $H(q) = 0$. Since R_2 becomes a unit matrix, the control is given by

$$u = \begin{bmatrix} -\lambda_3(N - N_0) \\ -\lambda_1 H \end{bmatrix} \tag{32}$$

By this control, q_1 and q_2 approach q_{r1} and q_{r2}, respectively. Since $F(q) = F_0$ is maintained as an invariant manifold in the second stage, q_3 converges to q_{r3}.

Fig.1 gives simulation results where we have used the following conditions. $q(0) = 0$, $q_r = (2,2,2)^T$, $\lambda_1 = 3$, $\lambda_2 = 5$ and $\lambda_3 = 5$. Choice of $T_s = 2$ (sec) results in a settling error of less than 10^{-4}.

3 Posture control of a hopping robot

We will deal with a hopping robot depicted in Fig. 2 and show a method to get around the problems P1-P3.

The robot is assumed to be in the air with zero angular momentum. It is composed of an extensible lower leg of length $l(t)$ with the concentrated mass of weight m_1 at the tip, an upper leg of length $2d$ with the concentrated mass of weight m_2 at the center, and a body whose weight and moment of inertia are M and J, respectively. The location of the waist joint is a distance of r from the center of the gravity (CG). The coordinate θ denotes the absolute body angle and ψ gives the relative angle between body and the upper leg. We will define that $q = (l, \psi, \theta)^T$, $u_1 = \dot{l}$ and $u_2 = \dot{\psi}$.

Then, the conservation law of angular momentum leads to the velocity constraint

$$A(q)\dot{q} = a(\psi, l)\dot{\theta} + b(\psi)\dot{l} + c(\psi, l)\dot{\psi} = 0 \tag{33}$$

corresponding to (1), where

$$\begin{aligned} a &= m_0 J + c_1 + 2c_2\cos\psi + c_3 \\ b &= m_1 M r \sin\psi, \quad c = c_1 + c_2\cos\psi \end{aligned} \tag{34}$$

$$\begin{aligned} : c_1 &= m_1(m_2 + M)l^2 + (m_1 m_2 + 4m_1 M + m_2 M)d^2 \\ &\quad + 2m_1(m_2 + 2M)ld \\ c_2 &= rM\{m_1(l + 2d) + m_2 d\} \\ c_3 &= M(m_1 + m_2)r^2, \quad m_0 = m_1 + m_2 + M \end{aligned}$$

In this case, (3) becomes

$$\dot{q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{b}{a} & -\frac{c}{a} \end{bmatrix} u \tag{35}$$

In this study, we will introduce

$$H(q) = l - l_r = 0 \tag{36}$$

as the control constraint. We take $l_r = 0$ in the sequel without loss of generality.

When (36) is assumed to be satisfied, (33) can be written as

$$\hat{A}\dot{q} = a(\psi, 0)\dot{\theta} + c(\psi, 0)\dot{\psi} = 0 \tag{37}$$

which can be rewritten as

$$\dot{\theta} = -\frac{\gamma + \eta\cos\psi}{\alpha + \beta\cos\psi}\dot{\psi} := -\frac{\hat{c}(\psi)}{\hat{a}(\psi)}\dot{\psi} \tag{38}$$

for appropriate constants α, β, γ and η, and its first integral becomes

$$F(q) = F_0 : \quad F(q) = \theta + g(\psi) \tag{39}$$

where [12] [1]

$$g(\psi) = \frac{\eta}{\beta}\psi + \frac{2(\gamma - \eta\alpha/\beta)}{\sqrt{\alpha^2 - \beta^2}} tan^{-1}\left(\sqrt{\frac{\alpha - \beta}{\alpha + \beta}} tan\frac{\psi}{2}\right) \tag{40}$$

Moreover, F_0 is chosen as

$$F_0 = \theta_r + g(\psi_r) \tag{41}$$

Then the control in the first stage is given by

$$u = \begin{bmatrix} -\lambda_1 l \\ -\frac{b\hat{a}\lambda_1}{a\hat{c} - \hat{a}c} l - \frac{a\hat{a}\lambda_2}{a\hat{c} - \hat{a}c}(F - F_0) \end{bmatrix} \tag{42}$$

Since l converges to $l_r = 0$ exponentially fast, a, $\hat{a}$ and b are bounded, we will investigate the following two terms in u_2 in checking that u remains bounded:

$$l/D, \quad (F - F_0)/D \tag{43}$$

where

$$\begin{aligned} D(l, \psi) &= a\hat{c} - \hat{a}c = -m_1 l(h_1 + h_2\cos\psi) \\ &: h_1 = \{m_0 J + M(m_1 + m_2)r^2\} \\ &\qquad \times\{(m_2 + M)l + 2(m_2 + 2M)d\} \\ h_2 &= Mrd\{(m_2 + M)(2m_1 + M)l \\ &\qquad +(3m_1m_2 + 2m_2^2 + 4m_1M + 3m_2M)d\} \\ &\qquad +Mr\{m_0J + M(m_1 + m_2)r^2\} \end{aligned}$$

[1] In order to avoid discontinuity in the calculation of the inverse tangent, $tan^{-1}(\ )$ in (40) must be replaced by

$$m\pi + arctan\left(\sqrt{\frac{\alpha - \beta}{\alpha + \beta}} tan\frac{\psi - 2m\pi}{2}\right)$$

according to the condition.

$$(2m - 1)\pi \le \psi \le (2m + 1)\pi$$

To solve the problem P1, let's expand D around $l = l_r = 0$ as

$$D(l, \psi) = L(\psi)l + L_{high}(\psi)l^2 + \ldots.. \tag{44}$$

where

$$L(\psi) = \left.\frac{\partial D(l, \psi)}{\partial l}\right|_{l=0}. \tag{45}$$

If $L(\psi) \neq 0(a.e)$ holds, then l/D converges to a constant value when l approaches zero; otherwise diverges since the power of l becomes negative. It will be shown in [13] that $L(\psi) \neq 0(a.e)$ is ensured because (3) is controllable. Then the term $(F - F_0)/D$ converges to zero if we choose

$$\lambda_2 > \lambda_1 \tag{46}$$

To solve the problem P2, we will propose to construct the robot to satisfy

$$h_1 > h_2 \tag{47}$$

Then there is no ψ and l which makes $D(l, \psi) = 0$ except for $l = 0$. However, $l = 0$ never happens before convergence since l decays exponentially fast. By this way, the problem P2 can be avoided. When $r = 0$, (47) trivially holds since $h_2 = 0$. We can confirm from simulations that (47) is not a restrictive condition even when $r \neq 0$.

Since the Jacobian matrix between q and $(l, \psi, F - F_0)^T$ always nonsingular, ψ satisfying $\dot{\psi} = u_2$ becomes the zero dynamics $N(q)$. Since u_2 cannot be ensured to converge to zero, the magnitude of u_2 becomes large as time passes. Therefore, we have to switch the first stage control at some short time T_s to solve the problem P3.

In the second stage, we aim for

$$N(q) = N_0 : \quad N_0 = \psi_r \tag{48}$$

while keeping $H = 0$. Then u is given by

$$u = -\begin{bmatrix} \lambda_1 l \\ \lambda_3(\psi - \psi_r) \end{bmatrix} \tag{49}$$

Fig.3 depicts the responses for a robot with parameters $M = 2$, $J = 2/3$, $r = 1$, $m_2 = 0.1$, $d = 1$ and $m_1 = 0.5$ which satisfies (47). Other conditions are $T_s = 0.2(sec)$, $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 4$ and

$$q(0) = (\pi/6, 2, \pi/3)^T, \quad q_r = (\pi/2, 0, 0)^T$$

4 Posture control of a space robot

The robot shown in Fig. 4 has two arms whose lengths and weights are l_1, l_2, m_1 and m_2, respectively; a body whose weight and moment of inertia are M and J; two joints for the arms which are at distance of r from the CG of the body. We define $q = (\psi_1, \psi_2, \theta)^T$, $u_1 = \dot{\psi}_1$ and $u_2 = \dot{\psi}_2$.

The conservation law of angular momentum of this robot is described by

$$A(q)\dot{q} = a(\psi_1, \psi_2)\dot{\theta} - b(\psi_1, \psi_2)\dot{\psi}_1 - c(\psi_1, \psi_2)\dot{\psi}_2 = 0 \tag{50}$$

$$
\begin{aligned}
: a &= m_0 J + m_1 M(l_1^2 + r^2) + m_1 m_2 (l_1^2 + l_2^2 + 4r^2) \\
&\quad + m_2 M(l_2^2 + r^2) + 2m_1 l_1 r(M + 2m_2)\cos(\psi_1) \\
&\quad + 2m_2 l_2 r(M + 2m_1)\cos(\psi_2) + 2m_1 m_2 l_1 l_2 \cos(\psi_1 - \psi_2) \\
b &= m_1(M + m_2) l_1^2 + m_1 l_1 r(M + 2m_2)\cos(\psi_1) \\
&\quad + m_1 m_2 l_1 l_2 \cos(\psi_1 - \psi_2) \\
c &= m_2(M + m_1) l_2^2 + m_2 l_2 r(M + 2m_1)\cos(\psi_2) \\
&\quad + m_1 m_2 l_1 l_2 \cos(\psi_1 - \psi_2) \\
m_0 &= m_1 + m_2 + M
\end{aligned}
$$

We have used

$$\psi_1 = \psi_{r1} \tag{51}$$

for the control constraint. When this constraint is satisfied, (50) can be described as

$$
\begin{aligned}
\dot{\theta} &= \frac{c(\psi_{r1}, \psi_2)}{a(\psi_{r1}, \psi_2)} \dot{\psi}_2 := \frac{\hat{c}(\psi_2)}{\hat{a}(\psi_2)} \dot{\psi}_2 \\
&= \frac{W_4 + W_5 cos\psi_2 + W_6 sin\psi_2}{W_1 + W_2 cos\psi_2 + W_3 sin\psi_2} \dot{\psi}_2
\end{aligned} \tag{52}
$$

where,

$$
\begin{aligned}
W_1 &= m_0 J + m_1 M(l_1^2 + r^2) + m_1 m_2 (l_1^2 + l_2^2 + 4r^2) \\
&\quad + m_2 M(l_2^2 + r^2) + 2m_1 l_1 r(M + 2m_2) cos\psi_{r1} \\
W_2 &= 2m_2 l_2 \{(M + 2m_1) r + m_1 l_1 cos\psi_{r1}\} \\
W_3 &= 2m_1 m_3 l_1 l_2 sin\psi_{r1}, W_4 = m_2(M + m_1) l_2^2 \\
W_5 &= 0.5 W_2, W_6 = 0.5 W_3
\end{aligned}
$$

Then with the first integral $F(q)$[12] from (52), the control in the first stage is given by

$$u = \begin{bmatrix} -\lambda_1(\psi_1 - \psi_{r1}) \\ \lambda_1 b\hat{a}(\psi_1 - \psi_{r1})/D - \lambda_2 a\hat{a}(F - F_0)/D \end{bmatrix} \tag{53}$$

where

$$D(\psi_1, \psi_2) = \hat{a}c - \hat{c}a \tag{54}$$

We can get around the problem P1 using $\partial(c/a)/\partial\psi_1 \neq 0$ (a.e) which comes from the controllability condition. To solve P2, we will draw ψ_1 and ψ_2 which satisfy $D = 0$ in Fig. 5 when $\psi_{1r} = -0.45\pi$ and $\psi_{2r} = 0.45\pi$. It follows from Fig. 5 that at least we know that the initial conditions $\psi_1(0)$ and $\psi_2(0)$ must be between the two singular curves to guarantee $D \neq 0$ before convergence. The control in the second stage will be given by

$$u = \begin{bmatrix} -\lambda_1(\psi_1 - \psi_{r1}) \\ -\lambda_3(\psi_2 - \psi_{r2}) \end{bmatrix} \tag{55}$$

since ψ_2 becomes the zero dynamics of the first stage.

Fig. 6 and Fig.7 show the response and the phase trajectory of the solution when $J = 1$, $M = 5$, $m_1 = 3.0$, $m_2 = 3.5$, $l_1 = 0.3$, $l_2 = 0.35$ and $r = 0.18$. Other conditions are $T_s = 0.5$ (sec), $\lambda_1 = 3$, $\lambda_2 = \lambda_3 = 6$ and

$$
\begin{aligned}
q(0) &= (-0.65\pi, 0.25\pi, -0.08\pi)^T \\
q_r &= (-0.45\pi, 0.45\pi, 0)^T
\end{aligned}
$$

5 Conclusion

We have proposed a control method for nonholonomic systems applicable to free flying robots. Remaining works include coping with singular problems for robots composed of revolutive joints. We are experimenting with jumping robots (http://www.mimi.ctrl.titech.ac.jp). The authors would like to appreciate Dr. Xin Xin for his help. This research work is supported by Titech COE.

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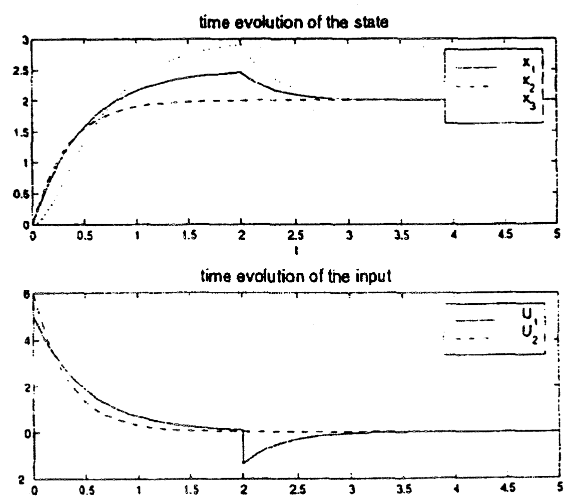

Fig.1 Response of the chained form

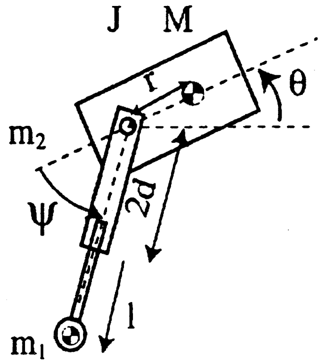

Fig.2 Hopping robot

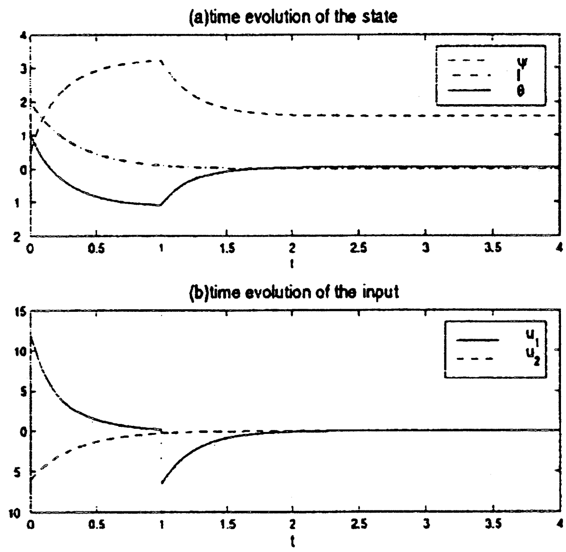

Fig.3 Response of hopping robot

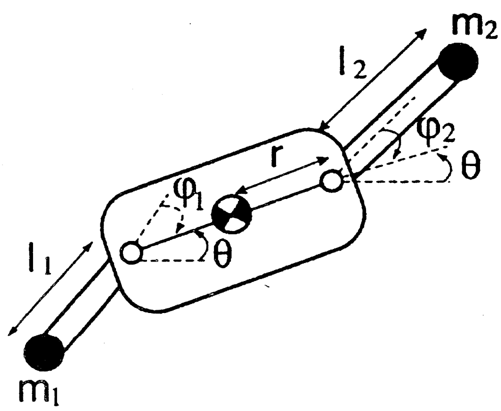

Fig.4 Space robot

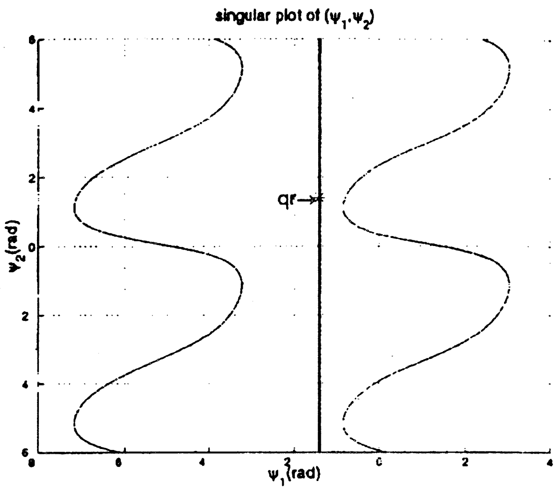

Fig.5 Singular Curve

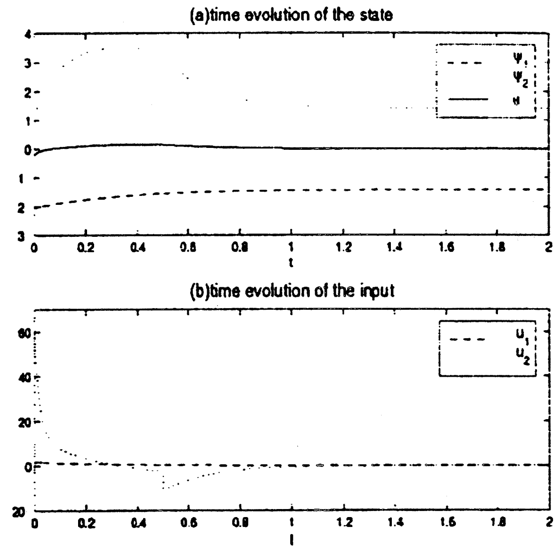

Fig.6 Response of the space robot

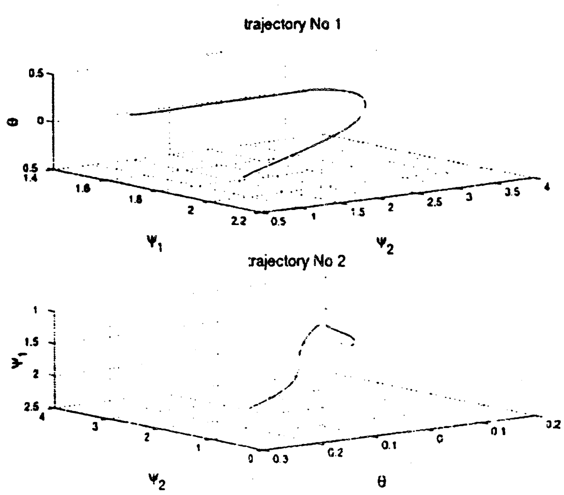

Fig.7 States trajectory of the space robot