Proceedings of the 38th
Conference on Decision & Control
Phoenix, Arizona USA • December 1999

Closed-Loop Output Error Identification Algorithms for Nonlinear Plants

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Abstract

A family of algorithms for the identification of continuous time nonlinear plants operating in closed-loop is presented. An adjustable output error type predictor is parametrized in terms of the existing controller and the estimated plant model. The algorithms are derived from stability considerations in the absence of noise and assuming that the plant model is in the model set. Subsequently the algorithms are analyzed in the presence of noise and when the plant model is not in the model set.

Keywords : closed-loop identification, nonlinear systems, adaptive systems, output-error

1 Introduction

The development of algorithms for plant model identification in closed-loop has been an important line of research in the last few years.

This line of research has been motivated by several factors:

1. The fact that in a number of situations identification in open-loop is difficult or simply not feasible. This includes the case of plants having an integrator or being unstable as well as the case of plants subject to significant drift in open-loop operation.
2. The presence of a controller in the loop (which has to be re-tuned).
3. The possibility of capturing the dynamic characteristics of the plant model which are critical for control design.

In the context of linear models, recursive and batch algorithms for plant model identification in closed-loop have been proposed, analyzed and evaluated experimentally. Such algorithms have already moved towards standard use in industry.

One of the successful ways to develop algorithms for identification in closed-loop is to consider "closed-loop output error" schemes. A closed-loop output-error-type predictor parameterized in terms of the existing controller and the estimated plant is used. The algorithm tries to minimize a quadratic criterion in terms of the closed-loop output error or tries to drive the closed-loop output error to zero; see [5] for results in a linear framework.

The problem of identification of nonlinear models in closed-loop is definitely of practical importance for the same reasons as those indicated previously for linear models. In addition, identifying nonlinear models in continuous time makes possible the direct estimation of physical parameters which have a clear significance for the end user.

In the present paper we focus on the recursive identification of *nonlinear* plants operating in closed-loop with a *nonlinear* controller using a closed-loop output error identification scheme. An important aspect is that we are addressing the problem of identifying nonlinear plants whose outputs **cannot** be expressed linearly in terms of the unknown parameters (i.e. $y \neq \theta_0^T \psi$ where y is the output, θ_0 is the vector of parameters and ψ is a vector of nonlinear functions of various variables). Our algorithm can also be used to identify linear plants that cannot be parametrized linearly in terms of the unknown parameters.

Another interesting aspect is that among the family of algorithms which will be proposed and analyzed, one of them can be interpreted as the batch algorithm proposed in [1]. The results of this paper allow the assessment of the properties of this batch algorithm.

As indicated in [1] and shown in Section 4, the Non-Linear Closed-Loop Output Error (NL-CLOE) algorithm presented in this paper cannot produce consistent estimates in the presence of noise. Therefore, the algorithm should be applied in low noise (high Signal-to-Noise Ratio (SNR)) situations. The main advantage of the NL-CLOE algorithm over its open-loop counterparts lies in the identification of low order models (undermodeling) in a high SNR situation. Indeed, in this situation one obtains a model that is well suited for control design since it is a good model of the plant with the controller that is operating. We refer to [3] for discussion of this problem in the linear case. Note that in practice there is always a level of undermodeling as any mathematical model is necessarily an abstraction of the real system.

Passivity properties of various linear time-varying input-output operators play an important role in assessing the convergence properties for the various algorithms.

The paper is organized as follows. The problem setting is given in Section 2. In Section 3, the derivation of the algorithms is done in continuous time and in a deterministic noise free environment assuming that the plant is in the model set for a particular value of the unknown parameter vector and that one can neglect the terms of power higher than one in certain Taylor expansions. The parameter adaptation algorithms which are used are the counterpart in structure of the discrete time parameter adaptation algorithms used in recursive identification. For this reason we still use the term "recursive identification" despite the fact that we operate in continuous time where typically the term "adaptive identification" is used. A stability analysis in this context is provided. The case when the plant model is perturbed by noise and possibly not in the model set as well as the case when the high order terms in the Taylor series expansions cannot be neglected is discussed in Section 4.

The proofs of the two main theorems have been omitted for conciseness reasons and can be found in the extended version of this paper together with a simulation example; see [4].

2 The Basic Equations and Problem Setting

The objective is to estimate the parameters of a single input single output (SISO) nonlinear time invariant system described by

$$S: \quad y = P_0(u,v) \tag{2.1}$$

where P_0 is an unknown causal nonlinear operator, u is the control input signal, y is the achieved output signal and v is the disturbance signal allowed to enter the system nonlinearly. It is not assumed that the output y can be expressed linearly in terms of some parameter vector θ_0. For ease of notation the time argument will be omitted when there are no ambiguities.

The plant is operated in closed-loop with a known nonlinear controller, i.e.

$$C: \quad u = -C(y,r) \tag{2.2}$$

where r is an external reference which is assumed to be quasi-stationary and uncorrelated with v. The controller C is a causal nonlinear operator of both r and y.

The closed-loop operator from the measured reference signal r to the measured output signal y, as defined in Figure 2.1 is denoted by

$$y = T_0(r,v). \tag{2.3}$$

It is required that the closed-loop system is Bounded Input Bounded Output (BIBO) stable. In the sequel we often make use of linearizations of some nonlinear operators around their operating trajectories. We therefore require that the plant, the model (to be defined subsequently), the controller and all closed-loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. This means that if the closed-loop operator is linearized around any (stable) trajectory, the resulting linear (time-varying) system is BIBO stable. See [2] for more details.

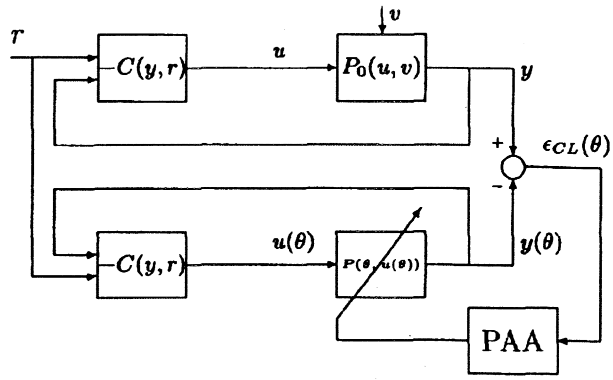

Figure 2.1: Closed-loop output error identification scheme

We consider the following adjustable predictor for the closed-loop system defined by (2.1) and (2.2) (See also Figure 2.1)

$$y(\theta) = P(\theta, u(\theta)) \tag{2.4}$$
$$u(\theta) = -C(y(\theta), r) \tag{2.5}$$

where $P(\theta, u)$ defines the adjustable plant model, $y(\theta)$ is the output of the closed-loop predictor and $u(\theta)$ is the plant model input.

The closed-loop prediction error is defined as

$$\epsilon_{CL} = y - y(\theta). \tag{2.6}$$

The following assumptions will be made in the sequel:

(i) $\exists\, \theta_0$ such that $P(\theta_0, u) = P_0(u, 0)$ for all $u \in \mathcal{L}_{2e}$.

(ii) **Notation:**
The operator $\partial P_u(\theta, u)$ is the linearization of $P(\theta, u)$ in response to a perturbation in u along the input trajectory u. The operator $\partial C_y(r, y)$ is the linearization of $C(y, r)$ in response to a perturbation in y along the trajectories produced r and y.

It is assumed that $\partial P_u(\theta, u)$ and $\partial C_y(r, y)$ exist for all allowable u, y and r. They are linear time-varying operators along the trajectories of the closed-loop system.

(iii) **Notation:**
The partial derivative of $P(\theta, u)$ with respect to θ_j is denoted by $P'_{\theta_j}(\theta, u)$ for $j = 1, \cdots, d$ where d is

the dimension of the parameter vector θ.

The operator $P'_{\theta_j}(\theta, u)$ and its time derivatives exist and are norm-bounded $\forall j$ along the trajectories of the closed-loop predictor which requires $\dot{r}$ to be bounded. This assumption is not particularly restrictive as P and $P(\theta)$ are assumed to be smooth operators.

(iv) Let us define

$$P_{CL}(\theta) = [I + \partial P_u(\theta, u(\theta))\partial C_y(r, y(\theta))]. \quad (2.7)$$

It is assumed that $P_{CL} = P_{CL}(\theta_0)$ and its inverse P_{CL}^{-1} exist along every trajectory of the closed-loop system encountered during the identification process. Both operators are BIBO linear time-varying operators.

(v) The reference r and the stochastic disturbance v are independent.

Assumption (i) means that at least for $\theta = \theta_0$, the plant is in the model set. (The case when this is not true will be discussed separately in Section 4).

Note that $y(\theta)$ and $u(\theta)$ in the closed-loop predictor (2.4) (2.5) will depend only upon the external excitation r for constant values of the estimated parameter vector θ (see also Figure 2.1). Since the stochastic disturbance v and the reference r are assumed to be independent, it will result that $u(\theta)$ and $y(\theta)$ are not correlated with v.

The generic parameter adaptation algorithm (PAA) which will be used throughout the paper is the continuous time version of the general PAA used in [6]:

$$\dot{\theta}(t) = F(t)\phi(t)\varepsilon_{CL}(t) \quad (2.8)$$
$$\dot{F}^{-1}(t) = -[1 - \lambda_1(t)]F^{-1}(t) + \lambda_2(t)\phi(t)\phi^T(t) \quad (2.9)$$
$$0 < \lambda_1(t) \le 1,\ 0 \le \lambda_2(t) < 2,\ F(0) > 0,$$
$$F^{-1}(t) > \alpha F^{-1}(0),\ 0 < \alpha < \infty$$

where $\theta(t)$ is the estimated parameter vector, $\varepsilon_{CL}(t)$ is the closed-loop output error, $\phi(t)$ is the observation vector, $F(t)$ is the adaptation gain matrix, $\lambda_1(t)$ is a time-varying forgetting factor and $\lambda_2(t)$ allows one to weight the rate of decrease of the adaptation gain. The two functions $\lambda_1(t)$ and $\lambda_2(t)$ allow one to have different laws of evolution of the adaptation gain. Some of the typical cases are:

1. $\lambda_1(t) \equiv 1$; $\lambda_2(t) \equiv 0$; $\dot{F}(t) = 0$; $F(t) = F(0)$ (the gradient algorithm);
2. $\lambda_1(t) \equiv 1$; $\lambda_2(t) \equiv 1$ (recursive least squares type algorithm);
3. $\lambda_1(t) = \text{const} < 1$; $\lambda_2(t) \equiv 1$ (least squares with forgetting factor);
4. $\lambda_1(t) < 1$; $\lim_{t\to\infty} \lambda_1(t) = 1$; $\lambda_2(t) \equiv 1$ (variable forgetting factor).

We will consider subsequently that the assumptions (i) through (iv) are valid and furthermore, for some analysis, that $v \equiv 0$. This will allow us to implement the appropriate parameter estimation algorithm to begin with (i.e. it allows us to find the observation vector $\phi(t)$) and to analyze its asymptotic properties. In the first stage we will use several expansions in Taylor series for the expression of the plant output and predictor output and we will neglect the terms of power higher or equal to 2. A subsequent analysis will discuss the case when these terms are not neglected. It will also treat the presence of disturbances and unmodeled dynamics, requiring Assumption (v).

3 Nonlinear Closed-loop Output Error Algorithms

In this section, we present the derivations of the algorithm and we provide a stability analysis in a deterministic environment assuming that the system can be modeled exactly and that one can neglect terms of power higher than one in certain Taylor series expansions. The results in this section heavily rely on concepts of strong strict passivity outlined in the appendix.

One has the following result (the NL-CLOE algorithm):

Theorem 3.1 *Under the assumptions (i) through (iv), assuming that $v(t) \equiv 0$ and neglecting the higher terms in certain Taylor expansions around the trajectories of the system one has for*

$$\phi(t) = [P'(\theta, u(\theta))]^T \quad (3.1)$$
$$= [P'_{\theta_1}(\theta, u(\theta)) \ \cdots \ P'_{\theta_d}(\theta, u(\theta))]^T \quad (3.2)$$

that

$$\lim_{t\to\infty} \varepsilon_{CL}(t) = 0 \quad (3.3)$$

if the linear time-varying operator

$$H = P_{CL}^{-1} - \frac{\lambda(t)}{2} I; \quad \lambda(t) > \lambda_2(t), \quad \forall t \quad (3.4)$$

is strongly strictly passive[1].

If furthermore P_{CL}^{-1} is of the form

$$\dot{x} = A(t)x(t) + B(t)u \quad (3.5)$$
$$y = C(t)x(t) + D(t)u \quad (3.6)$$

one has also

$$\lim_{t\to\infty} \phi^T(t)(\theta(t) - \theta_0) = 0. \quad (3.7)$$

Remark 1:

1. The Taylor series referred to in the theorem statement involve expansions in powers of $(u - u(\theta))$, $(y - y(\theta))$ and $(\theta_0 - \theta)$.

[1] It is assumed here that H has the form (A.2)-(A.3). See Definition A.1 in the appendix for a definition of strong strict passivity.

2. For the particular case when one can write

$$y(\theta) = P(\theta, u(\theta)) = \phi^T(t)\theta$$

where $\phi(t)$ is a vector of linear or nonlinear functions of $y(\theta)$ and $u(\theta)$ one has

$$[P'(\theta, u(\theta))]^T = \phi(t).$$

3. The condition (3.4) assures that the prediction error goes asymptotically to zero, and that the estimated parameter vector θ, converges to a set defined as

$$\mathcal{D}_c = \{\theta : \phi^T(t)\,(\theta - \theta_0) = 0\}\cdot \quad (3.8)$$

If

$$\phi^T(t)\,(\theta - \theta_0) = 0 \quad (3.9)$$

has a unique solution $\theta = \theta_0$, the parameter vector will converge toward this value. In fact this condition is a "persistence of excitation" condition for the nonlinear case.

4. The passivity condition of Theorem 3.1 can be relaxed by making other choices for $\phi(t)$, as will be indicated later.

5. The P_{CL}^{-1} operator is a linear time-varying operator.

Proof of Theorem 3.1: The proof can be found in [4] and is done in two steps.

Step I: Establishing the following lemma:

Lemma 3.1 *Neglecting the higher order terms in the Taylor expansion around the trajectories of the closed-loop system and higher order terms in the Taylor expansion of $P(\theta, u(\theta)) - P(\theta_0, u(\theta))$ in powers of $(\theta - \theta_0)$, the closed-loop output error is given by*

$$\varepsilon_{CL} = P_{CL}^{-1}\, P'(\theta, u(\theta))\; [\theta_0 - \theta(t)]\,. \quad (3.10)$$

Step II: proof of stability.

Algorithm AFNL-CLOE

Neglecting the swapping correction terms which anyway become negligible when one uses decreasing adaptation gains ($\lambda_2(t) > 0$, $\lim_{t\to\infty} \lambda_1(t) = 1$), (3.10) can be also written as

$$\varepsilon_{CL} = P_{CL}^{-1} P_{CL}(\theta)(P_{CL}^{-1}(\theta)\, P'(\theta, u(\theta))\,[\theta_0 - \theta(t)]) \quad (3.11)$$

In this case, following the same procedure as for the NL-CLOE algorithm one has to choose

$$\phi(t) = P_{CL}^{-1}(\theta)\, P'(\theta, u(\theta)) \quad (3.12)$$

as a regression vector. In this case one filters $P'(\theta, u(\theta))$ through a linear time-varying closed-loop system which depends on the current parameter estimate $\theta(t)$. The regression vector $\phi(t)$ can also be viewed as an approximation of the gradient of a quadratic criterion in terms of ε_{CL}; see [1] for further details. The corresponding strongly strictly passive condition will become

$$H = P_{CL}^{-1}\, P_{CL}(\theta) - \frac{\lambda(t)}{2} I;\ \lambda(t) > \lambda_2(t),\ \forall t > t_0, \quad (3.13)$$

should be strongly strictly passive for all θ encountered during the identification procedure. This of course requires that at each instant $P_{CL}^{-1}(\theta)$ derived by (2.7) is stable. If this is not the case, then as in the identification of linear models (e.g. recursive maximum likelihood, adaptive filtered closed-loop output error) one uses the last stable estimated filter $P_{CL}^{-1}(\theta)$.

Clearly in the vicinity of θ_0, condition (3.13) is much more likely to be satisfied, than condition (3.4).

4 Robustness Analysis

In Section 3, the stability of the closed-loop identification schemes has been guaranteed in the absence of noise and under the hypothesis that at least for $\theta = \theta_0$, the plant is in the model set (i.e. $P(\theta_0, u) = P_0(u, 0)$). It has also been assumed that the higher order terms in the various Taylor series expansions around the nominal trajectory can be neglected.

It is important to analyze the robustness of the identification schemes when the plant is not in the model set, when the output is affected by a disturbance that is allowed to enter the system nonlinearly and when the higher terms in the Taylor series expansion around the nominal trajectory cannot be neglected.

The objective of the analysis is to show that norm boundedness and mean square boundedness of all signals is assured for a certain type of characterization of the mismatch between the model and the plant and of the terms of higher order in the Taylor series expansion. The results which will be presented are extensions to the nonlinear case of the results given in [5] for the case of closed-loop output error algorithms for identification of linear plant models.

The plant will be described by

$$y_p = P_0(u, v) + \Delta P(u, v) \quad (4.1)$$

where $P_0(u, v)$ is the "reduced" order plant, $v(t)$ is a zero mean bounded disturbance, and $\Delta P(u, v)$ is a BIBO operator that is due to the unmodeled part of the system. Note that the BIBO assumption might be unnecessarily restrictive.

The estimated model is assumed to be represented by:

$$y(\theta) = P(\theta, u) \quad (4.2)$$

with the property that $P_0(u, 0) = P(\theta_0, u)$.

The true input u and the estimated input $u(\theta)$ are generated by (2.2) and (2.5) respectively.

To start with, we show that the effect of the noise and the unmodeled dynamics upon the closed-loop system can be considered to be additive. Denote by

$$\begin{aligned} y &= P(\theta_0, u, 0) && (4.3)\\ u &= -C(y,r) && (4.4)\end{aligned}$$

the values of the input and output obtained for the reduced order plant in the absence of noise.

Denote by

$$\begin{aligned} \bar{y} &= P(\theta_0, \bar{u}, v) + \Delta P(\bar{u}, v) && (4.5)\\ \bar{u} &= -C(\bar{y}, r) && (4.6)\end{aligned}$$

the values of the plant input and output, i.e. in the presence of noise and with the unmodeled dynamics.

Define

$$\begin{aligned} \bar{y} &= y + y_p && (4.7)\\ \bar{u} &= u + u_p && (4.8)\end{aligned}$$

where y_p and u_p are the perturbations coming from the noise v and the unmodeled plant dynamics.

Then

$$\begin{aligned} \bar{y} = {} & P(\theta_0, u, 0) + \partial P_u(\theta_0, u, 0)\, u_p + \partial P_v(\theta_0, u, 0)\, v \\ & + \Delta P(u, 0) + \partial \Delta P_u(u, 0)\, u_p + \partial \Delta P_v(u, 0)\, v \quad (4.9)\end{aligned}$$

and

$$\bar{u} = -C(y + y_p, r) = -C(y, r) - \partial C_y(r, y)\, y_p. \quad (4.10)$$

Here, $\partial P_v(\theta_0, u, 0)$ denotes the linearization of P_0 in response to a perturbation in v around the trajectory u and $v = 0$. Note that terms of order higher than one in the Taylor series expansion have been neglected; these are taken care of subsequently. Also, $\partial \Delta P_u(u, 0)$ and $\partial \Delta P_v(u, 0)$ denote the linearization of ΔP, respectively, in response to a perturbation in u and v around the trajectory u and $v = 0$. Therefore

$$\begin{aligned} y_p = {} & [\partial P_u(\theta_0, u, 0) + \partial \Delta P_u(u, 0)]\, u_p \\ & + [\partial P_v(\theta_0, u, 0) + \partial \Delta P_v(u, 0)]\, v + \Delta P(u, 0) \quad (4.11)\\ u_p = {} & -\partial C_y(r, y)\, y_p \quad (4.12)\end{aligned}$$

and combining (4.11) and (4.12) one gets

$$y_p = \widetilde{P}_{CL}^{-1}[(\partial P_v(\theta_0, u, 0) + \partial \Delta P_v(u, 0))\, v + \Delta P(u, 0)]. \quad (4.13)$$

where

$$\widetilde{P}_{CL}^{-1} = [I + (\partial P_u(\theta_0, u, 0) + \partial \Delta P_u(u, 0))\, \partial C_y(r, y)]^{-1}$$

is assumed to be a BIBO (asymptotically) stable I/O operator.

On the other hand the neglected terms in the developments leading to (3.10) for the closed-loop output error and (4.13) for the perturbation term have also to be taken into account. Therefore the equation of the closed-loop output error will take the form

$$\varepsilon_{CL} = P_{CL}^{-1}\, \phi(t, \theta)^T\, [\theta_0 - \theta(t)] + w(t) \quad (4.14)$$

where w reflects the perturbation due to the unmodeled part of the plant and the possible bounded output disturbances, i.e. y_p, and the effect of the high order terms in all Taylor series expansions.

One has the following result

Theorem 4.1 *Assume that the closed-loop output error is described by:*

$$\varepsilon_{CL} = H\, \phi^T(t)\, (\theta_0 - \theta(t)) + w(t) \quad (4.15)$$

where $w(t)$ represents the combined effect of unmodeled dynamics, bounded disturbances and of the high order term in the Taylor expansions around the nominal trajectories. Here, H and ϕ depend on the algorithm used.

- *Assume that H is a linear time-varying operator.*
- *Assume that the true closed-loop system is stable.*
- *Assume that $C(y, r)$, $\partial C_y(r, y)$, ΔP and $\widetilde{P}_{CL}^{-1}$ are BIBO operators.*
- *Assume that the P.A.A. of (2.8), (2.9) with $\lambda_1(t) \equiv 1$ and $\phi^T(t) = P'(\theta, u(\theta))$ is used.*
- *Assume that the external excitation $r(t)$ and the equivalent disturbance $w(t)$ are norm bounded, i.e.*

$$\begin{aligned} \lim_{t\to\infty} \int_{\tau=0}^{t} r^2(\tau)\, d\tau &\le \alpha^2; \quad \alpha^2 < \infty, && (4.16)\\ \lim_{t\to\infty} \int_{\tau=0}^{t} w^2(\tau)\, d\tau &\le \beta^2; \quad \beta^2 < \infty. && (4.17)\end{aligned}$$

Then the closed-loop output error $\varepsilon_{CL}(t)$, the predicted output $y(\theta, t)$ and the predicted input $u(\theta, t)$ are norm bounded if

$$\bar{H} = H - \frac{\lambda(t)}{2} I; \quad \lambda(t) > \lambda_2(t)\ \forall t \quad (4.18)$$

is a strongly strictly passive linear time-varying operator.

In fact this theorem of which a proof can be found in [4] says that even when one uses simplified nonlinear models, provided that the error between the true plant and a nominal reduced model is small in some sense, the boundedness of the signals is assured by the passivity conditions of Theorem 3.1, now evaluated for the nominal reduced model.

Remark 2:

- Suppose that $\Delta P(u,v) = 0$ for simplicity, i.e. the system can be modeled exactly. Then (4.13) reduces to

$$y_p = [I + \partial P_u(\theta_0, u, 0)\partial C_y(r,y)]^{-1}\,\partial P_v(\theta_0, u, 0)\,v. \quad (4.19)$$

  Note that if the noise is additive, $\partial P_v(\theta_0, u, 0) = 1$ in the equation above.

- It follows from (4.13) that $w(t)$ depends on u and y and it results that both $w(t)$ and $\phi(t,\theta)$ depend on the reference signal r. This shows that $w(t)$ and $\phi(t,\theta)$ are not independent and this causes the NLCLOE algorithm to produce biased estimates.

- The situation is different in the linear case where a consistent estimate is obtained when the system is in the model set and the reference and noise signal are independent; see e.g. [5]. Indeed, it follows that (4.19) reduces to

$$y_p = (I + PC_y)^{-1}\,v \quad (4.20)$$

  which is independent of the reference signal r. In the linear case and with the system in the model set, $w = y_p$ is therefore independent of $\phi(t,\theta)$.

5 Conclusion

The key contribution of this paper has been to show that the framework for a number of closed-loop output error identification algorithms can be pushed out from linear systems to nonlinear systems. One cannot ask for the impossible, and in nonlinear systems, the impossible usually corresponds to wanting a result when linearizations of the true and the estimated systems are very different. Hence our results, not surprisingly, for the most part assume that the high order terms can be neglected in certain Taylor series expansions, or we assume that they are at least small. Other than that, both the noisy and noiseless case are captured, as is the possibility that the true plant may not lie in the model set.

Acknowledgement: The second and third authors wish to acknowledge the funding of the US Army Research Office, Far East and the Office of Naval Research, Washington.

A Appendix

Passive operators play an important role both in deriving the algorithms and analyzing their properties. In particular the concept of a ***strongly strictly passive*** system is very useful.

Consider the system

$$y = Hu \quad (A.1)$$

and assume that it accepts a state space representation

$$\dot{x} = f(x,u,t) \quad (A.2)$$
$$y = h(x,t) \quad (A.3)$$

with $x \in \mathbf{R}^n$, $y \in \mathbf{R}^m$, $u \in \mathbf{R}^m$, f, h continuous in t and smooth in x. Suppose $f(0,0,t) = 0$ and $h(0,t) = 0$ for all $t \geq 0$.

Definition A.1 *The system H is said to be* ***strongly strictly passive*** *if there exist a positive definite (storage) function $V(x,t)$ which satisfies*

$$\gamma_1(|x|) \leq V(x,t) \leq \gamma_2(|x|) \quad (A.4)$$
$$V(0,t) = 0, \quad \forall t \geq 0 \quad (A.5)$$

where $\gamma_1(|x|)$ and $\gamma_2(|x|)$ are class $\mathcal{K}_\infty$ functions, and there exists a positive definite function (dissipation rate) $\psi(x) \geq \gamma_3(|x|)$; $\gamma_3(\cdot) \in \mathcal{K}_\infty$ such that

$$\int_{t_0}^{t} y^T(\tau)u(\tau)\,d\tau \;\geq\; V(x(t),t) - V(x(t_0),t_0) + \int_{t_0}^{t}\psi(x(\tau))\,d\tau \quad \forall t, t_0 \text{ with } t \geq t_0. \quad (A.6)$$

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