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# IMPLEMENTATION ISSUES FOR A NONLINEAR VERSION OF THE HANSEN SCHEME*

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## Abstract

In this paper, we propose a method for the identification of a nonlinear plant under possibly nonlinear feedback. This procedure is a nonlinear extension of a method known as the Hansen scheme in the literature. It is shown that using nonlinear left fractional descriptions one can convert a general nonlinear closed-loop identification problem to one of open-loop identification by parametrizing the model using a Youla-Kucera parameter. The open-loop problem can be implemented by parametrizing the nonlinear Youla parameter in terms of a model of the plant. We provide gradient expressions for implementation in a steepest descent algorithm.

**Keywords:** estimation, closed-loop identification, nonlinear system, left coprime fractional representation.

## 1 Introduction

It has been shown in [1] that the set of all nonlinear plants stabilised by a known linear controller, which also stabilises a linear nominal model of the plant, can be parametrised by a stable operator known as the Youla-Kucera parameter.

By utilising this description it is possible to convert the closed-loop plant identification problem to one of open-loop identification. This paper extends previous work by allowing the model of the nominal plant and the controller in the above scenario to be nonlinear. The ideas rely on a concept of differential coprimeness for nonlinear fractional system descriptions.

We consider the setting shown in Figure 1.1, where $P$ is a nonlinear plant to be identified, $C$ is a nonlinear controller, and $H$ is a linear stable output measurement noise generating system, driven in turn by the zero mean, white, stationary noise process $e$. It is assumed that $C$ internally stabilises the unknown plant $P$. While we restrict attention to time-invariant $C$ and $P$, there would seem to be no difficulty in extending the ideas to the time-varying case, as in [1].

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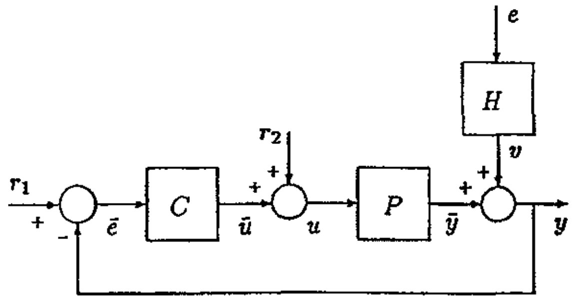


Figure 1.1: The closed-loop system.

We also show how the open-loop identification problem can actually be implemented by parametrizing the Youla-Kucera parameter in terms of a model of the plant. We provide gradient expressions that can be used to implement a steepest descent algorithm.

The theoretical results are backed up by a simulation with a plant that has a nonlinear input backlash followed by linear dynamics. This simulation identifies a model from data collected while the plant is operating in closed-loop with a linear controller.

The construction of a nonlinear closed-loop identification scheme is the first building block towards a nonlinear extension of an iterative identification and control procedure known as the "windsurfer" approach in the literature; see [5] for further details.

## 2 Preliminaries

**Right coprimeness (linear or nonlinear)** let $N_r$, $D_r$ be a right factorisation for a well-posed $P_0$, i.e. $P_0 = N_r D_r^{-1}$ where $N_r$ and $D_r$ are BIBO stable. Then $(N_r, D_r)$ is a right coprime factorisation of $P_0$ if there exists a BIBO operator $\mathcal{L}_l$ for which

$$\mathcal{L}_l \begin{bmatrix} N_r \\ D_r \end{bmatrix} = I. \tag{2.1}$$

Here $I$ denotes the identity operator.

**Left coprimeness (linear or nonlinear):** let $N_l$, $D_l$ be a left factorisation for a well-posed $P_0$, i.e. $P_0 = D_l^{-1} N_l$, where $N_l$ and $D_l$ are BIBO stable. Then $(N_l, D_l)$ is a left coprime factorisation of $P_0$ if there exists a BIBO

$\mathcal{L}_r$ for which

$$[N_l \; D_l]\, \mathcal{L}_r = I. \tag{2.2}$$

Again, (2.2) is a Bezout identity for left coprimeness. The existence of a left coprime factorisation is not always guaranteed; we refer to Remark 3.1 below.

**Differential coprimeness:** if the pair $(N_l, D_l)$ is left coprime and globally Lipschitz continuous, then we can write

$$N_l U_r + D_l V_r = W \tag{2.3}$$

where $N_l$, $D_l$, $U_r$ and $V_r$ are all nonlinear and $W$ is a unit.

Then, one can define well-posed operators

$\partial N_{l(x)}(.) = N_l(x+.) - N_l(x)$ and $\partial D_{l(z)}(.) = D_l(z+.) - D_l(z)$

for all signals $x$ and $z \in L_{2e}$. If $N_l$ and $D_l$ are linear, $\partial N_{l(x)}(.) = N_l(.)\ \forall x$ and $\partial D_{l(z)}(.) = D_l(.)\ \forall z$; then $(\partial N_{l(x)})U_r + (\partial D_{l(z)})V_r$ is a unit, i.e. $W$.

When $N_l$ and $D_l$ are nonlinear, we shall say that they are differentially coprime if and only if the unit property continues to hold, though now the unit will not usually be $W$. Formally, $N_l$ and $D_l$ are left differentially coprime if for all $x$, $z \in L_{2e}$, there exists BIBO $U_r$ and $V_r$ such that

$$(\partial N_{l(x)})U_r + (\partial D_{l(z)})V_r = W_{x,z},$$

where $W_{x,z}$ is a unit operator.

**Remark 2.1** We say that $N_l$ and $D_l$ are *uniformly* left differentially coprime if there exists $K$ such that $\|W_{x,z}\| < K$ and $\|W_{x,z}^{-1}\| < K$ independently of $x$ and $z \in L_{2e}$.

**Remark 2.2** Note that if $N_l$ and $D_l$ are known to be left differentially coprime in the sense that for some bounded $U_r$ and $V_r$, $\partial N_{l(x)}U_r + \partial D_{l(z)}V_r$ is a unit for any $x$ and $z$, then by taking $x = 0$, $z = 0$ we recover the standard coprimeness relation

$$N_l U_r + D_l V_r = W \text{ for some unit } W.$$

## 3 Assumptions

In the results which follow, we shall invoke the following assumptions. In the sequel, $P$ is the real system to be identified and $P_0$ is a nominal model for $P$.

**Assumption 3.1** *The nonlinear plant $P$ is weakly Lipschitz and well-posed.*

**Assumption 3.2** *(i) The controller $C$ is weakly Lipschitz and there exists $U_l$, $V_l$, $U_r$, $V_r$ all stable, well-posed operators with $V_l$, $V_r$ invertible such that*

$$C = U_r V_r^{-1} = V_l^{-1} U_l. \tag{3.4}$$

*$(U_r, V_r)$ are right coprime factors of the controller that are differentially coprime and globally Lipschitz continuous, and $(U_l, V_l)$ are left coprime factors of the controller that are globally Lipschitz continuous and* uniformly *differentially coprime.*

*(ii) The nominal plant model $P_0$ is smoothing, and there exists $N_l$, $D_l$, $N_r$, $D_r$ all stable, well-posed operators with $D_l$, $D_r$ invertible such that*

$$P_0 = N_r D_r^{-1} = D_l^{-1} N_l. \tag{3.5}$$

*$(N_r, D_r)$ are right coprime factors of $P_0$ that are differentially coprime and globally Lipschitz continuous, and $(N_l, D_l)$ are left coprime factors of $P_0$ that are globally Lipschitz continuous and* uniformly *differentially coprime.*

*It is further assumed that $N_l$ and $U_l$ are smoothing and $D_l$ and $V_l$ are of the form $aI + S$ where $aI$ is the scaled identity operator and $S$ is a smoothing operator.*

**Remark 3.1** Assumption 3.2 can be quite restrictive; especially the requirement of existence of left coprime factorisations for the nominal plant and the controller. The concept of kernel representation introduced in [7] might prove to be useful to alleviate these restrictions. In a sense, left coprime realizations are a special case of kernel representations

**Assumption 3.3** *The controller $C$ stabilises the nominal plant model $P_0$.*

## 4 Characterisation and Identification of nonlinear plants using a left coprime factor based description

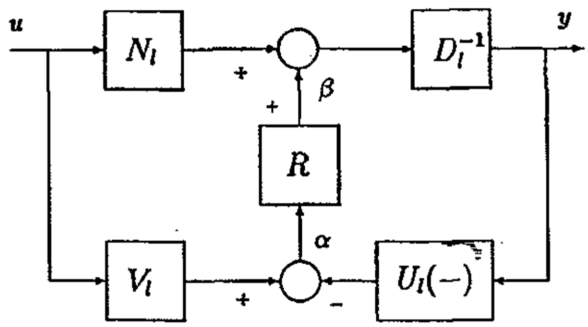


Figure 4.1: Left coprime factorisation based description of $P$.

This section shows that all nonlinear plants stabilised by a nonlinear controller $C$ can be represented by the setting depicted in Figure 4.1, with $R$ a nonlinear BIBO, smoothing, well-posed operator known as the Youla-Kucera parameter. Conversely, if the setup of Figure 4.1 defines a well-posed, smoothing $P$ for some BIBO, smoothing, well-posed $R$, then $P$ is stabilised by $C$.

The theorems of Subsection 4.1 argue that the representation of Figure 4.1 depicts the set of all plants stabilised by a given controller and hence shows how the closed-loop identification problem can be converted to an open-loop problem in the presence of noise. Subsection 4.2 is broken into two parts. The first part treats the noiseless situation, i.e. $v = 0$. The second part treats the case when the noise is no longer zero, and it describes how the disturbance can be incorporated into the identification algorithm. A formal proof of these theorems can be found in [6].

### 4.1 Describing the structure of the set of all plants stabilised by a given controller in a noise free setting.

**Lemma 4.1** *Adopt the assumptions in Section 3. Suppose that $R$ is a well-posed, bounded operator. Then if $R$ is smoothing, $P$ is smoothing. Also the closed-loop of Figure 4.2 is well-posed and internally stable.*

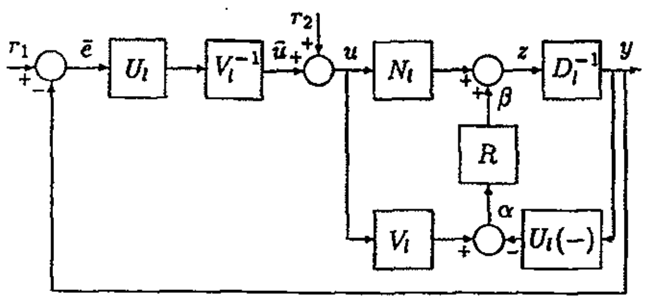


Figure 4.2: Closed-loop of Figure 1.1 with plant $P$ as in Figure 4.1 and $v = 0$.

The converse result is as follows.

**Lemma 4.2** *Adopt the assumptions in Section 3 and suppose the closed-loop in Figure 1.1 is well-posed and internally stable. Then there exists a well-posed, bounded $R$ given by*

$$R = (D_l P - N_l)(V_l - U_l(-P))^{-1}, \tag{4.1}$$

*such that in Figure 4.1*

$$y = P u.$$

*Further, if $P$ is smoothing then $R$ is smoothing.*

In summary, we now have

**Theorem 4.1** *Adopt the assumptions in Section 3. Then the closed-loop in Figure 1.1 is well-posed and internally stable if and only if $P$ has a description of the form of Figure 4.1, with $R$ a well-posed, stable, smoothing operator. Further $P$ is smoothing if and only if $R$ is smoothing.*

**Proof.** Theorem 4.1 follows from Lemmas 4.1 and 4.2. ∎

### 4.2 Conversion to open-loop identification and incorporation of measurement noise

This subsection demonstrates how the measurement noise can be incorporated in order to enable identification. The conversion to open-loop identification requires a small noise assumption (high SNR) so that $R$ may be linearised around its operating trajectory. As in [1], it is shown that instead of identifying the plant $P$, we can identify the Youla-Kucera parameter, $R$.

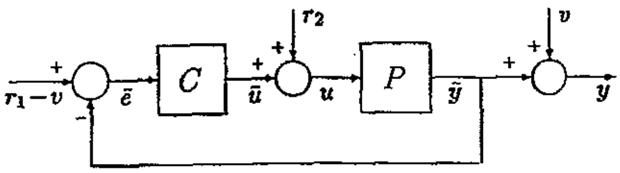


Figure 4.3: Rearrangement of the closed-loop system of Figure 1.1.

Refer first to Figure 4.2. In the noise free case, i.e. with $v = 0$, there holds

$$\alpha = V_l\, u - U_l(-y) = \partial U_{l(-y)}\, r_1 + \partial V_{l(\bar{u})}\, r_2.$$

Also, if $\beta = R\alpha$, then

$$\beta = -N_l\, u + D_l\, y. \tag{4.2}$$

Stability of the closed-loop ensures that $\alpha$ and $\beta$ are (in principle) computable (boundedly) from $r_1$, $r_2$ and $u$, $y$ respectively. It is now possible to identify $R$ in a standard open-loop fashion. The next paragraph examines how to take measurement noise into account.

**a) Incorporation of measurement noise**

When $v \neq 0$, similar equations hold provided we replace $r_1$ and $y$ by $r_1 - v$ and $y - v$ respectively in determining the input and output of $R$. This can be seen by examining Figure 4.3. Put another way, we now have

$$\beta_v = R\,\alpha_v$$

with

$$\begin{aligned}
\alpha_v &= V_l\, u - U_l(-y + v) = V_l(r_2 + \bar{u}) - U_l(-y + v) \\
&= V_l\, \bar{u} + \partial V_{l(\bar{u})}\, r_2 - U_l(-y) - \partial U_{l(-y)}\, v \\
&= U_l\,[(r_1 - y)] + \partial V_{l(\bar{u})}\, r_2 - U_l(-y) - \partial U_{l(-y)}\, v \\
&= \partial U_{l(-y)}\, r_1 + \partial V_{l(\bar{u})}\, r_2 - \partial U_{l(-y)}\, v \\
&= \alpha - \partial U_{l(-y)}\, v
\end{aligned} \tag{4.3}$$

and

$$\begin{aligned}
\beta_v &= -N_l\, u + D_l(y - v) = -N_l\, u + D_l\, y + \partial D_{l(y)}(-v) \\
&= \beta + \partial D_{l(y)}(-v).
\end{aligned} \tag{4.4}$$

As before, $\alpha$ and $\beta$ are given by (4.2) and (4.2) and are effectively measurable. That is, the closed-loop identification problem has been transformed into a nonstandard

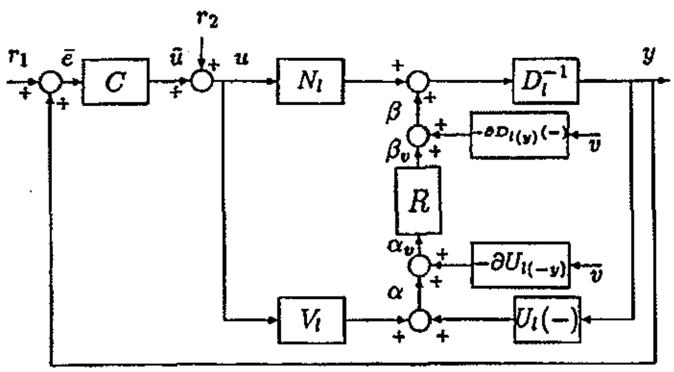


Figure 4.4: Incorporation of the noise in the left coprime factorisation based description and conversion to a non-standard open-loop identification problem.

open-loop identification problem. From $\beta_v = R\alpha_v$ we now obtain Figure 4.4.

As in [1] where the conversion process is considered for a nonlinear plant with a linear nominal plant model and a linear controller, the noise enters the structure in two places. This is opposed to the case where the plant, nominal plant model and controller are all linear. In such a case the noise enters in only one place.

**b) Conversion to a standard open-loop identification problem.**
Again assuming a high SNR, there exists a linearisation $\Delta R$ of $R$ around the trajectory produced by the input signal $\alpha$ which yields

$$\begin{aligned}\beta &= R\alpha + \Delta R(-\partial U_{l(-y)}\, v) - \partial D_{l(y)}(-v)\\ &= R\alpha - (\Delta R \partial U_{l(-y)} + \partial D_{l(y)}(-))\, v.\end{aligned}$$

The closed-loop identification problem has been transformed into a standard nonlinear open-loop identification problem as shown in Figure 4.5. This method requires both reference signals to be non-zero; see [1] for further details.

## 5 Implementation issues

Conventionally, at this point, a set $\mathcal{R}$ of permissible $R(\theta)$ is defined. As an alternative, we propose postulating a simple parametrization via

$$\mathcal{M} : y(\theta) = P(\theta, u). \tag{5.1}$$

It is of primary importance that the identified model is stabilized by the known controller $C$, i.e. we assume that the closed-loop system $[P(\theta)\ \ C]$ is uniformly stable over all $\theta \in D_\theta$. In a nonlinear context, the reparametrization of the Youla parameter using a plant model is appealing

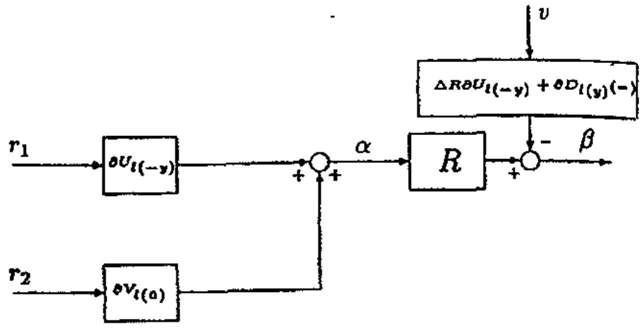


Figure 4.5: Conversion to a standard open-loop identification problem.

because it is difficult to find an appropriate model structure for the Youla parameter itself. It now follows from (4.1) that

$$\beta(\theta) = (D_l P(\theta) - N_l)(V_l - U_l(-P(\theta)))^{-1}\,\alpha. \tag{5.2}$$

This is illustrated in the upper part of Figure 5.1.

### 5.1 Generation of the gradient $\beta'(\theta)$

Let us first consider the following equations

$$\begin{aligned}\beta(\theta) &= D_l(P(\theta, z(\theta))) - N_l(z(\theta)), &(5.3)\\ z(\theta) &= V_l^{-1}(\alpha + U_l(-P(\theta, z(\theta))). &(5.4)\end{aligned}$$

The gradients of these two signals w.r.t the j-th entry of $\theta$ are, respectively, denoted by $\beta'_{\theta j}(\theta)$ and $z'_{\theta j}(\theta)$. They are the j-th component of the vectors $\beta(\theta)'$ and $z(\theta)'$ and they satisfy, for $j = 1, \cdots, n$,

$$\begin{aligned}\beta'_{\theta j}(\theta) &= \Delta D_l[P'_{\theta j}(\theta, z(\theta)) + \Delta P\, z'_{\theta j}(\theta)]\\ &\qquad -\Delta N_l\, z'_{\theta j}(\theta), &(5.5)\\ z'_{\theta j}(\theta) &= -\Delta V_l^{-1}\Delta U_l[P'_{\theta j}(\theta, z(\theta)) + \Delta P\, z'_{\theta j}(\theta)]. &(5.6)\end{aligned}$$

Here

$$\begin{aligned}\Delta D_l &= \Delta D_l(P(\theta, z(\theta))),\quad \Delta P = \Delta P(\theta, z(\theta)),\\ \Delta N_l &= \Delta N_l(z(\theta)),\quad \Delta U_l = \Delta U_l(-P(\theta, z(\theta))),\\ \Delta V_l^{-1} &= \Delta V_l^{-1}(\alpha + U_l(-P(\theta, z(\theta)))\end{aligned}$$

where $\Delta X(x)$ denotes the linearization of $X$ around its operating trajectory $x$. We refer to [3] for a full treatment of the linearization problem. It now follows that $\beta'_{\theta j}(\theta)$ can be obtained from Figure 5.1, at least for a stable $P(\theta)$. We refer to [4] for a stable implementation of the gradient when $P(\theta)$ is unstable. A similar procedure had already been proposed in [2] in a linear context.

### 5.2 Identification criterion

In the sequel, we restrict attention to Single-Input-Single-Output systems. The restriction to scalar systems is inessential but notationally convenient. Here, we make use of the identification criterion

$$V_N(\theta) = \frac{1}{2N}\sum_{t=1}^{N} \epsilon(\theta)^2 \tag{5.7}$$

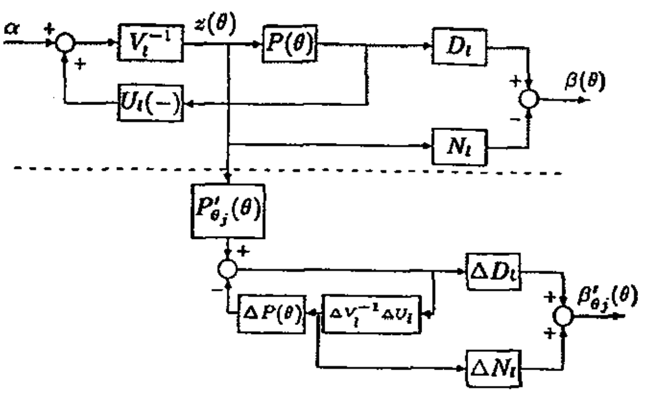


Figure 5.1: Generation of the gradient $\beta'_{\theta j}(\theta)$

where the prediction error is given by $\epsilon(\theta) = L[\beta - \beta(\theta)]$. Here $L$ can be any stable design data filter. To facilitate notations, we assume that $L = 1$. To minimize (5.7) with respect to the model parameter vector $\theta$, it is standard that one can iteratively seek a solution for $\theta$ to

$$V'_N(\theta) = -\frac{1}{N}\sum_{t=1}^{N}[\beta - \beta(\theta)]\,\beta'(\theta) = 0 \qquad (5.8)$$

by taking steps in the negative gradient direction

$$\theta[i+1] = \theta[i] - \gamma_i\, H_i^{-1}\, V'_N(\theta[i]) \qquad (5.9)$$

where $H_i$ is some appropriate positive definite matrix, typically an estimate of the Hessian of $V_N$. A good (but biased estimate) of the Hessian is obtaining using the following expression

$$H_i = \frac{1}{N}\sum_{t=1}^{N}\beta'(\theta[i])\,[\beta'(\theta[i])]^T. \qquad (5.10)$$

It is assumed earlier that closed-loop system $[P(\theta)\ \ C]$ is uniformly stable over all $\theta \in D_\theta$, i.e. it is assumed that stability of the predictor is preserved while iterating. This is a reasonable assumption since the step size $\gamma_i$ can be used effectively to control how much the model is allowed to change per iteration.

## 6 Numerical example

This section contains a discussion of simulations performed using the Hansen method documented in Section 4. We have chosen to illustrate this method with a plant that has a nonlinear input backlash followed by linear dynamics. This simulation identifies a nonlinear plant ($P_{NL}$) connected in closed-loop with a stabilising controller ($C$). This controller also stabilises a *linear* nominal model of the plant ($P_0$). This simulation was implemented in discrete-time.

The nonlinear plant is described by

$$y_t = P\,u_t + v_t = \frac{b}{z+a}\,\phi\, u_t + v_t \qquad (6.1)$$

where the disturbance signal $v_t$ is a zero mean white noise signal of variance $\sigma^2$ and $\phi$ is a nonlinear backlash operator defined using the following equations.

$$p_t = \phi(q_t) = \begin{cases} q_t - \frac{w}{2} & \text{if } q_t > s_{t-1}, \\ q_t + \frac{w}{2} & \text{if } q_t < s_{t-1} - w, \\ p_{t-1} & \text{if } s_{t-1} - w < q_t < s_{t-1} \end{cases} \qquad (6.2)$$

$$s_t = \begin{cases} q_t & \text{if } q_t > s_{t-1}, \\ q_t + w & \text{if } q_t < s_{t-1} - w, \\ s_{t-1} & \text{if } s_{t-1} - w < q_t < s_{t-1} \end{cases} \qquad (6.3)$$

Here $w$ is the width of the backlash deadzone and $s_t$ is a setpoint that is used because a backlash element has memory. We refer to [8] for further details.

We have taken the following plant parameters $a = 0.2$, $b = 0.5$ and $w = 0.1$. The stabilising controller $C$ is the one degree of freedom linear controller

$$u_t = -y_t + r_t, \qquad (6.4)$$

i.e. in Figure 1.1, $r_1 = \frac{r}{2}$, $r_2 = \frac{r}{2}$ and $C = I$. Thus, possible left coprime factors of the controller are $V_l = I$ and $U_l = I$, i.e.

$$u_t = -V_l^{-1}\,U_l\,y_t + r_t. \qquad (6.5)$$

The nominal plant model has a left coprime factorisation given by

$$P_0 = D_l^{-1} N_l \qquad (6.6)$$

where

$$D_l = \frac{z+0.1}{z+0.7}, \quad N_l = \frac{0.6}{z+0.7}. \qquad (6.7)$$

The choice (6.5) satisfies the Bezout identity $N_l U_l + D_l V_l = I$ which implies that $P_0$ is stabilised by $C$. We construct the plant in terms of left coprime factors of the linear controller, the *linear* nominal plant model, and a nonlinear operator known as the Youla-Kucera parameter. Identifying this parameter is equivalent to identifying the plant with the advantage that it can be written as an open-loop identification problem as shown in Section 4. Recall from Figure 4.1, that one can calculate $\alpha$ and $\beta$ from the data collected on the plant. These signals are of interest in the open-loop identification of the Youla-Kucera parameter. They are given by

$$\alpha = (U_l\, r_1 + V_l\, r_2), \qquad (6.8)$$

$$\beta \ = -N_l\, u + D_l\, y \qquad (6.9)$$

where $\beta = R\alpha + \Delta R(-\partial U_{l(-y)} v) - \partial D_{l(y)}(-v)$ in a high SNR situation. The reference signals $r_1 = r_2 = \frac{r}{2}$ were chosen to be known filtered unit variance and zero mean white noise signals independent of the process disturbance signal $v$. Note that this corresponds to an input signal $u$ that is of similar magnitude to the backlash width, $w$. With an input signal of much greater magnitude than $w$ this quantity would be hard to identify, the effect of the nonlinearity being swamped by the signal; if $u$ is of smaller magnitude there would also be a problem.

We have used a reparametrization of the Youla-Kucera parameter using the model structure

$$P(\theta) = \frac{\hat{b}}{z+\hat{a}}\,\hat{\phi}(\hat{w}) \qquad (6.10)$$

where $\hat{\phi}(\hat{w})$ is the backlash defined in (6.2) with $w$ replaced by $\hat{w}$. Here the parameter vector is $\theta = [\hat{a}, \hat{b}, \hat{w}]$, i.e. three parameters are identified. The $j$th component of the derivative $\beta'(\theta)$ for $j = 1, 2, 3$ can be obtained from (5.5)-(5.6) or, equivalently, using

$$\beta'_{\theta j}(\theta) = -N_l\, z'_{\theta j}(\theta) + D_l(P(\theta)\, z'_{\theta j}(\theta) + P'_{\theta j}(\theta)\, z(\theta)),$$
$$z'_{\theta j}(\theta) = -C(P(\theta)\, z'_{\theta j}(\theta) + P'_{\theta j}(\theta)\, z(\theta))$$

where

$$P'_{\hat{a}}(\theta) = \frac{-\hat{b}}{(z+\hat{a})^2}\,\hat{\phi},\; P'_{\hat{b}}(\theta) = \frac{1}{z+\hat{a}}\,\hat{\phi},\; P'_{\hat{w}}(\theta) = \frac{\hat{b}}{z+\hat{a}}\,\frac{\delta\hat{\phi}}{\delta\hat{w}}$$

with

$$\frac{\delta\hat{\phi}}{\delta\hat{w}} = \begin{cases} -0.5 & \text{if } q_t > s_{t-1}, \\ 0.5 & \text{if } q_t < s_{t-1} - w, \\ 0 & \text{if } s_{t-1} - w < q_t < s_{t-1} \end{cases} \qquad (6.11)$$

Using the previous closed-loop system, we have generated a data set $\{r_t,\ u_t,\ y_t\}$ with signals of length $N = 2000$. We started with initial parameter estimates

$$\theta[0] = [\hat{a}[0]\ \hat{b}[0]\ \hat{w}[0]] = [0.1\ 0.4\ 0]. \qquad (6.12)$$

When the simulation was run in a noise free situation the parameter estimates converged to the true values. Due to the type of nonlinearity implemented, the identification process was very sensitive to noise. Figure 6.1 shows the results when there is no noise ($\sigma^2 = 0$) and when the noise level is increased slightly ($\sigma^2 = 0.000005$). (A typical output level was 0.04, and so the SNR was approximately 24dB). The accuracy of the final parameter estimates decreases with $\sigma^2$ increasing (SNR decreasing).

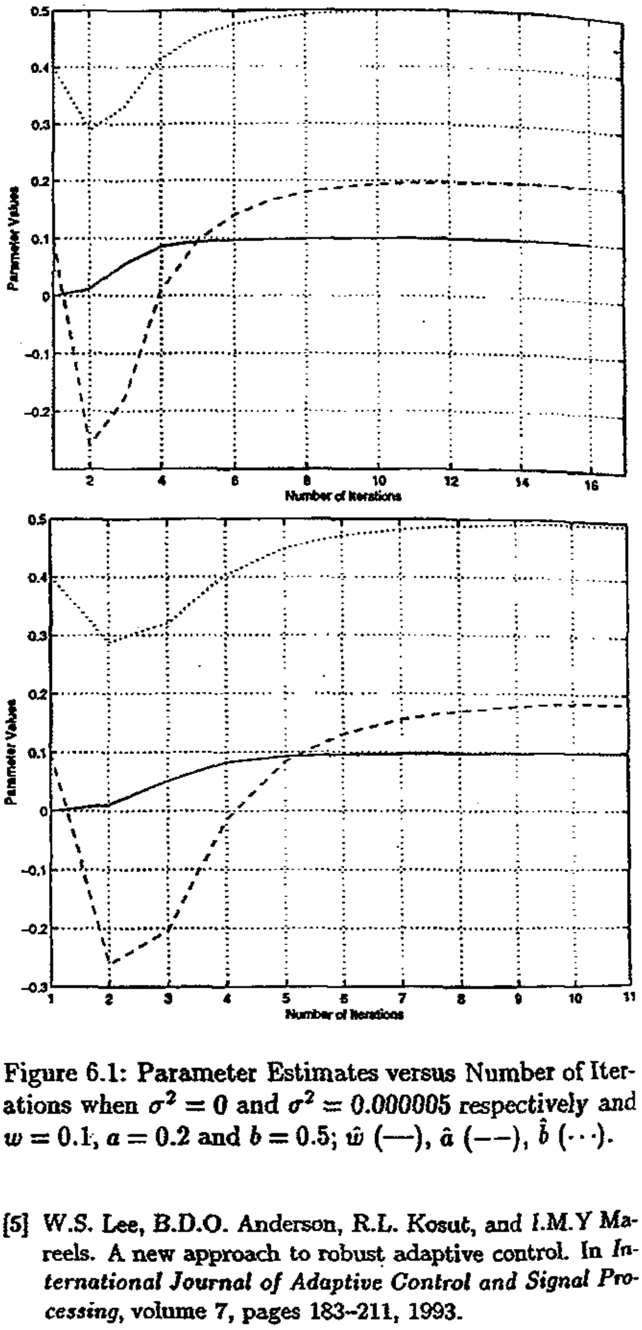


Figure 6.1: Parameter Estimates versus Number of Iterations when $\sigma^2 = 0$ and $\sigma^2 = 0.000005$ respectively and $w = 0.1$, $a = 0.2$ and $b = 0.5$; $\hat{w}$ (—), $\hat{a}$ (– –), $\hat{b}$ (···).

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