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Proceedings of the 37th IEEE
Conference on Decision & Control
Tampa, Florida USA • December 1998

THE HANSEN SCHEME REVISITED

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Abstract

In this paper, we present a modification of a closed-loop identification scheme known as the "Hansen scheme". The advantage of the alternative procedure is that the order of the resulting model is tunable by the user.

1 Introduction

Consider the setting shown in Figure 1.1, where P is a linear plant to be identified, C is a known linear controller and v is disturbance signal that can be modeled as filtered zero mean white noise. It is assumed here that the controller C internally stabilizes the unknown system P.

There are two major problems associated with closed-loop identification. The first one is that the measurement noise is correlated with the input signal, and what is more this correlation, being dependent on the unknown plant, cannot be determined a priori. The second problem is that closed-loop identification is hampered by the need to unravel the closed-loop operator to obtain the model. Even when the plant and the controller are linear, the plant appears in a nonlinear fashion in the closed-loop quantities.

Most closed-loop identification techniques have in common the ability to identify approximate models of the open-loop plant on the basis of closed-loop data, while the asymptotic bias distribution of the estimated plant transfer function at each frequency remains independent of the noise and is thus explicitly tunable by the user. This result is obtained by turning the closed-loop identification problem into an open-loop-like identification problem. One such closed-loop identification method is known as the "Hansen scheme"; see [2]. It builds on the Youla parametrization of all plants that are stabilized by a given controller. A major drawback of this method is that the order of the resulting model is not simply tunable due to the required re-parametrization. In this paper, we present a modified scheme that does not exhibit this drawback while maintaining the advantages of the method. We show that the procedure presented in this paper shows some similarities with closed-loop identification with a tailor-made parametrization studied in [1, 7]. In Section 2, we recall the Hansen closed-loop identification procedure and we present the modified method in Section 3. Section 4 offers conclusions.

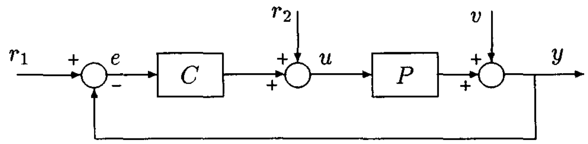

Figure 1.1: The identification setup: stabilized feedback loop showing the plant P and the controller C.

2 The Hansen scheme for closed-loop identification

The basic idea behind this method is introduced by Hansen in [2] in view of closed-loop experiment design. It uses the dual Youla parametrization of all linear time invariant (LTI) plants that are stabilized by a given known controller. In order to describe this method, we need the following concepts.

Proposition 2.1 [8] *Let P_0 and C have factorizations $P_0 = D_{P_l}^{-1} N_{P_l} = N_{P_r} D_{P_r}^{-1}$ and $C = D_{C_l}^{-1} N_{C_l} = N_{C_r} D_{C_r}^{-1}$, where $N_{P_l}, D_{P_l}, N_{P_r}, D_{P_r}, N_{C_l}, D_{C_l}, N_{C_r}$ and D_{C_r} belong to $\mathbf{M}(\mathbf{S})$, the set of matrices with elements in $\mathbf{S}$, the ring of proper stable transfer functions. Assume that the following Bezout equation holds*

$$\begin{aligned} \begin{bmatrix} D_{C_l} & N_{C_l} \\ -N_{P_l} & D_{P_l} \end{bmatrix} \begin{bmatrix} D_{P_r} & -N_{C_r} \\ N_{P_r} & D_{C_r} \end{bmatrix} &= \begin{bmatrix} D_{P_r} & -N_{C_r} \\ N_{P_r} & D_{C_r} \end{bmatrix} \begin{bmatrix} D_{C_l} & N_{C_l} \\ -N_{P_l} & D_{P_l} \end{bmatrix} && (2.1) \\ &= \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. && (2.2) \end{aligned}$$

This equation expresses both the fact that the factors are coprime and that the feedback loop formed by the plant P_0 and the controller C is internally stable. For any $R \in \mathbf{M}(\mathbf{S})$, define

$$\begin{aligned} P(R) &= (D_{P_l} - RN_{C_l})^{-1}(N_{P_l} + RD_{C_l}) && (2.3) \\ &= (N_{P_r} + D_{C_r}R)(D_{P_r} - N_{C_r}R)^{-1}. && (2.4) \end{aligned}$$

- *Then $P(R)$ is stabilized by C.*
- *Furthermore,* **any** *LTI plant stabilized by C has a left and right fractional representation (2.3) and (2.4) for some $R \in \mathbf{M}(\mathbf{S})$.* ■

The above proposition shows a parametrization of the class of all LTI plants that are stabilized by the given controller C. Note that P_0 is just any nominal auxiliary system that is stabilized by C. The interpretation is that P_0 is a known but imperfect model of the true plant P which is also stabilized by C. Applying Proposition 2.1 as shown in [2], there exists $R \in \mathbf{M}(\mathbf{S})$ such that the feedback system of Figure 1.1 can be recast as shown in Figures 2.2 and 2.3.

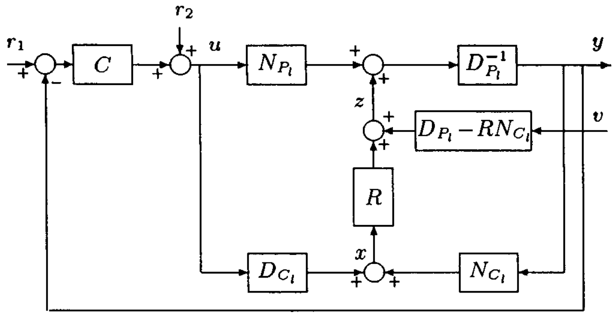

Figure 2.2: Alternative representation of the closed-loop system of Figure 1.1 using left coprime factors.

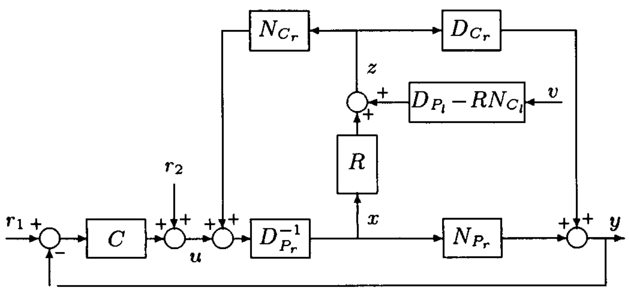

Figure 2.3: Alternative representation of the closed-loop system of Figure 1.1 using right coprime factors.

Defining the signals $x(t)$ and $z(t)$ as indicated in Figures 2.2 and 2.3, it follows that

$$x(t) = D_{C_l}\, u(t) + N_{C_l}\, y(t) = N_{C_l}\, r_1(t) + D_{C_l}\, r_2(t), \quad (2.5)$$

$$z(t) = D_{P_l}\, y(t) - N_{P_l}\, u(t) \quad (2.6)$$

and

$$z(t) = R\,x(t) + (D_{P_l} - RN_{C_l})\, v(t). \quad (2.7)$$

We can make the following observations:

- The signals $x(t)$ and $z(t)$ can be reconstructed from the data through (2.5) and (2.6), provided that the controller C is known.
- The signal $x(t)$ is uncorrelated with the noise signal $v(t)$, since it is a filtered version of $r_1(t)$ and $r_2(t)$, which are uncorrelated with $v(t)$.

This shows that we can identify a model of P through the identification of R from measurements

$$Z^N = \{z(1), x(1), \cdots, z(N), x(N)\}$$

reconstructed according to (2.5) and (2.6). Since $x(t)$ and $e(t)$ are uncorrelated, identification of R is an open-loop identification problem as shown in Figure 2.4.

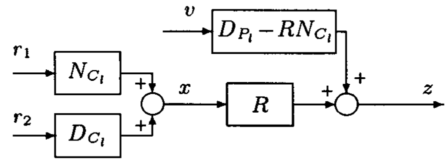

Figure 2.4: Conversion to open-loop identification

Due to the properties of the dual Youla parametrization, the identified model is guaranteed to be stabilized by the controller C. One of the drawbacks of this method is that the order of the resulting model is not simply tunable due to the required re-parametrization as presented in Proposition 2.1.

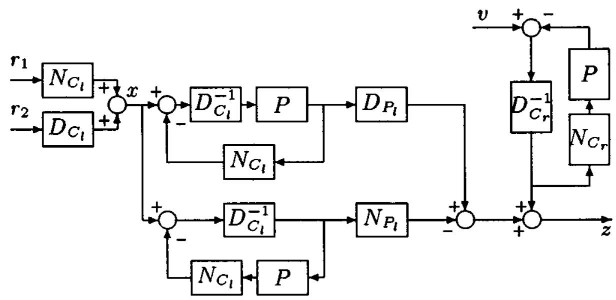

Figure 2.5: Conversion to open-loop identification: redrawing of Figure 2.4 using (2.8) and (2.9) for the noise.

2.1 Conversion to open-loop identification

In this subsection, we provide an interpretation for the conversion from a closed-loop identification problem to an open-loop identification problem.

Using (2.3) and (2.4), it is straightforward to see that the "true" Youla parameter is given by

$$R = (D_{P_l} P - N_{P_l})(N_{C_l} P + D_{C_l})^{-1} \quad (2.8)$$

$$= (P N_{C_r} + D_{C_r})^{-1}(P D_{P_r} - N_{P_r}). \quad (2.9)$$

It now easily follows that we can rewrite (2.7) using (2.8) or (2.9). Also, Figure 2.4 can be redrawn as shown in Figures 2.5 and 2.6, respectively, using (2.8) and (2.9).

Note that both the loop of Figure 2.5 and 2.6 are stable since C is a stabilizing controller for P.

3 A modified Hansen scheme

In this section, we present a modified Hansen scheme.

3.1 A closed-loop relevant predictor $z(\theta)$

Conventionally, at this point, a set $\mathcal{R}$ of permissible $R(\theta)$ is defined. For example, $\mathcal{R}$ might comprise all finite im-

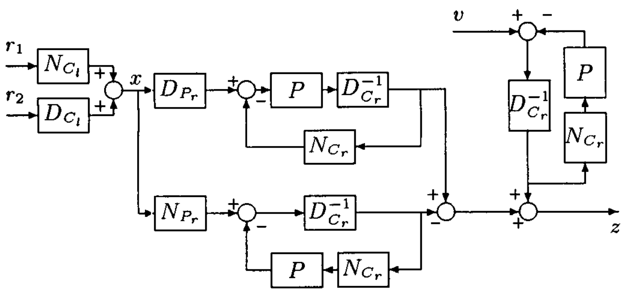

Figure 2.6: Conversion to open-loop identification: redrawing of Figure 2.4 using (2.9).

pulse transfer functions of order up to p, with θ the corresponding impulse response coefficients; again, $\mathcal{R}$ might comprise third order rational transfer functions with all poles in some prescribed region, and θ then corresponds to the numerator coefficients. This might result in a set of models $P(\theta)$ with twice, or more, the order of the nominal plant P_0. As an alternative, we propose postulating a simple parametrization via

$$\mathcal{M} : y(\theta) = P(\theta)\, u \tag{3.1}$$

parametrized by a parameter vector $\theta \in D_\theta \subset \mathbb{R}^n$ where D_θ is some prescribed domain. Note that, unless an explicit temporary assumption is made to the contrary, it is not assumed that the true system (even without noise) is in the model set. It is of primary importance that the identified model is stabilized by the known controller C, i.e. we assume that the closed-loop model set

$$\mathcal{G} : \left\{ G(\theta) = \frac{P(\theta)}{1 + CP(\theta)}, \quad \theta \subset D_\theta \right\} \tag{3.2}$$

is uniformly stable over all $\theta \in D_\theta$; we refer to [4] for further details. Note that uniform stability requires the stability of $G'(\theta)$ for all $\theta \in D_\theta$. We show subsequently that the uniform stability assumption is not particularly restrictive. We now postulate that

$$\begin{aligned} P(\theta) &= (D_{P_l} - R(\theta)N_{C_l})^{-1}(N_{P_l} + R(\theta)D_{C_l}) & (3.3) \\ &= (N_{P_r} + D_{C_r}R(\theta))(D_{P_r} - N_{C_r}R(\theta))^{-1} & (3.4) \end{aligned}$$

for some $R(\theta) \in \mathbf{M}(\mathbf{S})$. Then, we regain $R(\theta)$ as being parametrized by

$$\begin{aligned} R(\theta) &= (D_{P_l}P(\theta) - N_{P_l})(N_{C_l}P(\theta) + D_{C_l})^{-1} & (3.5) \\ &= (P(\theta)N_{C_r} + D_{C_r})^{-1}(P(\theta)D_{P_r} - N_{P_r}), & (3.6) \end{aligned}$$

i.e. we use the model structure

$$\mathcal{R} : z(\theta) = R(\theta)x \tag{3.7}$$

for R with $R(\theta)$ defined in (3.5) or (3.6). Now $R(\theta)$ is a possibly complicated function of θ, and it may be of quite high degree. Also, unless the parametrization (3.1) is cleverly chosen, i.e. unless we assume uniform stability, it may not be the case that all $P(\theta)$ are stabilized by C or, equivalently, not all $R(\theta)$ in (3.5)-(3.6) may be stable. The first point means that the gradients used in the identification are likely to be messy to compute. The second indicates that not only one has to consider stability issues carefully, but also if the set of all models $P(\theta)$ stabilized by C is not connected for all $\theta \subset D_\theta$, the identification algorithm, based as it is on making a series of small steps in the estimated value of θ, may run into difficulties; we refer to [7] for more details. It now easily follows that we can rewrite (3.7) using (3.5) and (3.6). Also, we can construct predictors for $z(\theta)$ as shown in Figures 3.1 and 3.2.

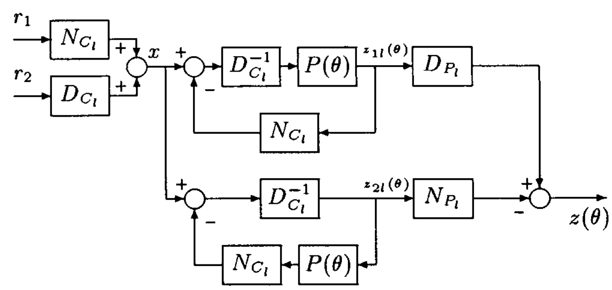

Figure 3.1: Predictor for $z(\theta)$ based on (3.5) and (3.7).

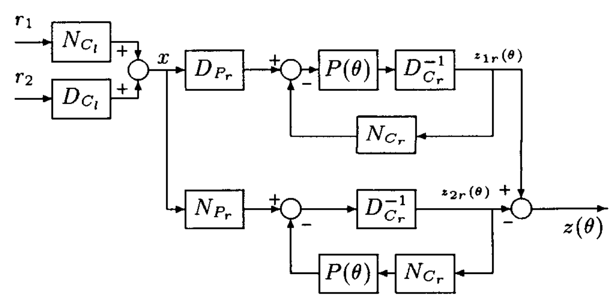

Figure 3.2: Predictor for $z(\theta)$ based on (3.6) and (3.7).

The predictors of Figures 3.1 and 3.2 are stable if the model $P(\theta)$ is stabilized by C. Both loops can be simplified as shown in the upper parts of Figures 3.3 and 3.4.

3.2 Generation of the gradient $z'(\theta)$

The symbol θ_j used below denotes the j-th entry of the vector θ. Let us first consider the equations that follow from Figure 3.1

$$\begin{aligned} z(\theta) &= D_{P_l}\, z_{1l}(\theta) - N_{P_l}\, z_{2l}(\theta), \\ z_{1l}(\theta) &= P(\theta) D_{C_l}^{-1}[x - N_{c_l}\, z_{1l}(\theta)], \\ z_{2l}(\theta) &= D_{C_l}^{-1}[x - N_{c_l} P(\theta)\, z_{2l}(\theta)]. \end{aligned}$$

The gradient of $z(\theta)$ w.r.t. θ_j satisfies, for $j = 1, \cdots, n$,

$$z'_{\theta_j}(\theta) = D_{P_l}\, z'_{1l,\theta_j}(\theta) - N_{P_l}\, z'_{2l,\theta_j}(\theta),$$

$$z'_{1l,\theta_j}(\theta) = P'_{\theta_j}(\theta)D_{C_l}^{-1}[x - N_{c_l}\, z_{1l}(\theta)] - P(\theta)D_{C_l}^{-1}N_{c_l}z'_{1l,\theta_j}(\theta),$$

$$z'_{2l,\theta_j}(\theta) = -D_{C_l}^{-1}N_{c_l}[P'_{\theta_j}(\theta)z_{2l}(\theta) + P(\theta)z'_{2l,\theta_j}(\theta)]$$

where $P'_{\theta_j}(\theta)$ is the derivative of $P(\theta)$ with respect to θ_j. It can easily be obtained since $P(\theta)$ has a known parametrization. The derivatives of $z(\theta)$, $z_{1l}(\theta)$ and $z_{2l}(\theta)$ with respect to θ_j are, respectively, denoted $z'_{\theta_j}(\theta)$, $z'_{1l,\theta_j}(\theta)$ and $z'_{2l,\theta_j}(\theta)$. Note that

$$D_{C_l}^{-1}[x - N_{c_l}\, z_{1l}(\theta)] = z_{2l}(\theta). \tag{3.8}$$

It now easily follows that each entry of $z'(\theta)$ can be obtained using the loop in Figure 3.3. The stability of this loop follows from the stability of the predictor, at least if $P'_{\theta_j}(\theta)$ is a stable transfer function. The contrary case is commented upon below.

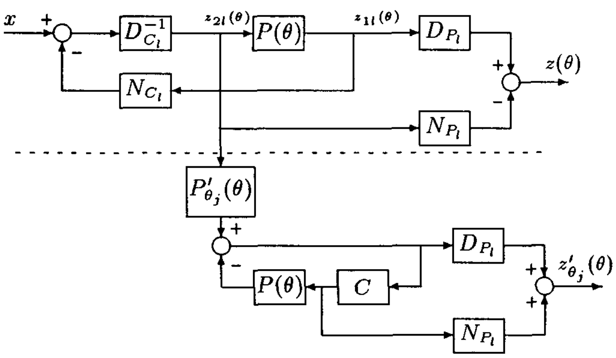

Figure 3.3: Generation of the gradient $z'_{\theta_j}(\theta)$

Similarly, let us consider the equations that follow from Figure 3.2

$$z(\theta) = z_{1r}(\theta) - z_{2r}(\theta),$$

$$z_{1r}(\theta) = D_{C_r}^{-1}P(\theta)[D_{P_r}\, x - N_{c_r}\, z_{1r}(\theta)],$$

$$z_{2r}(\theta) = D_{C_r}^{-1}[N_{P_r}x - P(\theta)N_{c_r}\, z_{2r}(\theta)].$$

The gradient of $z(\theta)$ w.r.t. θ_j satisfies, for $j = 1, \cdots, n$,

$$z'_{\theta_j}(\theta) = z'_{1r,\theta_j}(\theta) - z'_{2r,\theta_j}(\theta),$$

$$z'_{1r,\theta_j}(\theta) = D_{C_r}^{-1}P'_{\theta_j}(\theta)[D_{P_r}\, x - N_{c_r}\, z_{1r}(\theta)] - D_{C_r}^{-1}P(\theta)N_{c_r}z'_{1r,\theta_j}(\theta),$$

$$z'_{2r,\theta_j}(\theta) = -D_{C_r}^{-1}[P'_{\theta_j}(\theta)N_{c_r}z_{2r}(\theta) + P(\theta)N_{c_r}z'_{2r,\theta_j}(\theta)]$$

where derivatives of $z(\theta)$, $z_{1r}(\theta)$ and $z_{2r}(\theta)$ with respect to θ_j are, respectively, denoted $z'_{\theta_j}(\theta)$, $z'_{1r,\theta_j}(\theta)$ and $z'_{2r,\theta_j}(\theta)$. It is straightforward to see that

$$z'_{\theta_j}(\theta) = D_{C_r}^{-1}P'_{\theta_j}(\theta)[D_{P_r}\, x - N_{c_r}\, z(\theta)] - D_{C_r}^{-1}P(\theta)N_{c_r}z'_{\theta_j}(\theta).$$

It now easily follows that each entry of $z'(\theta)$ can be obtained using the loop in Figure 3.4. The stability of this loop follows from the stability of the predictor, at least if $P'_{\theta_j}(\theta)$ is a stable transfer function. The contrary case is commented upon below.

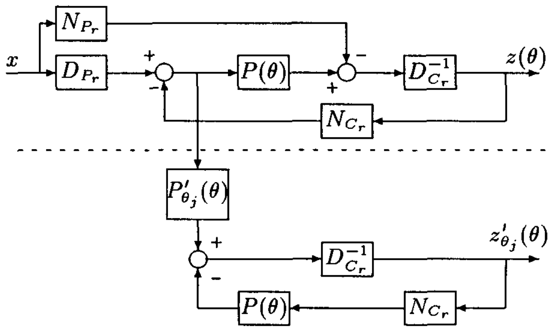

Figure 3.4: Generation of the gradient $z'_{\theta_j}(\theta)$

3.3 Stability issues

The loops of Figures 3.3 and 3.4 can always be implemented in a stable way if $P(\theta)$ is stabilized by C. Indeed, let

$$P(\theta) = [D_l(\theta)]^{-1}N_l(\theta) = N_r(\theta)[D_r(\theta)]^{-1}$$

be coprime factorizations of $P(\theta)$. Then, one can redraw Figure 3.3, for example, as shown in Figure 3.5.

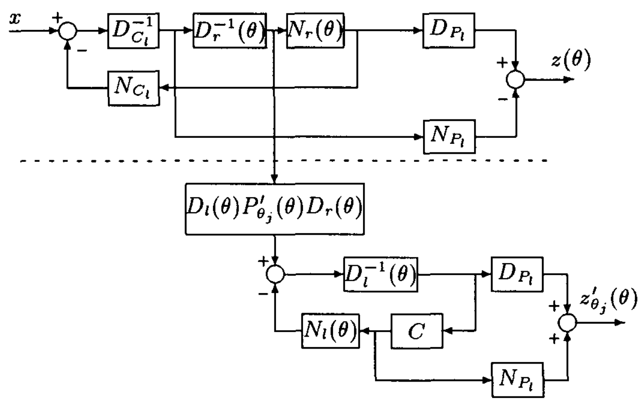

Figure 3.5: Generation of the gradient $z'_{\theta_j}(\theta)$

The stability of Figure 3.5 follows from the fact that $P(\theta)$ is stabilized by C and that (as is easily verified) $D_l(\theta)P'_{\theta_j}(\theta)D_r(\theta)$ is a stable transfer function for all $\theta \subset D_\theta$ and $j = 1, \cdots, n$.

3.4 Identification criterion

In the sequel, we restrict attention to Single-Input-Single-Output systems. The restriction to scalar systems is inessential but notationally convenient.

Here, we make use of the identification criterion

$$V_N(\theta) = \frac{1}{2N}\sum_{t=1}^{N} \epsilon(\theta)^2 \tag{3.9}$$

where the prediction error is given by $\epsilon(\theta) = L[z - z(\theta)]$. Here L can be any stable design data filter. To facilitate notations, we assume that $L = 1$. To minimize (3.9) with respect to the model parameter vector θ, it is standard that one can iteratively seek a solution for θ to

$$V_N'(\theta) = -\frac{1}{N}\sum_{t=1}^{N} [z - z(\theta)]\; z'(\theta) = 0 \qquad (3.10)$$

by taking steps in the negative gradient direction

$$\theta[i+1] = \theta[i] - \gamma_i\, H_i^{-1}\, V_N'(\theta[i]) \qquad (3.11)$$

where H_i is some appropriate positive definite matrix, typically an estimate of the Hessian of V_N. A good (but biased estimate) of the Hessian is obtaining using the following expression

$$H_i = \frac{1}{N}\sum_{t=1}^{N} z'(\theta[i])\, [z'(\theta[i])]^T. \qquad (3.12)$$

We refer the reader to [4] for more information on the choice (3.12). Here $\theta[i]$ denotes the parameter vector θ at iteration i and $\{\gamma_i\}$ is a sequence of positive numbers that determine the step size. It is assumed here that (3.11) is a batch type of adjustment.

It is assumed earlier that $\mathcal{G}$ is uniformly stable over all $\theta \in D_\theta$, i.e. it is assumed that stability of the predictor is preserved while iterating. This is a reasonable assumption since the step size γ_i can be used effectively to control how much the model is allowed to change per iteration.

3.5 Algorithm

We now present the algorithm for the closed identification procedure of P:

- **Step 0:** Perform one experiment on the actual system of Figure 1.1 and collect the data set $\{r_1, r_2, y\}$.
- **Step 1:** Start with some model $P_0 = P(\theta[0])$ that is stabilized by C. Note the initial estimate could for example be obtained using direct identification techniques. Select coprime factorizations for P_0 and C as shown in Proposition 2.1. Construct the signals x and z as shown in (2.5) and (2.6).
- **Step 2:** Simulate either one of the loops of Figures 3.1 or 3.2 with the signal x and collect the signal $z(\theta[i])$.
- **Step 3:** Compute a realization of $z - z(\theta[i])$.
- **Step 4:** Simulate the gradient vector $z'(\theta[i])$ as shown in either one of Figures 3.3 or 3.4.
- **Step 5:** Compute $V_N'(\theta[i])$ and H_i as shown in (3.10) and (3.12) with $z - z(\theta[i])$ simulated in Step 3 and $z'(\theta[i])$ simulated in Step 4.
- **Step 6:** Update the parameter vector using (3.11) while optimizing over γ_i at each iteration using a line search procedure. Go to Step 2.

3.6 Remarks

- Note that the second derivatives $z''(\theta)$ of $z(\theta)$ can be obtained very similarly. These could, for instance, be utilized to obtain a better estimate of the Hessian.
- Due to the used parametrization, the identification problem is a nonlinear optimization problem. The function $V_N(\theta)$ and the parameter set D_θ can be extremely non convex making it difficult to apply gradient search methods. To alleviate these problems, it is important to have good initial estimates, e.g. estimates obtained using direct identification techniques, and a good strategy to select the step size γ_i in (3.11).
- Most of the ideas outlined in this paper remain valid when both the plant and the controller are allowed to be nonlinear; see [3]. In the nonlinear context, the reparametrization of the Youla parameter using a plant model is appealing because it is difficult to find an appropriate model structure for the Youla parameter itself.

3.7 Links with tailor-made parametrizations

It is shown in [6] that the identification of the Youla parameter R is equivalent to the identification of a closed-loop transfer function as present in the first step of an indirect identification. Quite similarly, we show in this subsection that the identification of the Youla parameter R is equivalent to identifying the plant P using a parametrization that is tailored to the closed-loop configuration. The parametrization is called tailor-made because it is specifically directed towards the closed-loop configuration at hand; it uses knowledge of the controller and the closed-loop configuration to parametrize the closed-loop transfer function in terms of the parameters of the open-loop transfer function. We refer the reader to [1, 7] for further details. This implementation of the Hansen scheme can therefore be seen as a generalization of closed-loop identification with a tailor-made parametrization to the identification setup of Figure 1.1.

Suppose that C is a stable controller. Then,

$$\begin{aligned} N_{P_l} &= 0, D_{P_l} = I, N_{C_l} = C, D_{C_l} = I, \\ N_{P_r} &= 0, D_{P_r} = I, N_{C_r} = C, D_{C_r} = I \end{aligned}$$

are valid coprime factorizations for $P_0 = 0$ and C. Note that (2.1) is satisfied and that P_0 is stabilized by C.

It easily follows from (2.5), (2.6) and (2.7) that

$$\begin{aligned} z = y &= Rx + (I - RC)\, v && (3.13) \\ &= R(C r_1 + r_2) + (I - RC)\, v. && (3.14) \end{aligned}$$

It follows from (2.9), the "true" Youla parameter is given by

$$R = (I + PC)^{-1} P \qquad (3.15)$$

which yields

$$z = y = (I+PC)^{-1}P\,(C\,r_1+r_2)+(I+PC)^{-1}\,v, \quad (3.16)$$

i.e. the identification problem has been recast as shown in Figures 2.5 or 2.6 or, equivalently, Figure 3.6.

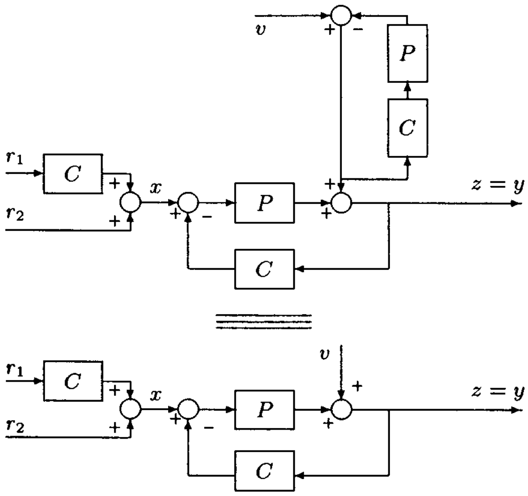

Figure 3.6: Transformation to an open-loop identification in the case of a stable controller C and coprime factors (3.7).

It is now straightforward to see that the use of the model structure (3.7) is equivalent to using the tailor-made parametrization shown in Figures 3.1 or 3.2 or, equivalently, Figure 3.7. The expressions of the gradients are the ones obtained in [1, 7], i.e. Figures 3.3 and 3.4 both reduce to Figure 3.8, at least if $P'_{\theta_j}(\theta)$ is a stable transfer function. Figure 3.8 can always be implemented in a stable way with the adjustment used in Figure 3.5.

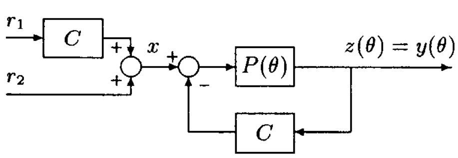

Figure 3.7: Tailor-made parametrization in the case of a stable controller C and coprime factors (3.7).

4 Some conclusions

In this paper, we have presented a modification of a closed-loop identification method that uses the Youla parametrization. This implementation of the "Hansen scheme" has the advantage that the order of the resulting method is tunable by the user. We have also shown that this alternative method shows some similarities with closed-loop identification using a tailor-made parametrization; i.e. in particular cases, the gradient signals are identical to the ones obtained [1, 7].

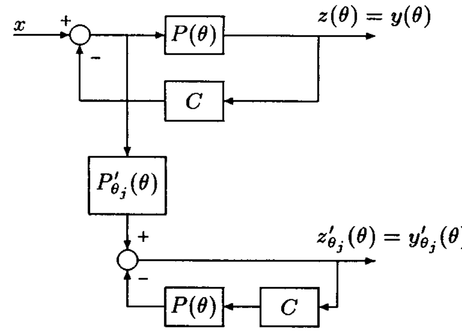

Figure 3.8: Generation of the gradient z'_{θ_j} in the case of a stable controller C and coprime factors (3.7).

Acknowledgement: The authors acknowledge the funding of this research by the US Army Research Office, Far East, the Office of Naval Research, Washington.

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