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ON CLOSED-LOOP IDENTIFICATION WITH A TAILOR-MADE PARAMETRIZATION

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Abstract

In this paper, we present gradient expressions for a closed-loop parametric identification scheme. The method is based on the minimization of a standard identification criterion and a parametrization that is tailored to the closed-loop configuration. It is shown that for both linear and nonlinear plants and controllers, the gradient signals can be computed exactly.

1 Introduction

In recent years, several new methods for the identification of approximate models of an open-loop plant on the basis of closed-loop data have been presented. This line of research follows from the fact that in reality, i.e. on many industrial processes, the data need to be collected in closed-loop either because the plant is unstable or because operating constraints do not allow one to open the control loop. Also there might be situations where it is wiser to identify the plant in closed-loop so that the identified model would capture the dynamical characteristics that are important for control design. We refer the reader to [4, 17] for a discussion of this problem in the linear case.

In the "identification for control" literature, the problem of identification of a linear system on the basis of data obtained from closed-loop experiments has received considerable attention, see e.g. the survey papers in [4, 17] with the many references therein. One can distinguish three main closed-loop identification procedures in the "linear" literature; see [8, 16, 18] for more details. These techniques have in common the ability to identify approximate models of the open-loop plant on the basis of closed-loop data, while the asymptotic bias distribution of the estimated plant transfer function at each frequency remains independent of the noise and is thus explicitly tunable by the user. The nonlinear extensions of these methods have been treated in [10, 11].

Another approach that has received less attention has been examined in [19]. Such an approach had already been mentioned as an exercise in [13] and further references include [1, 9, 14]. The authors of [19] consider the closed-loop identification of a linear system subjected to a linear controller by minimization of a closed-loop criterion, using a tailor-made parametrization of the plant. The method uses knowledge of the controller; it minimizes an error between the closed loop transfer functions of the true closed-loop and the model closed-loop. The parametrization is called tailor-made because it is specifically directed to the closed-loop configuration at hand. The main result of [19] is to show that, provided the model order is higher than the order of the controller, the parameter set is connected. The paper provides consistency results and gradient expressions.

Our paper uses the same closed-loop matching criterion as in [19] with a tailor-made parametrization, but it extends the results in two ways. First, in the linear case, we show that the gradient signals of [19] can be generated very simply on closed loop simulation models. This observation then leads us to show that this simulation method for the computation of the gradients can be extended to the case of nonlinear systems and/or systems with nonlinear controllers.

The ideas in this paper heavily rely on data-driven model-free control design methods that have recently been proposed in [3, 6, 15]. Indeed, we treat closed-loop identification with a tailor-made parametrization as a dual of direct controller optimization.

The organization of the paper is as follows. In Section 2, we describe the problem at hand. In Section 3 we present expressions of the gradient signals in the linear case. Section 4 considers the general case where both the plant and the known controller are possibly nonlinear. Section 5 presents consistency results in the nonlinear case. We conclude in Section 6.

2 General problem setting

For ease of notation, we will omit the time argument of the signals. Let us assume that the true system is the Single-Input Single-Output (SISO) nonlinear time-invariant system described by

$$\mathcal{S} : y = P_o(u, v) \tag{2.1}$$

where P is an unknown causal nonlinear operator. The restriction to scalar plants is inessential, but notationally convenient. Here u is the control input signal, y is the achieved output signal and v is a process disturbance signal. Note that the disturbance signal v is allowed to enter the system nonlinearly. The input signal is determined according to a known controller

$$\mathcal{C} : u = C(r, y) \tag{2.2}$$

where r is an external reference which is assumed to be quasi-stationary and uncorrelated with v. The controller C is a causal nonlinear operator of both r and y. The closed-loop operator from measured reference signal r to measured output signal y, as defined in Figure 2.1, can be written as follows,

$$y = T_o(r, v). \tag{2.3}$$

We require that the closed-loop system is Bounded-Input-Bounded-Output (BIBO) stable. In the sequel we often make use of linearizations of some nonlinear operators around their operating trajectories. We therefore require that the plant, the model, the controller and all closed-loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. We refer the reader to [2] for more details on such smoothness assumptions and a full treatment of the linearization problem. Note that, as opposed to the nonlinear methods described in [10, 11], there is no restriction on the Signal-to-Noise-Ratio (SNR) when using the method; consistency however may require a high SNR as discussed later. Also, all signals can either be continuous or discrete in time.

The basic idea is that the closed-loop operator from the reference signal r to the output signal y is identified using a parametrized output predictor

$$y(\theta) = T(\theta, r) \tag{2.4}$$

obtained from the feedback interconnection of an open-loop plant model

$$\mathcal{M} : y(\theta) = P(\theta, u) \tag{2.5}$$

for P_o, parametrized by a vector $\theta \in D_\theta \subset \mathbb{R}^n$ where D_θ is some prescribed domain, and the possibly nonlinear controller C in (2.2). We assume that the output predictor (2.4) or, equivalently, the loop in Figure 2.2 has the BIBO and smoothness properties of the true closed-loop system, for all values of $\theta \in D_\theta$. Note that, unless an explicit temporary assumption is made to the contrary, it is not assumed that the true system (even without noise) is in the model set.

Suppose that a data set $\{r, y\}$ has been collected on the actual system of Figure 2.1. The problem that is addressed in this paper is the one of selecting the model for P_o in (2.5) that best explains this data set in a closed-loop sense.

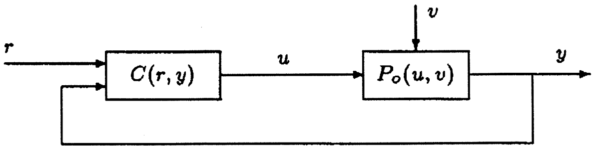

Figure 2.1: The actual loop

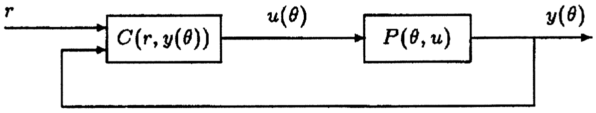

Figure 2.2: The simulation loop

We make use of the identification criterion

$$V_N(\theta) = \frac{1}{2N} \sum_{t=1}^{N} [L(y - y(\theta))]^2. \tag{2.6}$$

Here L can be any causal BIBO stable design operator. Besides the intuitively reasonable aspect of (2.6), it is shown in [19] that this criterion allows a consistent identification of a linear plant under linear feedback, when the input-output dynamics are in the model set. In Section 5, we give some insights on how this result generalizes when both the plant and the controller are allowed to be nonlinear. In any case, the linear consistency result adds greater weight to the selection of the identification criterion (2.6). We also refer the reader to [5] for variance considerations in the linear case.

Note that, provided the input signal u is measured, the generalization to the nonstandard identification criterion

$$V_N(\theta) = \frac{1}{2N} \sum_{t=1}^{N} \{[L_y(y - y(\theta))]^2 + \lambda [L_u(u - u(\theta))]^2\}$$

that was introduced by the authors of [20] is straightforward. Again, L_y and L_u are causal BIBO stable design operators.

The preceding parameter estimation problem is typically solved using gradient search techniques such as Gauss Newton; we refer the reader to [13] for more details on initial estimates, convergence, local minima, etc. We refer to [19] for a discussion on the connectedness of the set of all models (2.5) stabilized by the controller (2.2) in the linear case.

To minimize (2.6) w.r.t. the model parameter vector θ, it is standard that one can iteratively seek a solution for θ to

$$V_N'(\theta) = -\frac{1}{N} \sum_{t=1}^{N} [(y - y(\theta))\, y'(\theta)] = 0 \tag{2.7}$$

by taking steps in the negative gradient direction

$$\theta[i+1] = \theta[i] - \gamma_i\, R_i^{-1}\, V_N'(\theta[i]) \tag{2.8}$$

where $V_N'(\theta)$ and $y'(\theta)$, respectively, denote the gradient of $V_N(\theta)$ and $y(\theta)$ w.r.t. θ, and where R_i is some appropriate positive definite matrix, typically an estimate of the Hessian of V_N. It is assumed that stability of the predictor is preserved while iterating. This is a very reasonable assumption since the step size γ_i can be used effectively to control how much the model is allowed to change per iteration.

The key technical step in this iterative algorithm is the computation of the gradient $y'(\theta)$. Our contribution here is to show that this gradient computation can be performed by feeding the signal $u(\theta)$ of Figure 2.2 as the input of a closed loop simulation system. For simplicity, our method is explained first in Section 3 for the linear case, in which case our method is a simple alternative to the gradient computation proposed in [19]. The real advantage of our computation method is that it allows a good understanding of the stability issue and a generalization to a nonlinear setting, as is shown in Section 4.

3 Gradient expressions in the linear case

In this section, we consider the simplified case where both the real system and the controller are linear, i.e. we suppose that (2.1), (2.2) and (2.5) reduce to

$$\mathcal{S}: y = P_o\, u + v, \quad \mathcal{C}: u = C_r\, r - C_y\, y, \quad \mathcal{M}: y(\theta) = P(\theta)\, u.$$

Let us first consider the following equations

$$y(\theta) = P(\theta)\, u(\theta) \quad \text{and} \quad u(\theta) = C_r\, r - C_y\, y(\theta). \tag{3.1}$$

The gradients of these two signals w.r.t. the j-th entry of θ are, respectively, denoted by $u'_{\theta_j}(\theta)$ and $y'_{\theta_j}(\theta)$. They are the j-th component of the vectors $u'(\theta)$ and $y'(\theta)$ and they satisfy, for $j = 1, \cdots, n$,

$$y'_{\theta_j}(\theta) = P'_{\theta_j}(\theta)\, u(\theta) + P(\theta)\, u'_{\theta_j}(\theta) \tag{3.2}$$
$$u'_{\theta_j}(\theta) = -\, C_y\, y'_{\theta_j}(\theta) \tag{3.3}$$

where $P'_{\theta_j}(\theta)$, the derivative of $P(\theta)$ w.r.t. θ_j, can easily be obtained since $P(\theta)$ has a known structure. It now easily follows that each entry of $u'(\theta)$ and $y'(\theta)$ can be computed as shown in the loop of Figure 3.1.

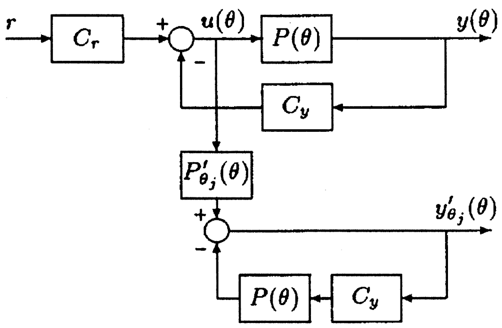

Figure 3.1: Generation of $y'_{\theta_j}(\theta)$

The scheme in Figure 3.1 can always be implemented in a stable way if $P(\theta)$ is stabilized by C. Indeed, let

$$P(\theta) = [D(\theta)]^{-1} N(\theta) = N(\theta)[D(\theta)]^{-1} \tag{3.4}$$

be a stable coprime factorization of $P(\theta)$. Then, one can redraw Figure 3.1 as shown in the loop of Figure 3.2.

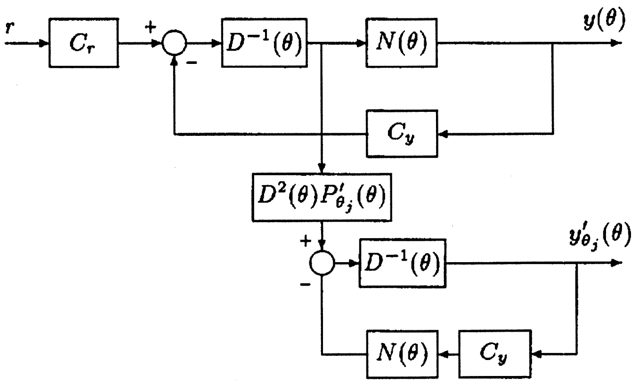

Figure 3.2: Stable implementation of Figure 3.1

The stability of Figure 3.2 follows from the stability of the predictor loop and the verifiable fact that $D^2(\theta)P'_{\theta_j}(\theta)$ is stable for $j = 1, \cdots, n$ and $\forall\, \theta \in D_\theta$.

It is now straightforward to see that

$$y'_{\theta_j}(\theta) = \frac{P'_{\theta_j}(\theta)}{[1 + P(\theta)C_y]}\, u(\theta) = \frac{P'_{\theta_j}(\theta)C_r}{[1 + P(\theta)C_y]^2}\, r, \tag{3.5}$$
$$u'_{\theta_j}(\theta) = -C_y\, y'_{\theta_j}(\theta). \tag{3.6}$$

Similar calculations show that

$$y''_{\theta_j}(\theta) = \frac{1}{1 + P(\theta)C_y}\left(P''_{\theta_j}(\theta)u(\theta) + 2P'_{\theta_j}(\theta)u'_{\theta_j}(\theta)\right) \tag{3.7}$$
$$= \frac{P''_{\theta_j}(\theta)C_r}{[1 + P(\theta)C_y]^2}\, r - 2\,\frac{[P'_{\theta_j}(\theta)]^2 C_r C_y}{[1 + P(\theta)C_y]^3}\, r, \tag{3.8}$$
$$u''_{\theta_j}(\theta) = -C_y\, y''_{\theta_j}(\theta). \tag{3.9}$$

Here $P''_{\theta_j}(\theta)$, $u''_{\theta_j}(\theta)$ and $y''_{\theta_j}(\theta)$, respectively, denote the second derivatives of $P(\theta)$, $u(\theta)$ and $y(\theta)$ w.r.t. θ_j.

4 Gradient expressions in the nonlinear case

Let us now consider the nonlinear case of Section 2, i.e. we have the following equations

$$y(\theta) = P(\theta, u(\theta)), \quad u(\theta) = C(r, y(\theta)). \tag{4.1}$$

As a tool for obtaining the gradient of V_N w.r.t. θ, we seek the gradients of $u(\theta)$ and $y(\theta)$ w.r.t. θ_j. If one of the parameter vector entries, say θ_j, is perturbed by a small $\delta\theta_j$, we obtain

$u(\theta_1, \cdots, \theta_j + \delta\theta_j, \cdots, \theta_n)$

$$= C[r, y(\theta_1, \cdots, \theta_j + \delta\theta_j, \cdots, \theta_n)],$$
$$\simeq C[r, y(\theta) + y'_{\theta_j}(\theta)\,\delta\theta_j],$$
$$\simeq C(r, y(\theta)) + \partial C_y(r, y(\theta))\, y'_{\theta_j}(\theta)\,\delta\theta_j,$$
$$= u(\theta) + \partial C_y(r, y(\theta))\, y'_{\theta_j}(\theta)\,\delta\theta_j \quad (4.2)$$

where $\partial C_y(r, y(\theta))$ is the linearization of C in response to a perturbation in y around the trajectory produced by r and by $y(\theta)$, i.e. the trajectory around which C is linearized depends on θ. The derivative of $y(\theta)$ w.r.t. θ_j is denoted $y'_{\theta_j}(\theta)$ and it is the j-th component of the vector $y'(\theta)$. It is straightforward to see that (4.2) yields

$$u'_{\theta_j}(\theta) = \partial C_y(r, y(\theta))\, y'_{\theta_j}(\theta) \quad (4.3)$$

where $u'_{\theta_j}(\theta)$ is defined in a similar fashion as $y'_{\theta_j}(\theta)$. A similar reasoning yields

$$y'_{\theta_j}(\theta) = P'_{\theta_j}(\theta, u(\theta)) + \partial P_u(\theta, u(\theta))\, u'_{\theta_j}(\theta) \quad (4.4)$$

where $\partial P_u(\theta, u(\theta))$ is the linearization of $P(\theta)$ in response to a perturbation in u around the trajectory produced by $u(\theta)$. The partial derivative of $P(\theta)$ w.r.t. θ_j is denoted by $P'_{\theta_j}(\theta, u(\theta))$. It can easily be obtained since $P(\theta)$ has a known structure.

The exact gradient signals can be obtained by feeding the signal $u(\theta)$ generated in the loop of Figure 2.2, filtered through $P'_{\theta_j}(\theta, u(\theta))$, as input of the (linear time-varying) linearized closed loop system of Figure 4.1. A similar observation had already been made in [3, 15] for an iterative feedback tuning scheme. The stability of the lower loop follows from the smoothness assumption on the nonlinear closed-loop operator and the stability of the predictor at each iteration. These two assumptions are equivalent to a small signal BIBO stability assumption, i.e. we assume that a small perturbation in the reference signal produces a small perturbation in the output signal.

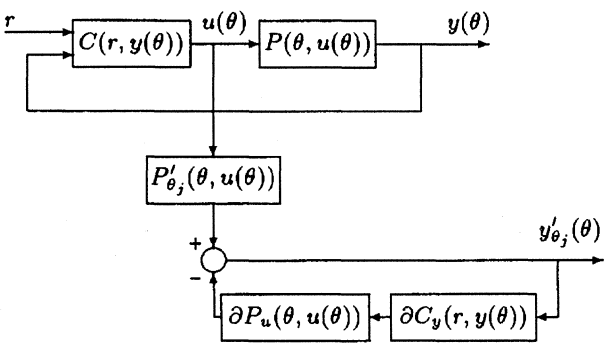

Figure 4.1: Generation of $y'_{\theta_j}(\theta)$ in the nonlinear case

The scheme shown in Figure 4.1 can be implemented in a stable way, even if $P(\theta, u(\theta))$ and thus $P'_{\theta_j}(\theta, u(\theta))$ are unstable, provided we can construct

$$y(\theta) = N_r(\theta, z_r(\theta)), \quad u = D_r(\theta, z_r(\theta)) \text{ and}$$
$$z_l(\theta) = D_l(\theta, y(\theta)), \quad z_l(\theta) = N_l(\theta, u)$$

respectively, as stable right and left coprime descriptions of (2.5); see [7] for further details. Then, one can redraw Figure 4.1 as shown in Figure 4.2. Here $\partial D_{l_y}(\theta, y(\theta))$ and $\partial N_{l_u}(\theta, u(\theta))$ are, respectively, the linearizations of $D_l(\theta, y(\theta))$ and $N_l(\theta, u(\theta))$ around their trajectory.

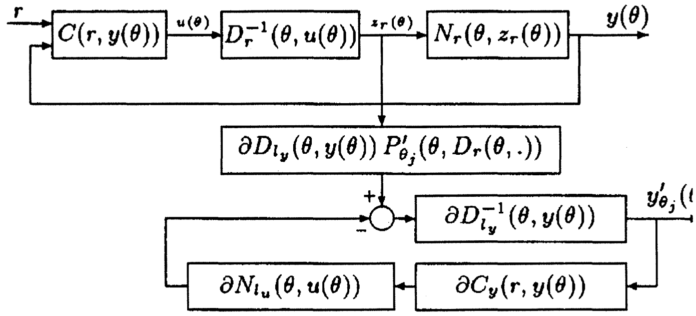

Figure 4.2: Stable implementation of Figure 4.1

The stability of Figure 4.2 follows from the stability of the predictor loop, the smoothness assumption on the closed loop system and the fact that (as is verifiable)

$$\partial D_{l_y}(\theta, z_l(\theta))\, P'_{\theta_j}(\theta, D_r(\theta, .))$$

is a stable operator for $j = 1, \cdots, n$ and $\forall\, \theta \in D_\theta$.

5 Consistency results

In the sequel, we assume that $\exists\, \theta_o$ such that the true system without noise lies in the model set, i.e.

$$P_o(u, 0) = P(\theta_o, u)\ \forall u \ \text{ or }\ T_o(r, 0) = T(\theta_o, r)\ \forall r.$$

In this situation, we would hope for consistent identification. Rewriting the identification error, we obtain

$$\begin{aligned} y - y(\theta) &= [y - y(\theta_o)] + [y(\theta_o) - y(\theta)] \\ &= [T_o(r, v) - T(\theta_o, r)] + [T(\theta_o, r) - T(\theta, r)], \end{aligned}$$

whence

$$\begin{aligned} &E\{[y - y(\theta)]^2\} = \\ &E\{[[T_o(r, v) - T(\theta_o, r)]^2\} + E\{[T(\theta_o, r) - T(\theta, r)]^2\} \\ &+ 2\,E\{[T_o(r, v) - T(\theta_o, r)][T(\theta_o, r) - T(\theta, r)]\}\cdot \quad (5.1) \end{aligned}$$

Here, the expected value is taken w.r.t. the probability distributions of the noise and the reference signals; the earlier assumption that r and v are independent is important. It is clear from (5.1) that a sufficient condition for consistency is given by

$$E\{[T_o(r, v) - T(\theta_o, r)][T(\theta_o, r) - T(\theta, r)]\} = 0. \quad (5.2)$$

It is easily established that this condition (not unexpectedly) is not satisfied in general, when T is nonlinear. A sufficient condition for (5.2) to hold is

$$T_o(r, v) - T(\theta_o, r) = v^{2k+1}\, R(\theta_o, r) \quad (5.3)$$

for some nonnegative integer k and some noise independent operator R; we can isolate several important situations where this holds.

Remarks

- Note that in a small noise situation, (5.3) approximately holds with $R(\theta_o, r) = \partial T_{ov}(r, 0)$ and $k = 0$. Here $\partial T_{ov}(r, 0)$ is the linearization of T_o in response to a perturbation in v around the trajectory produced by r and $v = 0$. We conclude that at SNR where linearization is valid, one has aproximate consistency.
- In many industrial processes, although the open-loop system is nonlinear, the controller has been designed in order for the closed loop system T_o to have a quasi-linear behaviour w.r.t. the reference signal r (and the disturbance signal v; at least if the noise signal v is additive). It is clear from (5.3) that consistency approximately holds in such cases.
- As is shown in [12], consistency can be recovered using direct identification, i.e. using the data set as if it had been collected in open-loop, if the system (input-output and noise dynamics) can be modeled exactly. However, the number of parameters increases, i.e. one has to estimate the noise dynamics. This can be problematic (because of variance considerations) in the case of small data sets.
- If the noise signal v enters the plant nonlinearly, direct identification of input-output and noise dynamics might be difficult to implement. In such a case, our approach offers a valid alternative.

The full version of this paper includes simulations.

6 Conclusions

In this paper, we have presented gradient expressions for a closed-loop identification scheme with tailor-made parametrization. The main advantage of these gradient expressions is that they can easily be extended to non-standard identification criteria and that the plant, the parametric model and the controller are allowed to be nonlinear.

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