CONTROL 97

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20-22 October 1997

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NCP 7/97
ISBN 1 85825 672
Published by the Institution of Engineers, Australia, 11 National Circuit, Barton ACT 2600, September 1996

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Printed by Vertifix Aust Pty Ltd

A TWO STEP METHOD FOR THE CLOSED LOOP IDENTIFICATION OF NONLINEAR SYSTEMS [1]

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Keywords: Closed Loop Identification, Nonlinear Systems

Abstract

In recent years, several new methods for the identification of approximate models of an open loop plant on the basis of closed loop data have been presented. In this paper, we extend such a method to the nonlinear case: we consider that both the plant and the controller can be nonlinear. The method that is considered is a two-step procedure, i.e. the sensitivity function of the closed loop system is identified through a high order nonlinear model and it is used in the second step to simulate a noise free input signal for an open loop like identification of the plant. We assume that the closed loop operators are smooth functions of both the reference signal and the disturbance signal and that the measurement noise enters the system under a high SNR assumption. We show that under these assumptions, the linear ideas of [9] carry over to a nonlinear framework. The validity of the theoretical results is verified using simulations.

1 Introduction

Consider the setting shown in Figure 1.1, where P_o is a nonlinear plant to be identified, and C is a nonlinear controller which is possibly unknown. We restrict attention to time-invariant C and P_o. There is no other restriction a priori on C and P_o except that we require P_o to be stable. Here $u(t)$ is the control signal, $y(t)$ is the achieved output signal, $v(t)$ is process disturbance signal and $r_1(t)$ and $r_2(t)$ are external reference or setpoint signals which we assume to be quasi-stationary; see [8].

We now consider that the data have been collected on the nonlinear process P_o while the nonlinear controller C was operating. This situation is typical of processes where the data need to be collected in closed loop either because the plant is simply unstable or because operating constraints do not allow one to open the control loop. Also there might be situations where it is wiser to identify the plant in closed loop so that the identified model will capture the dynamical characteristics that are important for control design. We refer the reader to [3, 10] for a discussion of this problem in the linear case.

There are two major problems associated with closed loop identification. The first one is that the measurement noise $v(t)$ is now correlated to $u(t)$, and what is more this correlation, being dependent on the unknown plant P_o, cannot be determined a priori. The second problem is that closed loop identification is hampered by the need to unravel the closed-loop operator to obtain P_o. Even when P_o and C are linear, P_o appears in a nonlinear fashion in the closed loop quantities. It can also be the case that if an estimate $\hat{P}_o$ of P_o is obtained by unravelling an estimate of the closed loop transfer function, then $\hat{P}_o$ and C have an unstable pole-zero cancellation; see [12] for further details.

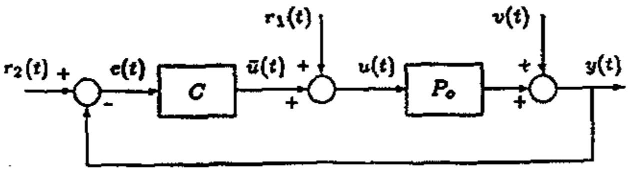

Figure 1.1: The closed Loop system

In the "identification for control literature", the problem of identification of a linear system on the basis of data obtained from closed loop experiments has received considerable attention, see for example the survey papers [3, 10] with the many references therein. Here, we will particularly focus on the three closed loop identification procedures that were first presented in

[1] Funding of activities is gratefully acknowledged of the Cooperative Research Centre for Robust and Adaptive Systems by the Australian Commonwealth Government under the Cooperative Research Centres Program.

[4, 9, 11]. These techniques have in common the ability to identify approximate models of the open loop plant on the basis of closed loop data, while the asymptotic bias distribution remains independent of the noise and is thus explicitly tunable by the user. This result is obtained by turning the closed loop identification problem into an open-loop-like identification problem. The way in which this is done is different for each method, which results in very different characteristic features for each of these methods. This paper endeavours to extend the current linear theory of [4, 9, 11] to the nonlinear setup of Figure 1.1.

The nonlinear extension of the Hansen identification scheme described in [4] has recently been treated in [1, 5, 6]. It will therefore not be repeated here. The main idea is that the identification of a possibly nonlinear time invariant plant using closed loop measurements with a known linear controller can be effectively tackled by regarding the unknown plant as a member of the set of all plants stabilized by the known linear controller. This set is parametrized by a Youla-Kucera parameter, itself a stable operator, which can be identified using open loop techniques under a high Signal-to-Noise Ratio (SNR) assumption.

The idea behind the *Two-Step* method described in [9] is to identify one of the closed loop operators in an open loop fashion and to use it in the second step to simulate a noise free input signal for an open loop like identification of the plant P_o. Note that this idea is only applicable for stable P_o. This method requires the measurement of the plant input and output signals $u(t)$ and $y(t)$ and either the measurement of both of the reference signals $r_1(t)$ and $r_2(t)$ or the measurement of either one of the reference signals $r_1(t)$ and $r_2(t)$ with the knowledge of C. As for the previous method, we will also impose an additional high SNR assumption.

The *Right Coprime Factor Identification* method described in [11] identifies a coprime factor pair of the plant in an open loop fashion. This method has not been treated in this paper. We refer the reader to [7] for a detailed nonlinear extension of this method.

Notice that the method described in this paper and the ones treated in [1, 5, 6, 7] deal with noise entering the system under a high SNR assumption. We will also require that the closed loop system is a smooth function of both the reference signal and the disturbance signal. Without these assumptions the analysis is much more involved.

Definitions
For a causal operator A:

A is Bounded Input - Bounded Output (BIBO) stable if

$$\|A\| = \sup_{\|x\|\neq 0} \frac{\|Ax\|}{\|x\|} \text{ is finite.}$$

For convenience the norm is taken to be the L_2 norm.

For a closed loop as shown in Figure 1.1, with the noise $v(t)$ identically zero and dropping the time dependence of the signals for convenience:

The closed loop is internally stable if $[e, \bar{u}, u, y]$ is bounded for all bounded r_1 and r_2.

If the noise v is non-zero, r_1 and r_2 must be replaced with r_1, r_2 and v in these definitions.

Lipschitz Continuity: A causal operator A is globally Lipschitz continuous if $\forall$ bounded y, we have

$$\|A(x+y) - A(x)\| \leq \mathcal{K}\|y\|$$

for $\forall\, x$ and for some constant $\mathcal{K}$. There is no requirement that x be bounded. Such a requirement can be relaxed since A is a causal operator, see [6] for more details.

Summary of Paper
In Section 2, we extend the *Two-Step Identification Method* of [9] to the nonlinear setup of Figure 1.1. In Section 3, we present a simulation example. Section 4 offers concluding remarks.

2 Two Step Method.

This method first estimates the operator from the external inputs $[r_1, r_2]^T$ to the input of the plant u. This gives rise to a noise free estimate of the plant input, $\hat{u}_r$. In the second step of this method, the operator from $\hat{u}_r$ to the plant output y is estimated to obtain a model of the plant. The measurement noise v is uncorrelated with $\hat{u}_r$, i.e. we have turned the closed loop identification problem into a standard open loop identification problem.

Assumptions

- The closed loop of Figure 1.1 is internally stable and the closed loop operators are smooth functions of both the reference signals and the disturbance signal.
- The plant P_o is stable.
- One or both of the reference signals r_1 and r_2 are measured in addition to u and y. Note that if r_1 is not measured, we need to impose the requirement that C is a known stable operator in order to be able to reconstruct $r_1 = u - C(r_2 - y)$. Similarly, if r_2 is not available for measurement, we have to impose the requirement that C is a known stably invertible operator to recover $r_2 = y + C^{-1}(u - r_1)$. When both r_1 and r_2 are measured there is no restriction on C, i.e. in such a case, we do not even require the knowledge of C.
- The data is collected under a high SNR assumption.

Step 1: Identification of the sensitivity function
In the general case, when both reference signals r_1 and r_2 are non-zero, we have the following relation between the signals u, r_1, r_2 and v:

$$u = F_o(r_1, r_2, v) \tag{2.1}$$

where F_o is some stable operator existing by internal stability of the closed loop system. Under a smoothness assumption on the operator F_o and a small signal assumption on v and with $\partial F_{ov}(r_1, r_2, 0)$ the linearization of F_o in response to a perturbation in v around the operating trajectory produced by r_1, r_2 and $v = 0$, we have that

$$u = F_o(r_1, r_2) + \partial F_{ov}(r_1, r_2, 0)\, v. \tag{2.2}$$

Since $[r_1, r_2]^T$ and v are uncorrelated signals and u and $[r_1, r_2]^T$ are available for computation, it follows that we can (in principle) obtain an estimate $\hat{F}_o$ of F_o using a Multiple-Input-Single-Output open loop identification. Thus, we obtain an estimate $\hat{u}_{r_1,r_2}$ of $u_{r_1,r_2} = F_o(r_1, r_2)$ with

$$\hat{u}_{r_1,r_2} = \hat{F}_o(r_1, r_2). \tag{2.3}$$

Note that, by definition, $\hat{u}_{r_1,r_2}$ is uncorrelated with the process disturbance signal v.

Step 2: Open loop like identification of the plant
The second step uses the simulated noise free input signal for an open loop identification of the plant.

From Figure 1.1 we have

$$y = P_o u + v. \tag{2.4}$$

By substituting (2.1) into (2.4) we obtain

$$y = P_o F_o(r_1, r_2, v) + v. \tag{2.5}$$

Again, under a smoothness assumption on the $P_o F_o$ and a small signal assumption on v and with $\partial[P_o F_o]_v(r_1, r_2, 0)$ the linearization of $P_o F_o$ in response to a perturbation in v around the operating trajectory produced by r_1, r_2 and $v = 0$, we have that

$$\begin{aligned} y &= P_o F_o(r_1, r_2) + \partial[P_o F_o]_v(r_1, r_2, 0)\, v + v, & (2.6)\\ &\simeq P_o\, \hat{u}_{r_1,r_2} + \partial[P_o F_o]_v(r_1, r_2, 0)\, v + v. & (2.7) \end{aligned}$$

The last equality follows from the stability of P_o and the smoothness assumption of the nonlinear closed loop operators. These two assumptions are equivalent to a small signal BIBO stability assumption, i.e. we assume that a small perturbation in the reference (or input) signal produces a small perturbation in the output signal. Since $\hat{u}_{r_1,r_2}$ and v are uncorrelated and available for computation, it is possible to obtain an estimate $\hat{P}_o$ of P_o in an open loop fashion.

Remark 1
This procedure will be greatly simplified when one of the reference signals r_1 or r_2 equals zero. Note that if r_2 equals zero, F_o represents the sensitivity operator S_o of the closed loop system, i.e. (2.2), (2.3), (2.6) and (2.7) reduce to

$$\begin{aligned} u &= S_o r_1 + \partial S_{ov}(r_1, 0)\, v, & (2.8)\\ \hat{u}_{r_1} &= \hat{S}_o r_1, & (2.9)\\ y &= P_o S_o r_1 + \partial[P_o S_o]_v(r_1, 0)\, v + v, & (2.10)\\ &\simeq P_o \hat{u}_{r_1} + \partial[P_o S_o]_v(r_1, 0)\, v + v. & (2.11) \end{aligned}$$

Similar simplifications occur when r_1 equals zero. In such a case, F_o represents the complementary sensitivity operator T_o of the closed loop system.

Remark 2
If the controller C is linear, we can without loss of generality restrict attention to the case with $r_1 = r$ and $r_2 = 0$. If P_o and C are linear, the linear theory described in [9] is captured.

Remark 3
In the linear case, a high order model structure is used to estimate the sensitivity function in Step 1 so that, when this estimate is used in Step 2, the simulated noise free input is as accurate as possible. This might give rise to difficulties in the nonlinear case for computational reasons.

Remark 4
Note that this method can tackle both the situation where one of the signals r_1 and r_2 is non-zero or where both r_1 and r_2 are non-zero. This is in contrast with the method described in [1, 5, 6] where both r_1 and r_2 are required to be non-zero.

Remark 5
Once the identification process has been completed it is wise to include a post identification validation step as in reality the operators may not be satisfactorily linearizable. This involves checking that all the assumptions were satisfied, i.e. that the nonlinearity in the system has not amplified the noise signal in such a way that it would interfere with the previous analysis.

Remark 6
In the previous derivations, we have linearized nonlinear closed loop operators around their operating trajectory making a small signal assumption on the noise and a smoothness on the closed loop operators. We refer the reader to [2] for more details on such smoothness assumptions and a full treatment of the linearization problem. Note that similar equations would have been obtained instead by imposing a Lipschitz continuity assumption on these operators.

3 Simulation example

In this section, we illustrate the theoretical results presented in this paper. We consider a nonlinear system operating in closed loop with some two degree of freedom linear controller. The nonlinear system is described by

$$y_t = \frac{q^{-1}(1+b_1 q^{-1})}{1+a_1 q^{-1}+a_2 q^{-2}} \mathrm{DZ}(u_t) + v_t \qquad (3.12)$$

where DZ is a nonlinear deadzone operator defined using the following equations

$$\mathrm{DZ}(u_t) = \begin{cases} u(t)-d_p & \text{if} \quad u_t \geq d_p \\ 0 & \text{if} \quad -d_m < u_t < d_p \\ u_t + d_m & \text{if} \quad u_t \leq -d_m \end{cases} \qquad (3.13)$$

with $|b_1| < 1$, $d_m > 0$ and $d_p > 0$. The disturbance signal is modeled as follows

$$v_t = \frac{1+c_1 q^{-1}+c_2 q^{-1}}{1+a_1 q^{-1}+a_2 q^{-1}} e_t \qquad (3.14)$$

where e_t is zero mean white noise of variance σ^2. The linear controller is the optimal two degree of freedom minimum variance controller for the linear system that is obtained from (3.12) by setting both d_m and d_p to zero. The controller is defined by

$$u_t = C_r(q^{-1})r_t - C_y(q^{-1})y_t, \qquad (3.15)$$

$$C_r(q^{-1}) = \frac{1+c_1 q^{-1}+c_2 q^{-1}}{1+b_1 q^{-1}}, \qquad (3.16)$$

$$C_y(q^{-1}) = \frac{(c_1-a_1)+(c_2-a_2)q^{-1}}{1+b_1 q^{-1}}. \qquad (3.17)$$

Here, we have taken the following plant parameters

$$b_1 = -0.9, \quad a_1 = -1.5, \quad a_2 = 0.7,$$
$$c_1 = -1, \quad c_2 = 0.2, \quad d_p = 0.7,$$
$$d_m = 0.2, \quad \sigma^2 = 0.3,$$

but of course these values are not provided to the identification algorithm but rather are to be identified, nor is the identification algorithm provided with the information as to how the controller is designed, though the algorithm is provided with the transfer functions C_r and C_y defining the controller. The reference signal r_t was chosen to be a known unit variance and zero mean white noise signal independent of the process disturbance signal v_t. Note that this corresponds to an input signal u_t that is of the same order of magnitude as d_p and d_m. With an input signal of much greater magnitude than d_p and d_m, these quantities would be hard to identify, the effect of the nonlinearity being swamped by the signal; if u_t is typically of much smaller magnitude, there is obviously also a problem. Using the previous closed-loop system, we have generated a data set $\{r_t, u_t, y_t\}$ with signals of length $N = 2000$.

For the identification of the plant itself, we have used the following model structure

$$\hat{y}_t(\theta) = \theta_1 \frac{q^{-1}(1+\theta_2 q^{-1})}{1+\theta_3 q^{-1}+\theta_4 q^{-2}} \bar{\mathrm{DZ}}(u_t) \qquad (3.18)$$

where $\bar{\mathrm{DZ}}$ is defined as in (3.13) with d_p and d_m, respectively, replaced by θ_5 and θ_6. Estimates of the parameters where obtained by minimizing

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}[y_t - \hat{y}_t(\theta)]^2 \qquad (3.19)$$

with respect to θ using a steepest descent method. Using the previously defined data set, we have applied two strategies:

Strategy I: direct standard open loop identification, i.e. we have used the data set $\{u_t, y_t\}$ as if it had been collected in open loop.

Strategy II: modified two-step method where $\hat{u}_r$ is obtained by using the sensitivity operator that is obtained by interconnecting the model (3.18) in feedback with the controller (3.15). The feedback loop was simulated with reference signal r_t, $e_t = 0$ and with the parameters identified using Strategy I, i.e. there is some approximation involved here.

We have obtained the following results

	True values	Initial values	Strat. I $\{u, y\}$	Strat. II $\{\hat{u}_r, y\}$
θ_1	1	1	0.82	1.02
θ_2	-0.9	-0.2	-0.88	-0.87
θ_3	-1.5	-1.3	-1.44	-1.50
θ_4	0.7	0.5	0.64	0.74
θ_5	0.7	0	0.62	0.67
θ_6	0.2	0	0.15	0.22
$V_N(\theta)$			0.41	0.31

Table 3.1: Identification cost and identified parameters using a one step procedure and a modified two step method.

Table 3.1 shows the results of estimating P_o with both procedures. In Figure 3.2, we have compared the magnitude Bode plots of the linear part of the identified models. The results clearly show the degraded performance of the direct identification scheme, i.e. this scheme is unable to produce bias free estimates. The indirect two step method gives more accurate results for the linear part of the (3.12). Note that the parameters characterizing the nonlinear part of the plant could also be identified more accurately with the modified two-step methods although the smoothness assumption on P_o is not satisfied here. The use of a nonlinear model structure for the direct identification of the sensitivity will most probably improve the identification accuracy of the parameters and the applicability of the method in general. The authors are presently experimenting with neural networks in order to obtain better noise free estimates of the control input. Note that the use of complicated model structures for the first step of the two step procedure, i.e. neural nets, is not a drawback here since their use is only in the generation of a noise free estimate $\hat{u}_r$.

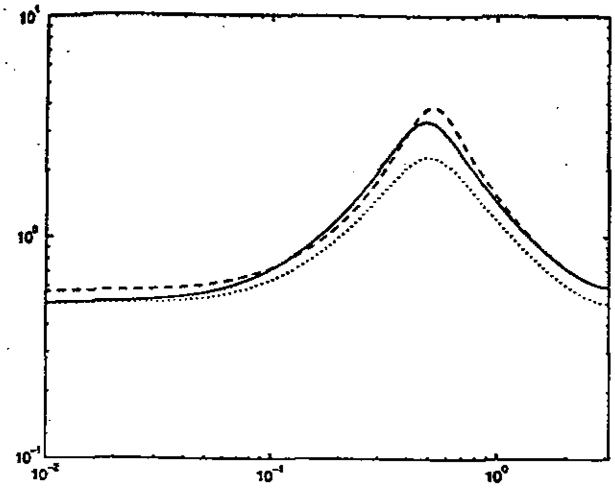

Figure 3.2: Amplitude Bode plots of the linear part of the nonlinear system (3.12) (—) and estimates of this same transfer function obtained using a one step method (···) and a modified two step method (-–-).

4 Conclusion

In this paper, we have extended a linear method for the identification of approximate models of an open loop plant on the basis of closed loop data to a general nonlinear setting. The method is an indirect two step method that is based on the identification of the sensitivity function. We have assumed that the noise signal enters the system under a high SNR and that the closed loop system is a smooth function of the reference signals and the disturbance signals. Preliminary simulation results with the two step method have shown to give very satisfactory results. The main advantage of the procedure is that one can allow for a nonlinear controller. The main drawback is that the possibly nonlinear plant is restricted to be stable.

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