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CLOSED LOOP IDENTIFICATION OF NONLINEAR SYSTEMS[1]

Natasha Linard, Brian D. O. Anderson and Franky De Bruyne

Department of Systems Engineering and Cooperative
Research Centre for Robust and Adaptive Systems, RSISE,
The Australian National University, Canberra ACT 0200, Australia

Abstract: In recent years, several new methods for the identification of approximate models of an open loop plant on the basis of closed loop data have been presented. In this paper, we extend two of these methods to the nonlinear case: we consider that both the plant and the controller can be nonlinear. The first method is a two-step procedure. The sensitivity function of the closed loop system is identified through a high order nonlinear model and it is used in the second step to simulate a noise free input signal for an open loop like identification of the plant. The second method identifies the right coprime factors of the plant through an open loop like identification of the filtered sensitivity and complementary sensitivity functions. For both methods, we assume that the measurement noise enters the system under a high SNR assumption.

Keywords: Closed Loop Identification, Nonlinear Systems, Coprime Factorizations

1 Introduction

Consider the setting shown in Figure 1.1, where P_o is a nonlinear plant to be identified, and C is a nonlinear controller. We restrict attention to time-invariant C and P_o. There is no other restriction a priori on C and P_o, i.e. both can be nonlinear and/or unstable.

We now consider that the data have been collected on the nonlinear process P_o while the nonlinear controller C was operating. This situation is typical of processes where the data need to be collected in closed loop either because the plant is simply unstable or because operating constraints do not allow one to open the control loop. Also there might be situations where it is wiser to identify the plant in closed loop so that the identified model will capture the dynamical characteristics that are important for control design. We refer the reader to [4, 10] for a discussion of this problem in the linear case.

There are two major problems associated with closed loop identification. The first one is that the measurement noise $v(t)$ is now correlated to $u(t)$, and what is more this correlation, being dependent on the unknown plant P_o, cannot be determined a priori. The second problem is that closed loop identification is hampered by the need to unravel the closed-loop operator to obtain P_o. Even when P_o and C are linear, P_o appears in a nonlinear fashion in the closed loop quantities. It can also be the case that if an estimate $\hat{P}_o$ of P_o is obtained by unravelling an estimate of the closed loop transfer function, then $\hat{P}_o$ and C have an unstable pole-zero cancellation; see [12].

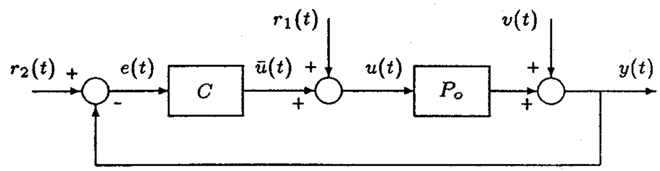

Figure 1.1: The closed Loop system

In the "identification for control literature", the problem of identification of a linear system on the basis of data obtained from closed loop experiments has received considerable attention, see e.g. the survey papers [4, 10] with the many references therein. Here, we will particularly focus on the three closed loop identification procedures that were first presented in [5, 9, 11]. These techniques have in common the ability to identify approximate models of the open loop plant on the basis of closed loop data, while the asymptotic bias distribution remains independent of the noise and is thus explicitly tunable by the user. This paper endeavours to extend the current linear theory of [5, 9, 11] to the nonlinear setup of Figure 1.1.

The nonlinear extension of the Hansen identification scheme described in [5] has recently been treated in [2, 6, 7]. It will therefore not be repeated here. The idea is that the identification of a possibly nonlinear time invariant plant using closed loop measurements with a known linear controller can be tackled by regarding the unknown plant as a member of the set of all plants stabilized by the known linear controller. This set is parametrized by a Youla-Kucera parameter, itself a stable operator, which can be identified using open loop techniques under a high Signal-to-Noise Ratio (SNR) assumption.

The idea behind the *Two-Step* method described in [9] is to identify one of the closed loop operators in an open loop fashion and to use it in the second step to simulate a noise free input signal for an open loop like identification of the plant P_o. Note that this idea is only applicable

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for stable P_o. This method requires the measurement of the plant input and output signals $u(t)$ and $y(t)$ and either the measurement of both of the reference signals $r_1(t)$ and $r_2(t)$ or the measurement of either one of the reference signals $r_1(t)$ and $r_2(t)$ with the knowledge of C. As for the previous method, we will also impose an additional high SNR assumption.

The *Right Coprime Factor Identification* method described in [11] identifies a coprime factor pair of the plant in an open loop fashion. The assumptions on $r_1(t)$, $r_2(t)$ and C are similar to the ones of the previous method. Again, we will make an additional high SNR assumption.

Notice that the two methods described in this paper and the one treated in [2, 6, 7] deal with noise entering the system under a high SNR assumption. We will also require that the closed loop system is a smooth function of both the reference signal and the disturbance signal. Without this assumption the analysis is much more involved.

Definitions

For a causal operator A:

A is Bounded Input - Bounded Output (BIBO) stable if

$$||A|| = \sup_{||x|| \neq 0} \frac{||Ax||}{||x||} \text{ is finite.}$$

For convenience the norm is taken to be the L_2 norm.

For a closed loop system as shown in Figure 1.1 and dropping the time dependence of the signals for convenience:

The closed loop is internally stable if $[e, \tilde{u}, u, y]$ is bounded for all bounded r_1, r_2 and v.

In this paper, we will use the following definition of right coprimeness valid for linear and nonlinear systems.

Right coprimeness: Let M, N be a right factorization for $G : \mathcal{U} \to \mathcal{Y}$, $G = NM^{-1}$, $N : \mathcal{C} \to \mathcal{Y}$, $M : \mathcal{C} \to \mathcal{U}$ where M and N are BIBO stable and M is invertible. Here $\mathcal{U}$, $\mathcal{Y}$ and $\mathcal{C}$ are, respectively, the input, output and partial state spaces. Then M, N is a right *coprime* factorization of G if there exists a BIBO $\mathcal{L}$ for which

$$\mathcal{L} \begin{bmatrix} M \\ N \end{bmatrix} = \mathcal{I}.$$

This definition is not universal, i.e. there is an alternative definition of right coprimeness based on set theoretic ideas. We refer the reader to [1] for more details.

Summary of Paper

In Section 2, we extend the *Two-Step Identification Method* of [9] to the nonlinear setup of Figure 1.1. This contribution has appeared in [8] in a slightly modified version. In Section 3, we document the extension of the *Right Coprime Factor Identification Method* described in [11]. In Section 4, we present a simulation example. Section 5 offers concluding remarks.

2 Two Step Method.

This method first estimates the operator from the external inputs $[r_1, r_2]^T$ to the input of the plant u. This gives rise to a noise free estimate of the plant input, $\hat{u}_r$. In the second step of this method, the operator from $\hat{u}_r$ to the plant output y is estimated to obtain a model of the plant.

Assumptions

- The closed loop of Figure 1.1 is internally stable and the closed loop operators are smooth functions of both the reference signals and the disturbance signal.
- The plant P_o is stable.
- The data is collected under a high SNR assumption.
- One or both of the reference signals r_1 and r_2 are measured in addition to u and y. Note that if r_1 is not measured, we need to impose the requirement that C is a known stable operator in order to be able to reconstruct $r_1 = u - C(r_2 - y)$. Similarly, if r_2 is not available for measurement, we have to impose the requirement that C is a known stably invertible operator to recover $r_2 = y + C^{-1}(u - r_1)$. When both r_1 and r_2 are measured there is no restriction on C.

Step 1: Identification of the sensitivity function
In the general case, when both reference signals r_1 and r_2 are non-zero, we have the following relation between the signals u, r_1, r_2 and v:

$$u = F_o(r_1, r_2, v) \tag{2.1}$$

where F_o is some stable operator existing by internal stability of the closed loop system. Under a smoothness assumption on the operator F_o and a small signal assumption on v and with $\partial F_{ov}(r_1, r_2, 0)$ the linearization of F_o in response to a perturbation in v around the operating trajectory produced by r_1, r_2 and $v = 0$, we have that

$$u = F_o(r_1, r_2) + \partial F_{ov}(r_1, r_2, 0)\, v. \tag{2.2}$$

Since $[r_1, r_2]^T$ and v are uncorrelated signals and u and $[r_1, r_2]^T$ are available for computation, it follows that we can (in principle) obtain an estimate $\hat{F}_o$ of F_o using a Multiple-Input-Single-Output open loop identification. Thus, we obtain an estimate $\hat{u}_{r_1,r_2}$ of $u_{r_1,r_2} = F_o(r_1, r_2)$ with

$$\hat{u}_{r_1,r_2} = \hat{F}_o(r_1, r_2). \tag{2.3}$$

Step 2: Open loop like identification of the plant
The second step uses the simulated noise free input signal for an open loop identification of the plant.

From Figure 1.1 we have

$$y = P_o\, u + v. \tag{2.4}$$

By substituting (2.1) into (2.4) we obtain

$$y = P_o F_o(r_1, r_2, v) + v. \tag{2.5}$$

Again, under a smoothness assumption on the P_oF_o and a small signal assumption on v and with $\partial[P_oF_o]_v(r_1, r_2, 0)$ the linearization of P_oF_o in response to a perturbation in v around the operating trajectory produced by r_1, r_2 and $v = 0$, we have that

$$\begin{aligned} y &= P_oF_o(r_1, r_2) + \partial[P_oF_o]_v(r_1, r_2, 0)\, v + v, && (2.6)\\ &\simeq P_o\, \hat{u}_{r_1,r_2} + \partial[P_oF_o]_v(r_1, r_2, 0)\, v + v. && (2.7)\end{aligned}$$

The last equality holds by stability of P_o. Since $\hat{u}_{r_1,r_2}$ and v are uncorrelated and since y and $\hat{u}_{r_1,r_2}$ are available for computation, it is possible to obtain an estimate $\hat{P}_o$ of P_o in an open loop fashion.

Remark 1
This procedure will be greatly simplified when one of the reference signals r_1 or r_2 equals zero. Note that if r_2 equals zero, F_o represents the sensitivity operator S_o of the closed loop system, i.e. (2.2), (2.3), (2.6) and (2.7) reduce to

$$\begin{aligned} u &= S_o\, r_1 + \partial S_{ov}(r_1, 0)\, v, && (2.8)\\ \hat{u}_{r_1} &= \hat{S}_o\, r_1, && (2.9)\\ y &= P_oS_o\, r_1 + \partial[P_oS_o]_v(r_1, 0)\, v + v, && (2.10)\\ &\simeq P_o\, \hat{u}_{r_1} + \partial[P_oS_o]_v(r_1, 0)\, v + v. && (2.11)\end{aligned}$$

Similar simplifications occur when r_1 equals zero.

Remark 2
Note that this method can tackle both the situation where one of the signals r_1 and r_2 is non-zero or where both r_1 and r_2 are non-zero. This is in contrast with the method described in [2, 6, 7] where both r_1 and r_2 are required to be non-zero.

Remark 3
Once the identification process has been completed it is wise to include a post identification validation step as in reality the operators may not be satisfactorily linearizable. This involves checking that all the assumptions were satisfied, i.e. that the nonlinearity in the system has not amplified the noise signal in such a way that it would interfere with the previous analysis.

Remark 4
In the previous derivations, we have linearized nonlinear closed loop operators around their operating trajectory making a small signal assumption on the noise and a smoothness on the closed loop operators. We refer the reader to [3] for more details on such smoothness assumptions and a full treatment of the linearization problem.

Note that similar equations would have been obtained instead by imposing a Lipschitz continuity assumption on these operators.

3 Right Coprime Factor Identification

Consider the system depicted in Figure 1.1.

Assumptions.
We will make the same assumptions as for the the two-step method except that we do not require the stability of P_o. In addition, we will assume that either one of the reference signals r_1 or r_2 are zero or that the reference signals r_1 and r_2 are filtered versions of the same signal r, i.e. $r_1 = D\, r$ and $r_2 = N\, r$ for some stable filters N and D.

Identification of the right factors of P
We will now show that it is possible to generalize the closed loop identification scheme of [11] to the nonlinear case. Using measurements of u and y together with measurements or reconstructions of r_1 or/and r_2, we can identify closed loop relations in an open loop fashion using (2.2) and (2.6). The corresponding factorization of P_o that can be estimated in this way is the factorization (P_oF_o, F_o). However, as shown in [11], this is only one of many factorizations of P_o and there is no guarantee that this factorization is coprime. By introducing auxiliary signals

$$x_1 = F_1\, r_1,\; x_2 = F_2\, r_2$$

with F_1, F_2 fixed stable and invertible operators, one can rewrite the system equations as

$$\begin{aligned} y &= P_oF_o(F_1^{-1}\, x_1, F_2^{-1}\, x_2) + \partial[P_oF_o]_v(r_1, r_2, 0)\, v + v,\\ u &= F_o(F_1^{-1}\, x_1, F_2^{-1}\, x_2) + \partial F_{ov}(r_1, r_2, 0)\, v.\end{aligned}$$

We will show that when C is either linear, nonlinear but stable or nonlinear but stably invertible, we can use these filters to identify a right coprime factorization of a nonlinear P_o.

Design of the data filters F_1 and F_2
Suppose that the controller C is linear and has left coprime factorization $C = D_C^{-1} N_C$. The plant P is allowed to be nonlinear and has a right coprime factorization $P = N_P D_P^{-1}$.

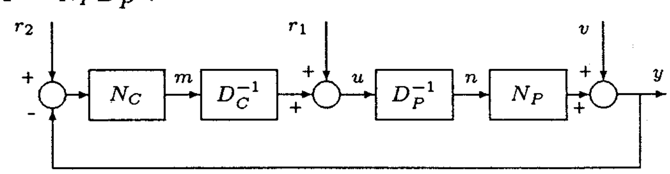

Figure 3.1: The closed loop system

Consider Figure 3.1. Using the linearity of N_C, D_C

$$m = D_C[D_P\, n - r_1] = D_C D_P\, n - D_C\, r_1, \tag{3.1}$$

and also

$$m = N_C[r_2 - N_P\, n - v] = N_C\, r_2 - N_C N_P\, n - N_C\, v. \quad (3.2)$$

Combining (3.1) and (3.2) yields

$$n = V^{-1}[N_C\, r_2 + D_C\, r_1 - N_C\, v]$$

where $V = N_C N_P + D_C D_P$ is a unit by internal stability of the closed loop system. Under a smoothness assumption on the closed loop system and a small signal assumption on v, define $\partial Y_v(r_1, r_2, 0)$ and $\partial U_v(r_1, r_2, 0)$ as the linearizations of the closed loop operators in response to a perturbation in v around the trajectory produced by r_1, r_2 and $v = 0$. In the sequel, we will, respectively, use ∂Y_v and ∂U_v as shorthand notations for $\partial Y_v(r_1, r_2, 0)$ and $\partial U_v(r_1, r_2, 0)$. We have that

$$\begin{aligned} y &= N_P\, V^{-1}[N_C\, r_2 + D_C\, r_1] + \partial Y_v\, v + v, \\ u &= D_P\, V^{-1}[N_C\, r_2 + D_C\, r_1] + \partial U_v\, v. \end{aligned}$$

Introducing the filters F_1 and F_2, we obtain

$$\begin{aligned} y &= N_P\, V^{-1}[N_C\, F_2^{-1}\, x_2 + D_C\, F_1^{-1}\, x_1] + \partial Y_v\, v + v, \\ u &= D_P\, V^{-1}[N_C\, F_2^{-1}\, x_2 + D_C\, F_1^{-1}\, x_1] + \partial U_v\, v. \end{aligned}$$

- Let us consider the case where $r_2 = 0$. The closed loop relations reduce to

$$\begin{aligned} y &= N_P\, V^{-1}\,(D_C\, F_1^{-1}\, x_1) + \partial Y_v\, v + v, \\ u &= D_P\, V^{-1}\,(D_C\, F_1^{-1}\, x_1) + \partial U_v\, v \end{aligned}$$

which can be used to provide an open loop identified estimate for a right coprime factorization of P_o. Indeed, it is easy to see that

$$\begin{bmatrix} \bar{N}_P \\ \bar{D}_P \end{bmatrix} = \begin{bmatrix} N_P\, V^{-1}\,(D_C F_1^{-1}) \\ D_P\, V^{-1}\,(D_C F_1^{-1}) \end{bmatrix} \quad (3.3)$$

is a right factorization of P_o if and only if $D_C F_1^{-1}$ is a stable operator. This same factorization is right coprime if and only if

$$F_1 = D_C W, \quad (3.4)$$

with W a unit operator. Indeed, premultiplication by the stable operator $V\,[X\;\; Y]$ of (3.3) yields $D_C F_1^{-1}$. The choice (3.4) implies that this quantity is a unit operator. Inversely, if (3.3) is a right coprime factorization then exists a unit operator W such

$$\begin{bmatrix} \bar{N}_P \\ \bar{D}_P \end{bmatrix} = \begin{bmatrix} N_P\, V^{-1} \\ D_P\, V^{-1} \end{bmatrix} W$$

which implies (3.4).

- A similar reasoning process in the case where $r_1 = 0$ shows that

$$F_2 = N_C W, \quad (3.5)$$

with W a unit operator, is a necessary and sufficient condition to produce a right coprime factorization

$$\begin{bmatrix} N_P\, V^{-1} N_C F_2^{-1} \\ D_P\, V^{-1} N_C F_2^{-1} \end{bmatrix}$$

of P_o. Here, we have to restrict attention to the case where N_C is square, i.e. the number of inputs u and outputs y is equal.

- Let us now assume that $F_1 = 1$, $F_2 = 1$, $r_1 = N\, r$ and $r_2 = D\, r$. The closed loop equations reduce to

$$\begin{aligned} y &= N_P\, V^{-1}[N_C N + D_C D] r + \partial Y_v\, v + v, \\ u &= D_P\, V^{-1}[N_C N + D_C D] r + \partial U_v\, v. \end{aligned}$$

It can easily be seen that if N and D have been chosen such that $N_C N + D_C D$ is a unit and, provided that u, y and r are available for computation, one can estimate a right coprime factorization of P_o using open loop identification techniques.

Remark 5
When C is a known stable operator (not necessarily linear), one can without loss of generality choose a left coprime factorization with $N_C = C$ and $D_C = 1$. Then, assuming that $r_2 = 0$, it is easy to show that, under a small signal assumption on v and a smoothness assumption on the closed loop operators, the equations reduce to

$$\begin{aligned} y &= N_P[D_P - N_C(-N_P)]^{-1}\, r_1 + \partial Y_v\, v + v, \\ u &= D_P[D_P - N_C(-N_P)]^{-1}\, r_1 + \partial U_v\, v. \end{aligned}$$

This allows an open loop like identification of right coprime factors of P_o. The filter F_1 can be chosen to be any unit transfer function. Similarly, if C is stably invertible and r_1 equals zero, one can choose $N_C = I$ and $D_C = C^{-1}$ without loss of generality. Using small signal arguments, one obtains the following closed loop equations

$$\begin{aligned} y &= N_P[D_C D_P + N_P]^{-1}\, r_2 + \partial Y_v\, v + v, \\ u &= D_P[D_C D_P + N_P]^{-1}\, r_2 + \partial U_v\, v. \end{aligned}$$

Again, it is possible to estimate a right coprime factorization of P_o using open loop like techniques. We can choose any unit transfer function for F_2.

Remark 6
It is not clear how to select the data filters F_1 or F_2 in the general case, i.e. when C can be nonlinear, unstable and/or non minimum phase.

Remark 7
In the linear case, the remaining freedom in F_1 and F_2 is used to estimate normalized coprime factors, i.e. the liberty in choosing the unit operator W is used to construct a normalized coprime factorization of P_o. We refer the

reader to [11] for more details. Similar ideas could be applied in the nonlinear case. We refer the reader to [13] for more details on normalized coprime factorizations in the nonlinear case.

4 Simulation example

In this section, we illustrate the results presented in this paper for the two step method. We consider a nonlinear system operating in closed loop with some two degree of freedom linear controller. The nonlinear system is described by

$$y_t = \frac{q^{-1}(1+b_1 q^{-1})}{1+a_1 q^{-1}+a_2 q^{-2}} \mathrm{DZ}(u_t) + v_t \qquad (4.6)$$

where DZ is a nonlinear deadzone operator defined using the following equations

$$\mathrm{DZ}(u_t) = \begin{cases} u(t) - d_p & \text{if} \quad u_t \geq d_p \\ 0 & \text{if} \quad -d_m < u_t < d_p \\ u_t + d_m & \text{if} \quad u_t \leq -d_m \end{cases} \qquad (4.7)$$

with $|b_1| < 1$, $d_m > 0$ and $d_p > 0$. The disturbance signal is modeled as follows

$$v_t = \frac{1+c_1 q^{-1}+c_2 q^{-1}}{1+a_1 q^{-1}+a_2 q^{-1}} e_t \qquad (4.8)$$

where e_t is zero mean white noise of variance σ^2. The linear controller is the optimal two degree of freedom minimum variance controller for the linear system that is obtained from (4.6) by setting both d_m and d_p to zero, i.e.

$$u_t = C_r(q^{-1})\, r_t - C_y(q^{-1})\, y_t, \qquad (4.9)$$

$$C_r(q^{-1}) = \frac{1+c_1 q^{-1}+c_2 q^{-1}}{1+b_1 q^{-1}}, \qquad (4.10)$$

$$C_y(q^{-1}) = \frac{(c_1-a_1)+(c_2-a_2)\, q^{-1}}{1+b_1 q^{-1}}. \qquad (4.11)$$

Here, we have taken the following plant parameters

$$\begin{aligned} b_1 &= -0.9, & a_1 &= -1.5, & a_2 &= 0.7, \\ c_1 &= -1, & c_2 &= 0.2, & d_p &= 0.7, \\ d_m &= 0.2, & \sigma^2 &= 0.3, \end{aligned}$$

but of course these values are not provided to the identification algorithm but rather are to be identified, nor is the identification algorithm provided with the information as to how the controller is designed, though the algorithm is provided with the transfer functions C_r and C_y defining the controller. The reference signal r_t was chosen to be a known unit variance and zero mean white noise signal independent of the process disturbance signal v_t. Note that this corresponds to an input signal u_t that is of the same order of magnitude as d_p and d_m. With an input signal of much greater magnitude than d_p and d_m, these quantities would be hard to identify, the effect of the nonlinearity being swamped by the signal; if u_t is typically of much smaller magnitude, there is obviously also a problem. Using the previous closed-loop system, we have generated a data set $\{r_t, u_t, y_t\}$ with signals of length $N = 2000$.

For the identification of the plant itself, we have used the following model structure

$$\hat{y}_t(\theta) = \theta_1 \frac{q^{-1}(1+\theta_2 q^{-1})}{1+\theta_3 q^{-1}+\theta_4 q^{-2}} \bar{\mathrm{DZ}}(u_t) \qquad (4.12)$$

where $\bar{\mathrm{DZ}}$ is defined as in (4.7) with d_p and d_m, respectively, replaced by θ_5 and θ_6. Estimates of the parameters where obtained by minimizing

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}[y_t - \hat{y}_t(\theta)]^2 \qquad (4.13)$$

with respect to θ using a steepest descent method. Using the previously defined data set, we have applied two strategies:

Strategy I: direct standard open loop identification, i.e. we have used the data set $\{u_t, y_t\}$ as if it had been collected in open loop.

Strategy II: modified two-step method where $\hat{u}_r$ is obtained by using the sensitivity operator that is obtained by interconnecting the model (4.12) in feedback with the controller (4.9). The feedback loop was simulated with reference signal r_t and with the parameters identified using Strategy I, i.e. there is some approximation involved here.

We have obtained the following results

	True values	Initial values	Strat. I $\{u, y\}$	Strat. II $\{\hat{u}_r, y\}$
θ_1	1	1	0.82	1.02
θ_2	-0.9	-0.2	-0.88	-0.87
θ_3	-1.5	-1.3	-1.44	-1.50
θ_4	0.7	0.5	0.64	0.74
θ_5	0.7	0	0.62	0.67
θ_6	0.2	0	0.15	0.22
$V_N(\theta)$			0.41	0.31

Table 4.1: Identification cost and identified parameters using a one step procedure and a modified two step method.

Table 4.1 shows the results of estimating P_o with both procedures. In Figure 4.2, we have compared the magnitude Bode plots of the linear part of the identified models. The results clearly show the degraded performance of the direct identification scheme, i.e. this scheme is unable to produce bias free estimates. The indirect two step method gives more accurate results for the linear part of the (4.6). Note that the parameters characterizing the nonlinear part of the plant could also be identified more accurately with the modified two-step methods although the smoothness assumption on P_o is not satisfied here. The use of a nonlinear model structure for the direct identification of the sensitivity will most probably improve the identification accuracy of the parameters and the applicability of the method in general. The authors are presently experimenting with neural networks in order to obtain better noise free estimates of the control input. Note that the use of complicated model structures for the first step of the two step procedure, i.e. neural nets, is not a drawback here since their use is only in the generation of a noise free estimate $\hat{u}_r$.

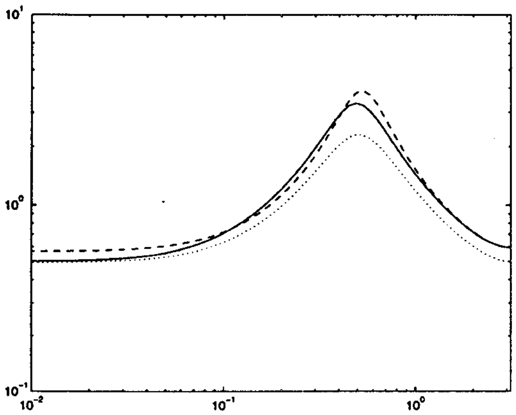

Figure 4.2: Amplitude Bode plots of the linear part of the nonlinear system (4.6) (—) and estimates of this same transfer function obtained using a one step method (···) and a modified two step method (−−)·

5 Conclusion

In this paper, we have extended two linear methods for the identification of approximate models of an open loop plant on the basis of closed loop data to the nonlinear case. The first method is based on an indirect two step method based on the identification of the sensitivity function. The second method identifies the right coprime factors through an identification of the sensitivity and the complementarity sensitivity function of the closed loop system. In both cases, we have assumed that the noise signal enters the system under a high SNR and that the closed loop system is a smooth function of the reference signals and the disturbance signals. Preliminary simulation results with the two step method have shown to give very satisfactory results.

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