ON THE ROBUSTNESS OF THE FRACTIONALLY-SPACED CONSTANT MODULUS CRITERION TO CHANNEL ORDER UNDERMODELING: PART I

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ABSTRACT

This paper studies the robustness properties of the constant modulus (CM) criterion specifically when the fractionally-spaced equalizer time span is less than that of the channel. Hence, there necessarily exists an error in the equalized signal. Noiseless, binary signalling is considered. The change in CM cost from a perfect equalization setting is derived for two cases: i) perturbations to the channel outside the time span of the equalizer, and ii) equalizer truncation. This CM cost is related to the mean squared error (MSE) cost and a design guideline for length selection is proposed. This guideline is shown by example to be robust when noisy, multi-level complex signalling is considered.

1. INTRODUCTION

The Constant Modulus (CM) criterion was first proposed by Godard in [2] and developed independently by Treichler and Agee in [8]. The stochastic gradient descent implementation, or Constant Modulus Algorithm (CMA), is widely used in practice (see [7]) though analytically lacks the maturity of LMS.

Reference [6] shows that under certain assumptions, fractionally-spaced CMA is globally convergent. One of these assumptions is that the length of the fractionally-spaced CMA equalizer (CMA-FSE) is at least as long as that of the channel. Analytic results addressing the violation of this length condition are rare ([10], [1] and this paper are exceptions).

One contribution examining CM robustness questions is the recent analysis in [9]. Using a geometric view, a CM local minimum (in the presence of noise and satisfying the length condition) is shown to lie in a neighborhood of a Wiener solution corresponding to an unknown system delay when sufficient conditions are satisfied. The MSE of the CM receiver above that of the MMSE receiver is bounded and approximated. An example shows the approximation is closer to the true MMSE performance than some quadratic approximations.

This geometric view is extended in [10] to consider singular channel matrices resulting from possibly a fractionally-spaced channel with common subchannel roots or violation of the length condition. Another result for violation of the length condition is [1], (Part II of this work) which extends the results of this paper to multi-level signalling and connects the results to previous work on CMA-FSE misadjustment.

This paper studies the robustness properties of the CM criterion specifically when the length condition is not satisfied. Hence, there necessarily exists an error in the equalized signal. Noiseless, binary signalling is considered. We take an algebraic approach to describe the CM error surface deformation, which is not wholly unrelated to the geometric approach proposed in [10]. The change in CM cost from a perfect equalization setting is derived for two cases: i) perturbations to the channel outside the time span of the equalizer, and ii) equalizer truncation. It is shown that the change in CM cost is small under certain assumptions, resulting in a small change in the MSE. A design guideline for FSE length selection is proposed based on these results so that the CMA-FSE can achieve a prescribed MSE threshold and be transferred to decision directed (DD) LMS. The design guideline suggests that the FSE length be chosen long enough to cover those channel coefficients whose magnitudes are greater than approximately 20% of the magnitude of the largest channel tap.

The paper is organized as follows: §2 describes the proposed analyses approaches in addressing the undermodeling problem. §3 applies these approaches to the binary CM criterion. §4 relates these results to the MSE criterion. §5 proposes an interpretation of the results, including a design guideline for FSE length selection, supported by examples. §6 contains concluding remarks. Please note that Part II of this work, [1], extends the results of this paper to multi-level signalling and connects these results with previous work on CMA-FSE misadjustment.

2. ANALYSES APPROACHES

2.1. Channel Perturbation

The first approach taken in addressing the robustness of a CM receiver to undermodeling is to consider those channel coefficients that are outside the time span of the FSE as channel perturbations, in order to study the CM cost incurred and infer a design guideline for FSE length selection. This CM criterion is specifically the one minimized by CMA-FSE in a stochastic gradient descent implementation, though MSE or even BER may be the ultimate performance measure. Let

$$\mathbf{c} = [c_0 \; c_1 \ldots c_{L_c-1}]^T \qquad (1)$$

be the length-L_c channel impulse response vector, which is zero outside this finite time support. Define two length-L_c vectors; vector $\mathbf{c_m}$ contains L_m ($L_m < L_c$) consecutive taps of $\mathbf{c}$ in the same positions as they occurred in $\mathbf{c}$ with zeros in the remaining $L_c - L_m$ positions, and vector $\mathbf{c_p}$ contains the $L_c - L_m$ taps of $\mathbf{c}$ that are not in $\mathbf{c_m}$ in the same positions as they occurred in $\mathbf{c}$, with zeros in the remaining L_m positions. Hence, $\mathbf{c} = \mathbf{c_m} + \mathbf{c_p}$; with a length-$L_m$ FSE, the "full length" channel is composed of a "modeled" portion

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which may be perfectly equalizable (baud-spaced, combined channel-FSE is a pure delay), and a "perturbation" portion which is potentially non-zero outside the time support of the FSE. For example, one such partitioning of the channel taps is

$$\mathbf{c_m} := [c_0\ c_1 \dots c_{L_m-1} \quad \underbrace{0\ 0 \dots 0}_{L_c-L_m\ zeros}\ ]^T$$

$$\mathbf{c_p} := [\underbrace{0\ 0 \dots 0}_{L_m\ zeros}\ c_{L_m}\ c_{L_m+1} \dots c_{L_c-1}]^T \qquad (2)$$

which considers the perturbation as appended channel taps with largest delay from the current symbol.

Further, let $\mathbf{C_m}$, $\mathbf{C_p}$ and $\mathbf{C}$ be the convolution matrices associated with $\mathbf{c_m}$, $\mathbf{c_p}$ and $\mathbf{c}$, respectively, and let $\mathbf{f_m}$ be the equalizer coefficient vector corresponding to a global minimum of the CM cost associated with channel $\mathbf{c_m}$. The combined channel-equalizer before decimation becomes

$$\mathbf{h} = \mathbf{C}\mathbf{f_m} \qquad (3)$$
$$= \mathbf{C_m}\mathbf{f_m} + \mathbf{C_p}\mathbf{f_m} \qquad (4)$$

and the decimated (baud-spaced) version can be written as

$$\mathbf{h}_\downarrow = \mathbf{C}_{\mathbf{m}\downarrow}\mathbf{f_m} + \mathbf{C}_{\mathbf{p}\downarrow}\mathbf{f_m} \qquad (5)$$
$$= \mathbf{h_m} + \mathbf{h_p} \qquad (6)$$

where $\mathbf{C}_{\mathbf{m}\downarrow}$ and $\mathbf{C}_{\mathbf{p}\downarrow}$ are appropriately row-decimated versions of $\mathbf{C_m}$ and $\mathbf{C_p}$, respectively. Observe that since $\mathbf{f_m}$ is a global minimum of the CM criterion with respect to channel $\mathbf{c_m}$, there is no error in the equalized signal due to the first term in (6) since the source kurtosis is less than 3 (see [5]). The second term is the effect of the channel perturbations outside the time span of the FSE.

2.2. Equalizer Truncation

A related approach to that above is to consider the effect on the CM cost due to equalizer taps lost in truncation from the FSE which achieves perfect equalization. Let

$$\mathbf{f}^* = [f_0^*\ f_1^* \dots f_{L_c-1}^*]^T \qquad (7)$$

be a CM global minimum for channel $\mathbf{c}$. Define two length-L_c vectors $\mathbf{f_t}$ and $\tilde{\mathbf{f}}$ such that $\mathbf{f_t} = \mathbf{f}^* + \tilde{\mathbf{f}}$. Vector $\mathbf{f_t}$ contains L_m consecutive taps of $\mathbf{f}^*$ in the same positions as they occurred in $\mathbf{f}^*$ with zeros in the remaining $L_c - L_m$ positions, and vector $-\tilde{\mathbf{f}}$ contains the $L_c - L_m$ taps of $\mathbf{f}^*$ that are not in $\mathbf{f_t}$ in the same positions as they occurred in $\mathbf{f}^*$, with zeros in the remaining L_m positions. For example, one such partitioning is

$$\mathbf{f_t} := [f_0^*\ f_1^* \dots f_{L_m-1}^* \quad \underbrace{0\ 0 \dots 0}_{L_c-L_m\ zeros}\ ]^T$$

$$\tilde{\mathbf{f}} := [\underbrace{0\ 0 \dots 0}_{L_m\ zeros}\ -f_{L_m}^*\ -f_{L_m+1}^* \dots -f_{L_c-1}^*]^T \qquad (8)$$

The baud-spaced, combined channel-equalizer response for the truncated equalizer $\mathbf{f_t}$ can be written as

$$\mathbf{h}_\downarrow = \mathbf{C}_\downarrow\mathbf{f_t} \qquad (9)$$
$$= \mathbf{C}_\downarrow\mathbf{f}^* + \mathbf{C}_\downarrow\tilde{\mathbf{f}} \qquad (10)$$

where $\mathbf{C}_\downarrow$ is the appropriately row-decimated version of $\mathbf{C}$. Observe that (10) is the same form as (6), where the first term is analogous to the "modeled" contribution, and second term is analogous to the "perturbation" contribution. Thus, the first term satisfies the length condition and will achieve perfect equalization since $\mathbf{f}^*$ is a global minimum of the CM criterion and the source kurtosis is less than 3 (see [5]). Our goal in addressing CMA's robustness properties is to study the effect of the second terms of (6) and (10) on the CM cost function.

3. CM CRITERION

Notice that the above analysis approaches do not specifically address the CM cost of the optimal (in CM-sense) length-constrained FSE. Instead, our analyses approaches change the problem slightly to one that is answerable. The CM-cost for this length-constrained optimum FSE, however, is bounded due to optimality by the cost we now derive.

The CM cost function for the BPSK, white, zero-mean, equiprobable source case can be written as (see [4])

$$J_{CM} = 1 - 2\sum_{i=0}^{P} h_i^2 + 3\left(\sum_{i=0}^{P}\sum_{j=0,j\neq i}^{P} h_i^2 h_j^2\right) + \sum_{i=0}^{P} h_i^4 \quad (11)$$

where the h_i are the elements of $\mathbf{h}_\downarrow$ and $P = \lceil (L_c + L_m - 2)/N \rceil$, with $\lceil\cdot\rceil$ denoting round-up to the nearest integer, and N is the oversample ratio (typically 2). Further, each h_i can be expressed in terms of the "modeled" and "perturbation" portions of the channel-equalizer combination, $h_i = m_i + p_i$, i.e., $m_i \in \mathbf{C}_{\mathbf{m}\downarrow}\mathbf{f_m}$ or $m_i \in \mathbf{C}_\downarrow\mathbf{f}^*$, and $p_i \in \mathbf{C}_{\mathbf{p}\downarrow}\mathbf{f_m}$ or $p_i \in \mathbf{C}_\downarrow\tilde{\mathbf{f}}$.

Now consider the three terms of (11) separately.

$$1 - 2\sum_{i=0}^{P} h_i^2 = 1 - 2\sum_{i=0}^{P}(m_i + p_i)^2 \qquad (12)$$
$$= 1 - 2\sum_{i=0}^{P}(m_i^2 + 2m_ip_i + p_i^2)$$
$$= -1 - 2(2p_\delta + p_\delta^2) - 2\sum_{i\neq\delta,i=0}^{P} p_i^2 \quad (13)$$

since the "modeled" part of the channel achieves perfect equalization. Hence, $m_\delta = 1$ and $m_i = 0\ \forall i \neq \delta$. (The case where $m_\delta = -1$ can be shown to be equivalent).

Similarly, express

$$\sum_{i=0}^{P} h_i^4 = \sum_{i=0}^{P}(m_i + p_i)^4$$
$$= 1 + 6p_\delta^2 + 4p_\delta + 4p_\delta^3 + \sum_{i=0}^{P} p_i^4 \qquad (14)$$

And for the double sum in (11)

$$\sum_{i=0}^{P}\sum_{j=0,j\neq i}^{P} h_i^2 h_j^2 = \sum_{i=0}^{P}\sum_{j=0,j\neq i}^{P}(m_i + p_i)^2(m_j + p_j)^2$$
$$= 2(1 + 2p_\delta)\sum_{i=0,i\neq\delta}^{P} p_i^2 + \sum_{i=0}^{P}\sum_{j=0,j\neq i}^{P} p_i^2 p_j^2 \quad (15)$$

since terms involving cross products of m_i and m_j are zero due to the summation limits and the perfect equalizability assumption.

Now collecting terms (13), (14), and (15), the CM cost incurred from undermodeled channel taps is expressed as

$$\Delta J_{CM} = \left[4\|\mathbf{h_p}\|_2^2\right] + p_\delta\left[4p_\delta^2 + 12\sum_{i\neq\delta,i=0}^{P} p_i^2\right]$$
$$+ \left[\sum_{i=0}^{P} p_i^4 + 3\sum_{i=0}^{P}\sum_{j=0,j\neq i}^{P} p_i^2 p_j^2\right] \qquad (16)$$

Observe that the terms in (16) are grouped according to powers of the perturbation elements p_i. Further observe that the cubic terms in the CM cost function are proportional to p_δ and therefore depend on the delay sought in $\mathbf{h_m}$ by the CMA-FSE, which is unknown due to the inherent phase invariance in the CM cost function.

4. RELATION TO MSE

CMA-FSE's performance as reported in field tests and simulation studies suggest broad capabilities of the CM criterion for blind update of a FSE. Recognize, however, a classical compromise: the user of CMA-FSE is minimizing one cost in hopes of minimizing another cost, namely MSE (which is equalivalent to minimizing BER under certain assumptions on the noise and source statistics). However, the CM cost can be related to the MSE cost in the vicinity of a global minimum. We next derive the change in the MSE cost due to channel perturbations outside the time span of the FSE, as was done in deriving (16).

$$J_{MSE} = E\{(y-s_\delta)^2\} = \sum_{i=0}^{P} h_i^2 + \sigma_s^2 - 2\sigma_s^2 h_\delta \quad (17)$$

where y is the FSE output, s_δ is a source symbol, and σ_s^2 is the source power. Hence, the change in MSE cost is

$$\Delta J_{MSE} = \sum_{i=0}^{P} (m_i + p_i)^2 + 1 - 2(m_\delta + p_\delta) \quad (18)$$

$$= ||\mathbf{h_p}||_2^2 \quad (19)$$

Therefore, when the higher-order terms in (16) can be neglected (specifically when the perturbation terms are "small") the CM and MSE costs are related by

$$\Delta J_{CM} \approx 4\Delta J_{MSE} \quad (20)$$

In this case, the CM and MSE costs incurred due to undermodeling are "small," and the CM cost is approximately a scaled version of the MSE cost. This behavior suggests a small deformation in both error surfaces due to undermodeling, so that the CM minimum stays in the neighborhood of the MSE minimum.

5. INTERPRETATION

5.1. Design Guideline: FSE Length Selection

The results thus far help to explain the robustness of the CM criterion to the nearly unavoidable situation of channel order undermodeling, which prevents perfect equalization. The objective of this section is to infer a design guideline for FSE length selection based on this preceding analysis.

The design guideline should at least attempt to keep the quantity in (16) "small". To this end, the "perturbation" coefficients should be small enough so that the cubic and quartic terms in (16) can be neglected and the CM cost is approximately a scaled version of the MSE cost. Hence, the FSE should span the "significant" portion of the channel impulse response. To address what "significant" means, we consider channel models derived from empirical measurements of digital microwave radio signals in the San Francisco Bay Area which are described in [3] and now available over the *www* from the database at *http://spib.rice.edu/spib/microwave.html.*

We evaluate (16) for the two approaches previously described in §2. The channel taps for the channel-perturbation approach of §2.1 are partitioned according to (2). The partitioning of FSE coefficients for the FSE-truncation approach of §2.2 is according to (8). The results are scaled (by a factor of $\frac{1}{4}$) to approximate the MSE cost. Figure 1 shows the results for channel 3 from the database. Results for other channels from the database can be found over the *www* at *www.ee.cornell.edu/~johnson/BERG.* Each figure contains two plots. The top plot contains the magnitude of the $T/2$ channel impulse response coefficients. The bottom plot contains the graphs of three functions: i) (solid) the approximation of MSE from a scaled version of (16) due to channel perturbations outside the FSE time span (§2.1), ii) (dashed) the approximation of MSE from a scaled version of (16) due to FSE truncation (§2.2), and iii) (dotted) the true MSE described by (19) according to the approach in §2.1–this quantity is precisely a scaled version of the quadratic contribution of (16). The graphs may be referenced to the dashed line of constant MSE which corresponds to a Transfer Level for which CMA is typically transferred to decision directed (DD) LMS when further error-rate reduction or tracking is needed. (Here, the Transfer Level corresponds to a 16-QAM, unit variance source).

These figures show that though an FSE length at least as long as the channel is needed for perfect equalization, far fewer CMA-FSE taps are needed for successful transfer to DD-LMS for this practical class of channels. The "significant" portion of the channel appears to be those coefficients whose magnitudes are greater than about 20% of the magnitude of the largest channel tap, since little improvement in the approximate MSE is observed by increasing the FSE length to span coefficients less than this threshold. Also, in this region, the true MSE as described in iii) above is essentially the same as the approximated MSE as described in i) above, i.e., the CM cost is essentially a scaled version of the MSE cost. Therefore, the CM minimum stays in a tight neighborhood of the MSE minimum. These figures also show that the two different but related approaches described in §2 are not order-able (one is not always greater than the other), suggesting the validity of both.

It should be noted that this design guideline is quite different than that typically used for the baud-spaced case, but quite similar in motivation. For example, side-bar C of [7] proposes for the baud-spaced case to first approximate the channel as a two-ray model, so that the channel inverse and hence sum of unequalized terms can be approximated and kept "small."

16-QAM Example:

We next test the FSE length-selection-guideline which was derived for noiseless, BPSK signalling and apply it to a 32-tap channel derived from data in [3] whose impulse response magnitudes are shown in the left plot of Figure 2, but use more realistic 16-QAM signalling with additive white gaussian noise of SNR 30 dB. The design guideline suggests that the FSE span the "significant" channel coefficients– those taps greater in magnitude than approximately 20% of the magnitude of the largest tap. This rule implies the FSE span the second and third (dominant) rays of this channel, about $27 - 12 = 15$ FSE taps. CMA-FSE is initialized with a unity center spike and the MSE trajectories are averaged over ten independent source sequences of length 10,000 $T/2$-spaced elements. The MSE trajectories for various FSE lengths are shown in the right plot of Figure 2.

The results agree remarkably well with our simple guideline; a length-16 FSE is barely sufficient to reach the Transfer Level, while a length-8 FSE fails and length-24 FSE is excessively long. Though our design guideline was derived for noiseless BPSK signalling, it appears robust when applied to higher-order, complex signalling with modest noise power.

5.2. Other Observations

Other observations on the analysis can be made which may (or may not) prove insightful for certain problems.

Observation 1:
When the error perturbation elements are small,

$$\mathbf{h_p} = [\epsilon_0\ \epsilon_1 \ldots \epsilon_P]^T \tag{21}$$

with $|\epsilon_i| \leq \epsilon_{max} \ \forall\ i$, then (16) is dominated by the quadratic terms and can be bounded:

$$\Delta J_{CM} \approx 4||\mathbf{h_p}||_2^2 \leq 4P\epsilon_{max}^2 \tag{22}$$

$$= (\frac{4}{N})(L_c + L_m + N - 1)\epsilon_{max}^2 \tag{23}$$

This bound may be helpful for analysis of certain known channel classes.

Observation 2:
When $L_m \approx L_c$, the vector $\mathbf{c_p}$ is predominantly filled with zeros, by definition (see (2) and (8)). Hence, the corresponding combined channel-equalizer response, $\mathbf{h_p}$, has many zero-valued elements. Depending on the value of δ sought by CMA-FSE, p_δ is likely to be zero. In this case, the cubic terms in (16) disappear, leaving only quadratic and quartic contributions. This dependence suggests a connection between the penalty paid due to undermodeling and "proper" CMA-FSE initialization, which remains unresolved.

Observation 3:
The terms in (16) can be expected to suffer a relative increase in magnitude when either the "full-length" channel, $\mathbf{c}$, or the "modelled" portion, $\mathbf{c_m}$, have nearly reflected roots. For example, suppose the length condition is violated according to the approach described in §2.1, so that $m_i \in \mathbf{C_{m\downarrow}f_m}$ and $p_i \in \mathbf{C_{p\downarrow}f_m}$. As the "modeled" portion of the channel ($\mathbf{c_m}$) loses disparity, the equalizer suffers noise gain enhancement (an increase in the relative magnitude of the elements of $\mathbf{f_m}$,) which in turn causes a magnification of the p_i and thus a relative increase in (16). Compare this idea with the approach discussed in §2.2 of truncated FSE coefficients, so that now $m_i \in \mathbf{C_\downarrow f^*}$ and $p_i \in \mathbf{C_\downarrow \tilde{f}}$. In this case, it is reflected roots in $\mathbf{c}$ rather than $\mathbf{c_m}$ which cause a relative increase in the coefficients of $\mathbf{f^*}$ and hence also in $\tilde{\mathbf{f}}$ and the p_i. The bounds derived in (16) may therefore become too large to be practical when $\mathbf{c}$ or $\mathbf{c_m}$ become singular. It should be noted, however, that the channel models derived from data in [3] and used in the previous examples all have *nearly* reflected roots, and still produce useful results in determining FSE length selection.

6. CONCLUDING REMARKS

Unlike many of its competitors, numerous comparison studies show CMA-FSE is robust to a variety of practical situations. This paper has studied the CM criterion robustness properties when the length condition is violated, a nearly unavoidable condition in practice for high data rate communications. It is shown that the effect on the CM cost and the MSE cost is small when channel coefficients outside the time span of the FSE are small, so that the CM minimum stays in a tight neighborhood around the MSE minimum. This analysis suggests a design guideline for FSE length selection based on noiseless, binary signalling which is shown by example to be accurate and robust when applied to higher-level, complex signalling with modest noise power. Part II of this work, [1], extends the results of this paper to multilevel signalling and connects FSE length selection with the resulting CMA-FSE misadjustment term.

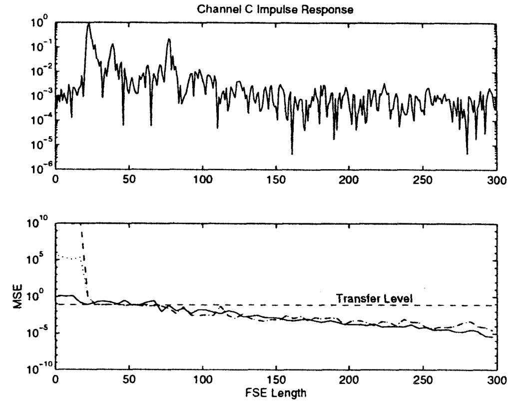

Figure 1. AppSigTec Channel 3

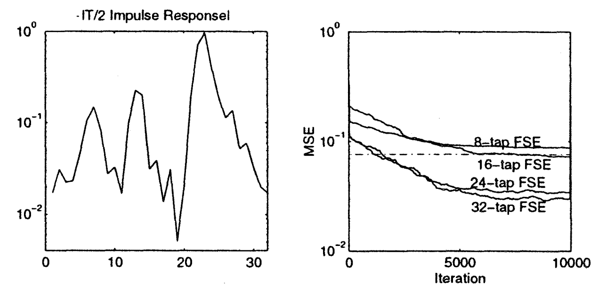

Figure 2. CMA-FSE on 32-tap Microwave Channel

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