IDENTIFICATION OF NONLINEAR PLANTS USING LEFT COPRIME FRACTIONAL BASED REPRESENTATIONS

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Abstract

It has been shown that the set of all nonlinear plants that can be stabilized by a known linear controller which also stabilizes a linear nominal model of the plant can be parametrized by a stable operator known as the Youla-Kucera parameter. This paper extends previous work by allowing the model of the nominal plant in the above scenario to be nonlinear, and hence explores the possibilities of converting the closed loop plant identification problem to one of open loop identification. The ideas rely on a concept of differential coprimeness for nonlinear fractional system descriptions.

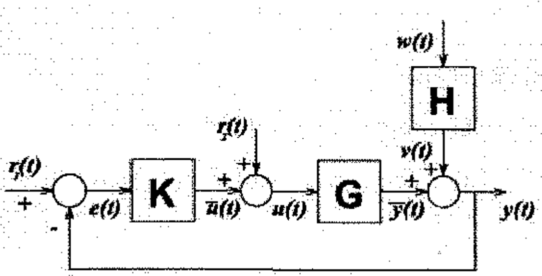

Figure 1: *Closed Loop*

1 Introduction

Consider the setting shown in Figure 1, where G is a nonlinear plant to be identified, K is a linear controller, and H is a linear stable output measurement noise generating system, driven in turn by the zero mean, white, stationary noise process $w(t)$. It is assumed that K internally stabilizes the unknown plant G. While we restrict attention to time-invariant K and G, there would seem to be no difficulty in extending the ideas to the time-varying case, as in [1].

This paper endeavours to extend the current theory of [1-4] relating to NLTV plants (which is based on the use of a Youla-Kucera parameter and is principally linked to right coprime realizations and/or a priori nominal linear plant models) to enable one to find a *left* coprime factorization based description of the set of all plants stabilized by a given controller, given a nominal plant model that is *not necessarily linear.*

Section 2 contains definitions and assumptions that are used throughout the paper, and a notion of *differential coprimeness.* This section also sets out some stability and operator existence requirements. Section 3 characterizes the set of all plants stabilized by a known linear controller. This is done using a Youla-Kucera parameter. It covers both the noise free case and then modifies this description to incorporate non-zero measurement noise into the system. The section is concluded by depicting how these model characterizations under a high SNR assumption can be used to identify the system. In general, identification requires r_1 and r_2 (see Figure 1) to be non zero. Section 4 offers concluding remarks.

2 The Class of Plants in the noise free case.

This Section states the definitions, assumptions and some stability characterizations that later results in the paper are based on.

2.1 Definitions

For an operator A:

A is well posed if given: $z = Ax$
a time function z can be causally, uniquely determined from a time function x for all bounded x, where any choice of norm may be used.

A is invertible if for all $z \in \mathcal{Y}$, x can be causally and uniquely determined.

A is bounded input - bounded output

(BIBO) stable if $\|A\| = sup_{\|x\| \neq 0} \frac{\|Ax\|}{\|x\|}$ is finite. (The norm is taken, for convenience to be the L_2 norm.)

For *a closed loop* as shown in Figure 1, with noise w identically zero:

The closed loop is well posed if $[e, \bar{u}, u, y]$ can be causally, uniquely determined from r_1 and r_2 for all bounded r_1, r_2.

The closed loop is internally stable if $[e, \bar{u}, u, y]$ is bounded for all bounded r_1 and r_2.

If the noise v is non-zero r_1 and r_2 must be replaced with r_1, r_2 and v in these definitions.

The next two definitions, of coprimeness in nonlinear fractional representations, are not universal. The definitions here, based on Bezout identities, have been used in eg. [[5], [6], [2]]. However, alternative definitions, based on set theoretic ideas, have been used in eg. [[7], [8], [9]]. In the linear case, the definitions are equivalent; in the nonlinear case, right coprimeness defined using a Bezout identity implies right coprimeness from a set theoretic view point.

Right coprimeness (linear or nonlinear) Let M, N be a right factorization for $G : \mathcal{U} \to \mathcal{Y}$, $G = NM^{-1}$, $N : \mathcal{C} \to \mathcal{Y}$, $M : \mathcal{C} \to \mathcal{U}$ where M and N are BIBO stable. Then M, N is a right *coprime* factorization of G if there exists a BIBO $\mathcal{L}$ for which

$$\mathcal{L} \begin{bmatrix} M \\ N \end{bmatrix} = I.$$

Left coprimeness (linear or nonlinear) Let $\tilde{M}$, $\tilde{N}$ be a left factorization for $G : \mathcal{U} \to \mathcal{Y}$, $G = \tilde{M}^{-1}\tilde{N}$, $\tilde{N} : \mathcal{U} \to \tilde{\mathcal{C}}$, $\tilde{M} : \mathcal{Y} \to \tilde{\mathcal{C}}$ where $\tilde{M}$ and $\tilde{N}$ are BIBO stable. Then $\tilde{M}$, $\tilde{N}$ is a left *coprime* factorization of G if there exists a BIBO $\mathcal{L}$ such that

$$\begin{bmatrix} \tilde{M} & \tilde{N} \end{bmatrix} \mathcal{L} = I.$$

Lipschitz Continuity F is differentially bounded by $\Theta_F, \epsilon_F \Leftrightarrow |a_1 - a_2| \leq \epsilon_F \to |Fa_1 - Fa_2| < \Theta_F$.

Now suppose A is globally Lipschitz continuous, ie $\forall$ bounded signals x and y, we have

$$\|A(x + y) - A(x)\| \leq \mathcal{K}\|y\|$$

for some constant $\mathcal{K}$. This means there exists an operator A_x, (causally) dependent on x and bounded independently of x with

$$A_x(y) = A(x + y) - A(x)$$

In case A is linear, the operator A_x is independent of x and

$$A_x(y) = A(y)$$

An important result is obtained as follows. Suppose that a Lipschitz $\mathcal{A}$ is causal. Let x be a possibly unbounded signal as $t \to \infty$, and let x_T denote the truncation of x at time T, ie. $x_T(t) = x(t)$ for $t \leq T$, $x_T(t) = 0$ for $t > 0$. Then x_T is obviously bounded. Evidently, for all T,

$$\|\mathcal{A}(x_T + y) - \mathcal{A}(x_T)\| \leq K_1\|y\|.$$

Since this holds for all T, it is evident that

$$\|\mathcal{A}(x + y) - \mathcal{A}(x)\| \leq K_1\|y\|,$$

ie. the requirement that x be bounded can be relaxed when $\mathcal{A}$ is causal.

Differential Coprimeness Let

$$NU + DV = \mathcal{I},$$

where N, D, U and V are all nonlinear. (Actually, in the immediate application, U and V will be linear). Let

$$\delta N_x(y) = N(x + y) - N(x) \quad (2.1)$$
$$\delta D_z(y) = D(z + y) - D(z)$$

If N, D, U and V are linear, then

$$(\delta N_x)U + (\delta D_z)V = \mathcal{I}.$$

In the nonlinear scenario, we say that N and D are *differentially coprime* if

$$(\delta N_x)U + (\delta D_z)V = \mathcal{W}_{xz},$$

where $\mathcal{W}_{xz}$ is a unit.

2.2 Assumptions

Assumption 1 *The nonlinear plant G is well posed.*

Assumption 2 *The plant has no direct feed-through term. That is, there is no delay-free transmission from the plant input to output.*

Assumption 3 *(i) The controller K is linear and has left and right coprime factors (U_l, V_l) and (U_r, V_r) respectively. That is, with U_l, V_l, U_r, V_r all linear, stable and well posed, and with V_l, V_r invertible, one has*

$$K = U_r V_r^{-1} = V_l^{-1} U_l.$$

(ii) There exists a nominal plant model G_o such that

$$G_o = N_r D_r^{-1} = D_l^{-1} N_l,$$

where N_l, D_l, N_r, D_r are well posed operators that are not necessarily linear, with D_l, D_r invertible, with D_r, N_r coprime, and with N_l and D_l globally Lipschitz continuous and totally differentially coprime. Further, G_o has no direct feedthrough term.

Assumption 4 *The controller K forms a well posed loop with the nominal plant model G_o.*

Assumption 5 *The controller K stabilizes the nominal plant model G_o.*

2.3 Stability and operator existence

In order to establish the main result, we shall need several characterizations of stability. The first is effectively inherent in the definition, and was established in [2].

Lemma 1 *Adopt Assumptions 3 and 4, save that G_o need not be linear and may have direct feedthrough. Then the closed loop system (G_o, K) is stable if and only if*

$$\begin{bmatrix} D_r & -U_r \\ N_r & V_r \end{bmatrix}^{-1}$$

exists and is bounded.

Actually, the validity of the above Lemma does not depend on the linearity of K (Assumption 3). This is in contrast to two of the following stability results, which replace one or the other of the right coprime realizations by left coprime realizations.

Lemma 2 *Adopt Assumption 3 and 4, save that G_o may have direct feedthrough. Then the closed-loop system (G_o, K) is stable if and only if*

$$(U_l N_r + V_l D_r)^{-1}$$

exists and is bounded.

Next, we use a left realization of the plant and a right realization of the controller to obtain a stability result. The following theorem appears to be of independent interest. It will also be used in describing the structure of the set of all plants stabilized by a given controller later in this paper.

Theorem 1 *Suppose that Assumptions 3 and 4 in Section 2.2 hold, save that U_r and V_r need not be linear. Then if $[N_l U_r - D_l(-V_r)]^{-1}$ exists and is bounded, the closed loop of Figure 2 is stable, and additionally $q = V_r^{-1}(r_1 - y)$ is bounded. Conversely, suppose the closed loop of Figure 2 is stable, with q bounded, then $[N_l U_r - D_l(-V_r)]^{-1}$ exists and is bounded.*

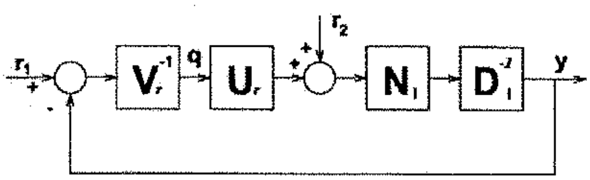

Figure 2: *Closed loop diagram with nominal plant G_o*

The final stability result is an easy consequence of the preceding Theorem and will be used to characterize the set of all plants stabilized by a linear controller.

Lemma 3 *Adopt Assumption 3 and 4. Then the closed-loop system (G_o, K) is stable if and only if*

$$\begin{bmatrix} V_l & U_l \\ -N_l & D_l \end{bmatrix}^{-1}$$

exists and is bounded.

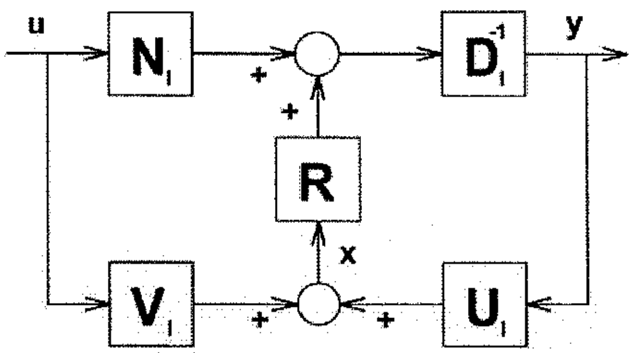

Figure 3: *Left coprime factorization based description of G*

3 Characterization and Identification of all Nonlinear Plants using a Left Coprime factor based Description

Consider the system in Figure 1 with the noise w assumed to be zero and given certain assumptions above. This section will show that all nonlinear plants stabilized by a controller K can be represented by the setting depicted in Figure 3, with R a nonlinear stable, well posed operator known also as the Youla-Kucera parameter. Conversely, if the set up of Figure 3 defines a well-posed G for some stable well-posed R, then G is stabilized by K.

As this description involves the left coprime factors of the controller, K, and the nominal plant, G_o it will be referred to as a left coprime factorization *based* description. Note that G (as opposed to G_0) does not always have a left coprime factorization due to the nonlinearity of the operators involved.

3.1 Describing the structure of the set of all plants stabilized by a given controller in a noise free setting.

We will use Theorem 1 and Lemma 3 in proving the following Lemma.

Lemma 4 *Adopt Assumptions 1, 3, 4 and 5 of Section 2.2. Suppose further that R is a well-posed bounded operator and that the operator $G : u \to y$ depicted in Figure 3 is also well-posed. Then the closed loop in Figure 4 is well posed and stable. Further R has no direct feedthrough term implies G has no direct feedthrough term.*

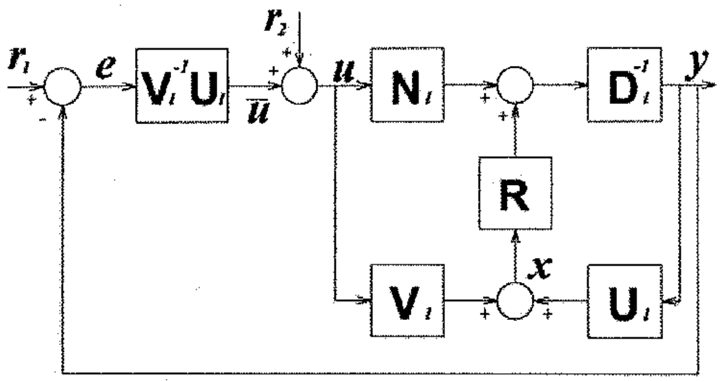

Figure 4: *Closed loop diagram with plant G as in Figure 2*

It now remains to show the converse result, namely that for a stable (K,G) interconnection, an R always exists which is bounded.

Lemma 5 *Under Assumptions 1, 3, 4 and 5 in Section 2.2, suppose the closed loop in Figure 1 is well posed and internally stable. Then there exists a well posed, bounded R given by*

$$R = (D_l G - N_l)(V_l + U_l G)^{-1},$$

such that in Figure 3

$$y = Gu.$$

Further, if Assumption 2 holds, R has no direct feedthrough.

In summary, we now have

Theorem 2 *Under Assumptions 1, 3, 4 and 5 the closed loop in Figure 1 is well posed and internally stable iff G has a description of the form of Figure 3, with R a well posed, stable operator. Further, G has no direct feedthrough if and only if R has this property.*

3.2 Incorporation of Measurement Noise and Conversion to Open Loop Identification

This section demonstrates two ways that measurement noise can be incorporated into the closed loop structure described earlier such that identification may be possible.

Both methods require a small noise assumption (high SNR) so that D_l and R may be linearized. This assumption can be simplified by assuming that while N_l may still be *nonlinear*, that D_l is *linear*. The requirement that D_l is linear may not be particularly restrictive. For example, if G_o is stable, $N_l = G_o$, $D_l = I$ is a coprime left coprime factorization!

With the small noise assumption both methods are able to convert the closed loop identification problem into one of open loop identification. As in [1] it is shown that instead of identifying the plant G, we can identify the Youla-Kucera parameter, R, for a given plant. From the previous sections it is easily noted that identifying R is equivalent to identifying G.

3.2.1 The First Method

The first method is similar to that found in [1] and shows how Figure 3 must be changed if the noise v is no longer zero.

a) Incorporation of measurement noise

Redraw Figure 1 as shown in Figure 5 so that the closed-loop part of Figure 5 is similar to that in Figure 1. Notice that the noise now enters in two places, at the output and via a noise modified input signal. Comparing Figures 1 and 5, note that the signals at similar points in the loop are related via:

$$\hat{\hat{u}} = \hat{u}, \hat{u} = u, \hat{\hat{y}} = \hat{y}, \hat{e} = e, y = \hat{\hat{y}} + v.$$

So by replacing the plant G by the structure in Figure 3, we can use the same equations with Figure 5 that we derived from Figure 1 with noise absent, replacing r_1 by $r_1 - v$ and y by $y - v$. Thus the relationship between the input and the output is given by:

$$\begin{aligned} D_l(y - v) &= R(V_l r_2 + U_l(r_1 - v)) \\ &\quad + N_l(r_2 + V_l^{-1} U_l(r_1 - y)) \end{aligned} \tag{3.1}$$

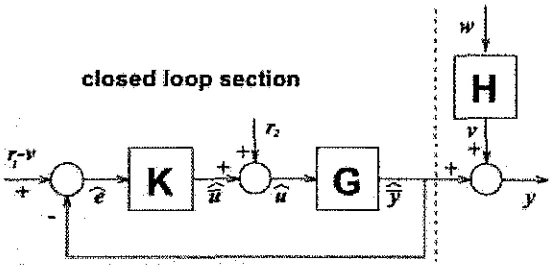

Figure 5: *Redrawn Figure 1*

In [1], the next step was to introduce noise into a modification of Figure 4. The modification is shown in Figure 6. Unfortunately, with a general nonlinear pair N_l, D_l describing G_o, this is in general not possible. The situation is summed up in Theorem 3.

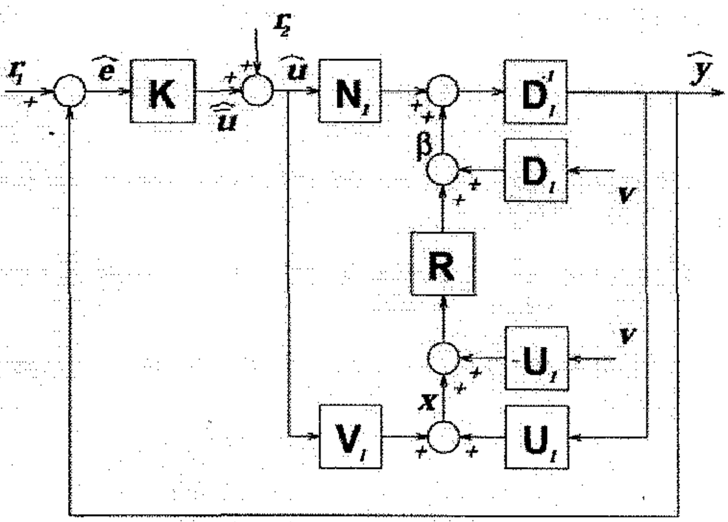

Figure 6: *Noise incorporated left coprime factorization based description with D_l linear.*

Theorem 3 *Consider the stable closed loop in Figure 1, with G as in Figure 3 and the Assumptions stated in Section 2.2. Then the signals in Figure 6 relate to those of Figure 1 by $\hat{u} = u, \hat{e} = e, \hat{\bar{u}} = \bar{u}$ and $\hat{y} = y$ iff the operator D_l is linear.*

Returning to the structure in Figure 5 we can note that y differs from $\hat{\bar{y}}$ only because of the noise v. So if D_l is nonlinear but it is legitimate to make a small noise assumption (high SNR) then we can linearize D_l in (3.1), assuming $D_l(y-v) = D_l y - \delta D_l v$:

$$\begin{aligned} D_l y - \delta D_l v &= R(V_l r_2 + U_l(r_1 - v)) \\ &\quad + N_l(r_2 + V_l^{-1} U_l(r_1 - y)). \end{aligned} \tag{3.2}$$

This is shown in Figure 7. Of course, if D_l is linear, $\delta D_l = D_l$ and there is no approximation involved in (3.2). Also, Figures 6 and 7 are identical, and $y = \hat{y}$.

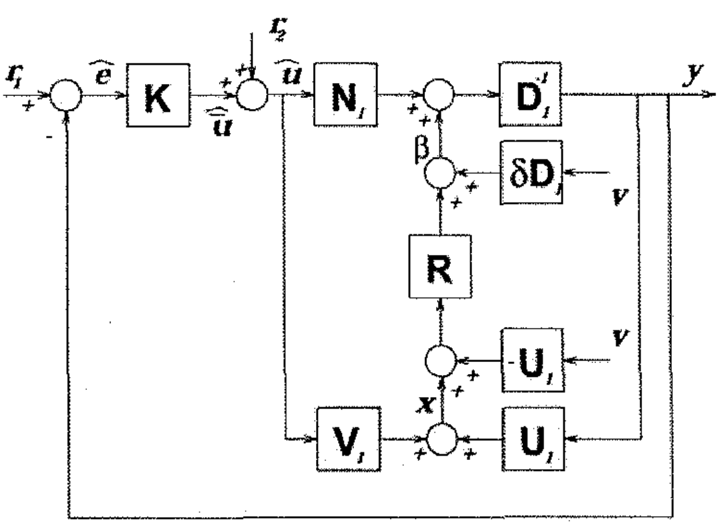

Figure 7: *Noise incorporated left coprime factorization based description with δD_l the linearization of D_l around y.*

b) Conversion to Open Loop Identification

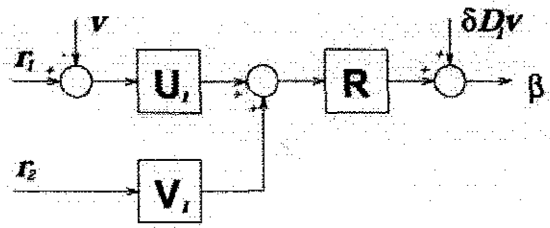

Figure 8: *Conversion to open loop*

Observe in Figure 7 that

$$\beta = \delta D_l v + R(U_l(r_1 - v) + V_l r_2), \tag{3.3}$$

where the r_i are uncorrelated with v, see Figure 8. In this Figure, U_l, V_l are known while R is unknown; the inputs r_1, r_2 are known, while the input noise v and measurement noise $\delta D_l v$ are unknown. The output β is known since, from Figure 7, we note that β can be measured using u and y and is bounded if the closed loop is stable, as

$$\beta = D_l y - N_l u.$$

A conventional open-loop identification problem would have no input noise; and some parametrization for R would almost certainly be adopted.

The difficulty caused by the input noise can be recovered under a small noise assumption on v.

With δR the linearization of R around the operating trajectory produced by x, (3.3) gives

$$\begin{aligned} \beta &= (\delta D_l - \delta R U_l)v + R(U_l r_1 + V_l r_2), \\ &= (\delta D_l - \delta R U_l)Hw + R(U_l r_1 + V_l r_2). \end{aligned}$$

This gives the setting of Figure 9 where the input to the system that needs be identified is independent with the measurement noise, which is true of the usual open loop setting of the identification problem. It should be noted that to identify R, both r_1 and r_2 may have to be nonzero.

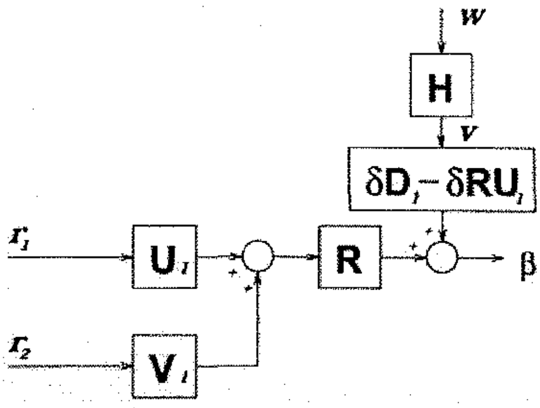

Figure 9: *Conversion to open loop*

3.2.2 The Second Method

The second method is similar to that in [4]. It again sets up an open-loop identification problem. Refer first to Figure 3.

In the noiseless case, (ie $v = 0$), there holds

$$x = V_l u + U_l y = V_l r_2 + U_l r_1.$$

Also, if $\beta = Rx$, then

$$\beta \;=\; -N_l u + D_l y.$$

Stability of the closed loop ensures that x and β are (in principle) computable (boundedly) from r_1, r_2 and u, y respectively.

a) Incorporation of measurement noise

When $v \neq 0$, the same equations hold provided we replace r_1 and y by $r_1 - v$ and $y - v$ respectively:

$$x = V_l r_2 + U_l(r_1 - v)$$
$$\beta = -N_l u + D_l(y - v)$$

The identification task is now, with known r_1, r_2, u and y and unknown v, to find a stable R such that $\beta = Rx$.

b) Conversion to Open Loop Identification

From $\beta = Rx$, we obtain

$$-N_l u + D_l(y - v) = R[V_l r_2 + U_l(r_1 - v)],$$

or

$$-N_l u + D_l y - \delta D_l v = R[V_l r_2 + U_l r_1] + \delta R(-U_l v)],$$

or, again assuming low noise,

$$[-N_l u + D_l y] = R[V_l r_2 + U_l r_1] + [\delta D_l - \delta R U_l]v.$$

This is now of the form

measured signal = R(*known signal*) + *noise*

ie it is an open loop identification problem, which is still nonlinear.

4 Conclusion

This paper has considered the identification of a nonlinear plant operating in a closed loop with a linear controller, allowing for nonlinearity of the initial plant model given that the actual nonlinear plant need not have a left coprime factorization.

Many of the results are analogous to those in [1], [3] and [4]. In particular, the requirement for high signal to noise ratio is still present. The results of this paper do rest on new stability results, and may even open the way to identification with known nonlinear controllers. To this end, the recent work of Paice and Van der Shaft ([10], [11]) establishing the utility of stable kernel representations may be helpful.

References

[1] Dasgupta S., Anderson B.D.O., *Automatica*, vol. 32 pp. 1349-1360, (1996).

[2] Verma M.S., *Int. Journal of Control*, vol 48, pp 897-918, (1988).

[3] Linard N., Anderson B.D.O., *Proceedings of CDC '96, CDROM*, (1996).

[4] Anderson B.D.O., *Proc. IFAC World Congress*, (1996).

[5] Chen G., de Figueiredo R.J.P., *Circuits, Sys. and Signal Proc.*, vol 11(2), pp 285-307 (1992).

[6] Danow R., Chen G., *Circuits, Sys. and Signal Proc.*, vol 12(3):489-492, (1993).

[7] Hammer J., *Int. J. Control*, vol 42, pp 1-20, (1985).

[8] Tay T.T., Moore J.B., *Int. J. Control*, vol 49, pp 1235-1248, (1989).

[9] Paice A.D.B., Moore J.B. Horowitz R., *J.Math Syst., Estimation and Control*, vol 2, pp 293-321, (1992).

[10] Paice A.D.B., van der Shaft A.J., *33rd CDC*, (1994).

[11] Paice A.D.B., van der Shaft A.J., *Report Nr 299, Inst. für Dynamische Syst.*, (November 1993).