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Identification of nonlinear plants under linear control using Youla-Kucera parametrizations

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Abstract

This paper treats the identification of a noise contaminated nonlinear plant operating in a closed-loop with a stabilizing controller. The a priori model of the plant is nonlinear, and its update is obtained via a Youla-Kucera parameter. The introduction of this parameter induces a nonstandard open-loop identification problem, which in a low noise situation can be formulated as a standard open-loop problem.

1 Introduction

This paper is concerned with the identification of a nonlinear plant, operating in a closed loop and in the presence of additive noise. As such, it extends results in [1].

Our key result applies in a high signal-to-noise ratio situation. It explains how, given an a priori nonlinear model of a plant, and a linear controller stabilising the model and the true plant, measurements on the closed loop system involving controller and plant can be used to identify the plant. A Youla-Kucera parametrization is used to parametrize the unknown plant, and the closed-loop identification is reduced to a conventional open loop problem.

All these ideas are described in Section 3. Section 2 serves to put the ideas in context, by recalling the key difficulty of closed-loop identification for linear plants, its resolution by Hansen et al [2-4], and an extension of the idea to nonlinear plants with linear a priori model and with linear controller in [1].

Section 4 contains remarks concerning possible extensions. An ultimate goal, noting possible applications to adaptive control (especially using the "windsurfer" approach of iterative controller design and identification with progressive expansion of bandwidth), is to be able to perform identification of a nonlinear plant with a nonlinear stabilising controller and a nonlinear a priori model of the plant also stabilised by the controller.

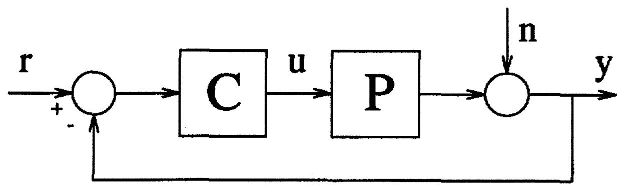

Figure 1: Closed Loop

2 Background: Linear System closed-loop identification using a Youla-Kucera parameter

Consider the linear system arrangement of Figure 1. The measurements of r, u and y are available; n is a noise process that is independent of r, and the closed-loop is stable. Now

$$y = Pu + n. \tag{1}$$

If u were simply r (so that the plant was operating in open-loop) one could cross correlate with u and then solve for P:

$$\Phi_{yu}(s) = P(s)\Phi_{uu}(s).$$

Most open-loop identification schemes operate somewhat like this (possibly in the time-domain, recursively, and with sample averages). In the closed-loop case, u and n are correlated and in fact

$$\Phi_{yu}(s) = P(s)\Phi_{uu}(s) - (1+C^*(s)P^*(s))^{-1}C^*(s)\Phi_{nu}(s). \tag{2}$$

So blind use of an open-loop identification method which implicitly evaluates $\Phi_{yu}(s)\Phi_{uu}^{-1}(s)$ is bound to give errors. This is the fundamental reason which makes closed-loop identification difficult. A secondary difficulty however is that software may presuppose that $P(s)$ is stable, normally a reasonable assumption in the

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open-loop case but an unwarranted assumption in the closed-loop case.

The contribution of [2-4] serves to replace the closed-loop problem by a conventional open-loop problem; the tool is to use a Youla-Kucera parameter to describe the unknownness of the plant. We review the multivariable version.

In particular, let $C(s) = XY^{-1}$ denote a right coprime realization of the controller; thus X, Y are matrices with entries which are stable, proper transfer functions (we shall say $X, Y \in \mathcal{S}$), and there exist $\tilde{D}$, $\tilde{N}$ for which

$$\tilde{D}Y + \tilde{N}X = I. \qquad (3)$$

It is well known that one can also find $\tilde{X}$, $\tilde{Y}$, N, $D \in \mathcal{S}$ such that

$$\begin{bmatrix} \tilde{Y} & \tilde{X} \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & -X \\ N & Y \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \qquad (4)$$

This double Bezout identity implies that the nominal plant $P_0(s) = \tilde{D}^{-1}\tilde{N} = ND^{-1}$ is stabilised by $C(s) = XY^{-1} = \tilde{Y}^{-1}\tilde{X}$. If $C(s)$ and $P_0(s)$ are both prescribed with $C(s)$ stabilising $P_0(s)$, it remains possible to choose fractional representations satisfying (2.4).

Suppose $C(s)$ also stabilises the true plant; then for some $S \in \mathcal{S}$, the true plant has a fractional representation $(N + YS)(D - XS)^{-1}$. Figure 2 shows the arrangement. It can be shown that using the figure and (2.4) that

$$y = (N + YS)(D - XS)^{-1}u + n.$$

Now it turns out that when the plant is connected in a closed-loop as in Figure 1, one can show that

$$\begin{aligned} x &= \tilde{X}r, \\ z &= \tilde{D}y - \tilde{N}u. \end{aligned} \qquad (5)$$

so that x and z are computable from measured quantities, and x is independent of n. Since, as Figure 2 implies,

$$z = Sx + (\tilde{D} - S\tilde{X})n, \qquad (6)$$

with S known to be stable, the identification of S using x and z is a standard open-loop identification problem.

In [1], much of the above linear thinking is carried across to the nonlinear plant case. A linear controller $C(s)$ and linear a priori model $P_0(s)$ for the plant are assumed - so that the Bezout identity (2.4) holds. Provided the controller $C(s)$ stabilises the true plant P, it is known that a Youla-Kucera parameter S exists, which is a stable, nonlinear causal operator. Using S, the plant P can be represented as

$$P = (N + YS)(D - XS)^{-1}, \qquad (7)$$

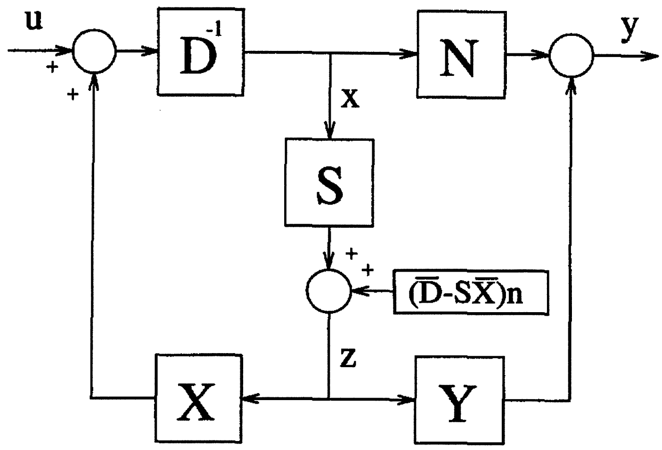

Figure 2: Plant representation using controller, a priori model of plant and Youla-Kucera parameter

or, in the absence of noise,

$$(D - XS)y = (N + YS)u. \qquad (8)$$

The introduction of noise is less straightforward. The replacement of Figure 2 is depicted in Figure 3. (The two figures are equivalent if S is linear, but not in general if S is nonlinear).

Actually, the use of N, D, X, Y in the above analysis and Figure 2 - all associated with right coprime realizations of linear objects - can be replaced by use of $\tilde{N}, \tilde{D}, \tilde{X}, \tilde{Y}$ with modest amendments to the equation and the diagram. However, the use of left coprime quantities, $\tilde{D}$ and $\tilde{X}$,which are associated with the noise cannot be replaced by use of right coprime quantities.

In this paper, we shall retain the linear controller, but we shall allow the a priori plant to be nonlinear. *The difficult issues to resolve are those associated with the appearance of quantities associated with left coprime realisations - in particular in (2.5) and (2.6);* a nonlinear plant in general does not have a left fractional representation (though it may have a kernel representation [5], which is a related object). As it turns out, the scheme of Figure 3 even has to be abandoned.

3 New results

We start by formulating our assumptions. A linear controller with coprime realisation XY^{-1} (Y, X matrices of stable transfer functions) is known to stabilize an unknown nonlinear plant P and a nominal, known nonlinear model P_0. The nonlinear model P_0 has a coprime factorization ND^{-1}; this means that D and N are causal nonlinear bounded operators (mapping

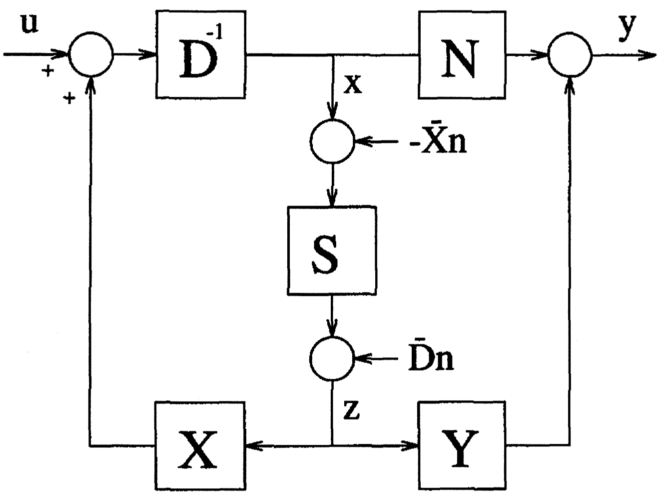

Figure 3: Nonlinear plant representation, using a linear controller, linear a priori model of plant and nonlinear Youla-Kucera parameter

L_2 signals causally into L_2 signals) and (expressing the coprimeness) there exists a bounded operator such that

$$L\begin{bmatrix} D \\ N \end{bmatrix} = I. \tag{9}$$

The fact that C stabilizes P and P_0 means that if r_1, r_2 are in L_2 (see Figures 4 and 5), all internal variables are well-defined and also $\in L_2$. In particular, w_1, w_2 given by

$$\begin{bmatrix} D & -X \\ N & Y \end{bmatrix}\begin{bmatrix} w_2 \\ w_1 \end{bmatrix} = \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} \tag{10}$$

are in L_2. This means that $\begin{bmatrix} D & -X \\ N & Y \end{bmatrix}^{-1}$ is a stable operator, [6].

According to [6], there exists a bounded operator S such that $P = (N + YS)(D - XS)^{-1}$. Thus neglecting the noise contribution in Figure 3, P is expressible in terms of the fractional description of C and P_0 via the arrangement of Figure 3.

Our goal is to explain how to take into account noise. We begin by obtaining a Bezout identity, but not a double one.

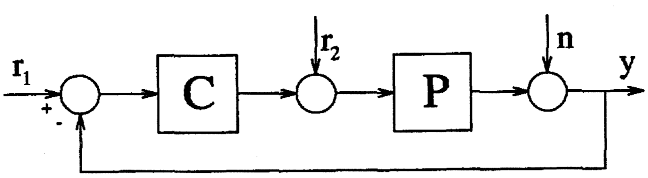

Figure 4: Nonlinear plant and linear controller in stable loop

Lemma 1 *Adopt the notation and assumptions above. Let $\bar{Y}^{-1}\bar{X}$ be a left coprime realization of $C(s)$, with*

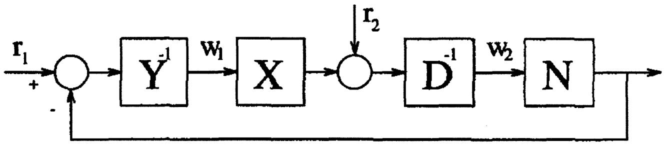

Figure 5: Nominal nonlinear model and linear controller forming a stable closed loop

$\bar{X}, \bar{Y} \in \mathcal{S}$. Then (with abuse of notation regarding $\bar{X}, \bar{Y}$ as operators), $R = \bar{Y}D + \bar{X}N$ has a bounded inverse. Further, if N and D are replaced by NR^{-1} and DR^{-1} to form a new fractional description of P_0 there holds (after this replacement)

$$\bar{Y}D + \bar{X}N = I. \tag{11}$$

Remark. One cannot replace $\bar{X}, \bar{Y}$ by $R^{-1}\bar{X}, R^{-1}\bar{Y}$ without sacrificing the linearity of the operators.

Proof: Recalling that $\bar{X}, \bar{Y}$ are both linear, apply the operator $\begin{bmatrix} \bar{Y} & \bar{X} \end{bmatrix}$ to both sides of (3.2). There results

$$(\bar{Y}D + \bar{X}N)w_2 = \bar{Y}r_2 + \bar{X}r_1. \tag{12}$$

Since $\bar{X}, \bar{Y}$ is a coprime pair, there exists $U, V \in \mathcal{S}$ such that

$$\bar{Y}U + \bar{X}V = I. \tag{13}$$

Take $r_1 = Vr$, $r_2 = Ur$. Then (3.4) can be rewritten as

$$(\bar{Y}D + \bar{X}N)w_2 = r. \tag{14}$$

Closed-loop stability of the controller and nominal model imply that for all $r_1, r_2 \in L_2$, $w_2 \in L_2$. Since $r \in L_2$ implies $r_1, r_2 \in L_2$, it follows that for all $r \in L_2$, $w_2 \in L_2$. Thus $(\bar{Y}D + \bar{X}N)$ has a bounded inverse. The remainder of the lemma is trivial.

Let us therefore suppose that (3.3) holds. ■

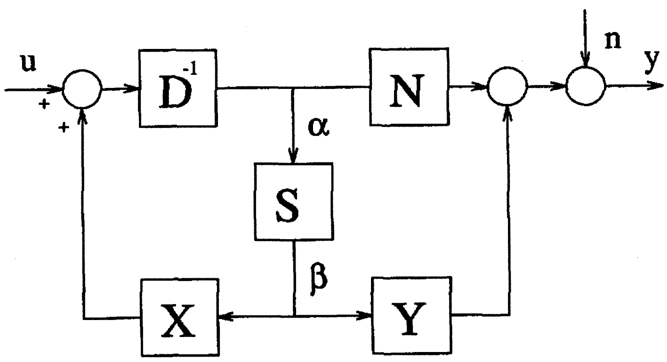

Figure 6: Nonlinear plant represented using a priori data and Youla parameter

Figure 6 depicts the nonlinear plant P using a Youla-Kucera parameter. Observe from Figure 6 that

$$\begin{bmatrix} D & -X \\ N & Y \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} u \\ y - n \end{bmatrix}. \tag{15}$$

Lemma 2 *Adopt notation and assumptions as above, with $\bar{X}, \bar{Y}$ satisfying (3.3). Using the coprimeness of X and Y, define linear operators $\bar{U}, \bar{V} \in S$ such that*

$$\bar{U}Y + \bar{V}X = I. \qquad (16)$$

Then (3.7) can be "solved" as

$$\begin{aligned} \alpha &= \bar{X}r_1 + \bar{Y}r_2 - \bar{X}n, & (17) \\ \beta &= -\bar{V}u + \bar{U}y + & (18) \\ & (\bar{V}D - \bar{U}N)(\bar{X}r_1 + \bar{Y}r_2 - \bar{X}n) - \bar{U}n. \end{aligned}$$

Proof: Operate on the left of (3.7) with the linear operator $[\ \bar{Y} \quad \bar{X}\ ]$. Since $\bar{Y}^{-1}\bar{X} = XY^{-1}$ and (3.3) holds, there follows

$$\alpha = \bar{Y}u + \bar{X}(y - n). \qquad (19)$$

The controller ensures that $u = r_2 + C(r_1 - y)$ or $\bar{Y}(u - r_2) = \bar{X}(r_1 - y)$ or $\bar{Y}u + \bar{X}y = \bar{X}r_1 + \bar{Y}r_2$. Then (3.9) is immediate.

Next operate on the left of (3.7) by the linear operator $[\ -\bar{V} \quad \bar{U}\ ]$, and use (3.8). There results

$$\beta - \bar{V}D\alpha + \bar{U}N\alpha = -\bar{V}u + \bar{U}(y - n),$$

or

$$\begin{aligned} \beta = \ & \bar{V}D(\bar{X}r_1 + \bar{Y}r_2 - \bar{X}n) - & (20) \\ & \bar{U}N(\bar{X}r_1 + \bar{Y}r_2 - \bar{X}n) - \bar{V}u + \bar{U}y - \bar{U}n. \end{aligned}$$

This is (3.10). ∎

Lemma 2 provides us with a nonstandard open-loop identification problem. For as Figure 6 shows, we have

$$\beta = S\alpha, \qquad (21)$$

and α and β are composed of measured signals (r_1, r_2, u and y), contaminated by noise (n). Moreover, the noise enters α (which is not standard), and enters β nonlinearly (which is also not standard).

In the high signal-to-noise ratio case, a more conventional problem can be obtained.

Lemma 3 *Consider the identification problem of Figure 4, with assumptions on, and fractional descriptions of, C, P and P_0 as previously. Suppose that P is modeled as in Figure 6. Suppose further that D_L, N_L and S_L represent linearizations of the operators D, N and S around the operating trajectory defined by the input function $\bar{X}r_1 + \bar{Y}r_2$. Then neglecting quantities of second order in n, there holds*

$$\begin{aligned} \beta &= -\bar{V}u + \bar{U}y & (22) \\ & \quad +(\bar{V}D - \bar{U}N)(\bar{X}r_1 + \bar{Y}r_2) \\ & \quad -(\bar{V}D_L - \bar{U}N_L)\bar{X}n - \bar{U}n, \\ &= S(\bar{X}r_1 + \bar{Y}r_2) - S_L\bar{X}n, \end{aligned}$$

or

$$z = Sx + v, \qquad (23)$$

where

$$\begin{aligned} x &= \bar{X}r_1 + \bar{Y}r_2, & (24) \\ z &= -\bar{V}u + \bar{U}y & (25) \\ & \quad +(\bar{V}D - \bar{U}N)(\bar{X}r_1 + \bar{Y}r_2), \\ v &= [(\bar{V}D_L - \bar{U}N_L)\bar{X} & (26) \\ & \quad +\bar{U} - S_L\bar{X}]n. \end{aligned}$$

Proof: By direct calculation based on (3.9), (3.10) and (3.11). ∎

Notice that (3.13) and (3.15) guarantee x and v are independent; (3.13) and (3.14) guarantee x and z are measurable; and (3.12) apart from the nonlinearity of S, is an equation defining a standard nonlinear identification problem, (stable or bounded operator, and measurement noise independent of input).

Remark. How does this result square up with that of [1]? In [1], it was the case that $P_0 = ND^{-1}$ was linear, and we could choose a left coprime realization $\bar{D}^{-1}\bar{N}$ of P_0 with the additional property $\bar{D}Y + \bar{N}X = I$ (this equation being part of the double Bezout identity). This means that in Lemma 2, we can identify $\bar{U}, \bar{V}$ with $\bar{D}, \bar{N}$ and then $\bar{V}D - \bar{U}N = 0$. This yields major simplification to (3.10) as:

$$\beta = -\bar{N}u + \bar{D}y - \bar{D}n, \qquad (27)$$

and then the equation

$$\beta = S\alpha,$$

says

$$-\bar{N}u + \bar{D}y = \bar{D}n + S(\bar{X}r_1 + \bar{Y}r_2 - \bar{X}n). \qquad (28)$$

In Figure 3, it is possible to argue that when the plant is connected to a controller that $x = \bar{X}r_1 + \bar{Y}r_2$, while $z = -\bar{N}u + \bar{D}y$ can also be established. Thus the scheme of Figure 3 is indeed recovered.

4 Further Developments

One of the goals of the present work is to devise a nonlinear adaptive control scheme, analogous to the "windsurfer" approach to the adaptive control of linear systems, achieved in fact not by continuous adjustment of the controller, but by an iterative process of controller design and identification. Successive controller designs achieve progressively wider closed-loop bandwidths.

For there to be a nonlinear analog, one would need a scheme for closed-loop identification of a nonlinear plant, given a nonlinear a priori model and a nonlinear stabilising controller of the plant and model. [One might need a number of other things too, including a scheme for approximation of a high order nonlinear model by a lower order model].

The work of [5] and [7] suggests that Youla parametrizations are best obtained for nonlinear systems using the analog of left coprime factorizations, viz stable kernel factorizations. [Actually [1] has some results on nonlinear system identification with a linear controller and a priori linear model described using left coprime factorizations, and the authors have preliminary results when the a priori model is nonlinear with a left fractional description (something which is *not* standard, even when a right fractional description exists).

Two other issues are: how should S be parametrized so the identification is efficacious, and how useful anyway is a nonlinear plant description obtained using a Youla parametrization, at least without well accepted procedures for reduction of order of state-variable descriptions of nonlinear systems.

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