ON SOME PRACTICAL ISSUES IN SYSTEM IDENTIFICATION FOR THE WINDSURFER APPROACH TO ADAPTIVE ROBUST CONTROL

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Abstract. In this paper we investigate some important issues in system identification for the windsurfer approach to robust adaptive control. In particular, correlation function estimates and power spectrum estimates are compared as methods for model validation. This comparison leads us to suggest a reliable procedure for deciding (1) when should we identify a better model, and (2) whether we have identified a good model for our purposes.

Key Words. Adaptive control; control system design; identification; ;model reference control; robustness

1. INTRODUCTION

1.1. *Background and Objectives of This Paper*

Iterative identification and control design is a topic of growing interest recently (see, for example, Anderson and Kosut (1991),Lee *et al.* (1993*a*), Partanen and Bitmead (1993),Schrama and den Hof (1992),and Zang *et al.* (1991)). One of the schemes proposed in this area is *the windsurfer approach* which attempts to increase the bandwidth of a closed-loop system while keeping the closed-loop frequency response approximately flat in the passband, given that the initial model of the plant may involve significant error in the high-frequency region. It was demonstrated recently in Lee *et al.* (1993*b*) that when the performance robustness of the closed-loop system deteriorates (while the closed-loop bandwidth is increased to a certain value), it is possible to improve the performance robustness of the closed-loop system through an iterative identification and controller design procedure.

In this paper we shall investigate some important issues in system identification for the windsurfer approach. In particular we would like to know *(1) when should we identify a better model, and (2) whether we have identified a good model for our purposes.*

1.2. *Structure of The Paper*

To establish notation and relevant equations, we describe in Section 2 the system identification method employed by the windsurfer approach. Necessary conditions for identifying a good model are also outlined. In Section 3 we present a frequency domain method for model validation. In Section 4 we describe a time domain method for model validation. In Section 5 the model validation methods are compared through case studies by simulations. In Section 6 we explain how the model validation methods can provide a reliable procedure for anwering the questions highlighted at the end of Subsection 1.1.

2. CONTROL-RELEVANT SYSTEM IDENTIFICATION

Given a stable strictly proper model G_0 of a stable strictly proper plant G, it was shown in Lee *et al.* (1993*b*) that we can design a sequence of controllers $\{K_0^j\}$ such that the designed closed-loop transfer functions

$$\bar{T}_0^j = \frac{G_0 K_0^j}{1 + G_0 K_0^j}$$

are well behaved and the sequence $\{\bar{T}_0^j\}$ has progressively increasing bandwidth λ_0^j. The performance robustness of the actual closed-loop transfer function,

$$T_0^j = \frac{G K_0^j}{1 + G K_0^j} \ ,$$

is measured by

$$J = \|v\|_2^2 \qquad (1)$$

where

$$v = \left(T_0^j - \bar{T}_0^j\right) r \ ; \quad r\text{=reference input} \ , \qquad (2)$$

is the tracking error induced by the model error associated with G_0 under closed-loop conditions.

Remark

- Note that the tracking error v cannot be measured directly. It can only be estimated from the closed-loop output error ξ shown in Fig. 1. It can be easily seen that

$$\xi = v + w \qquad (3)$$

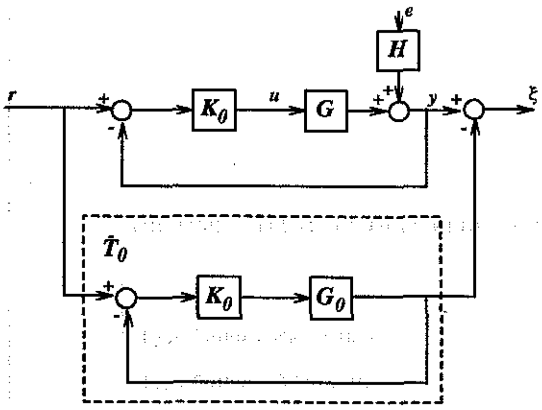

Fig. 1. Closed-loop output error ξ

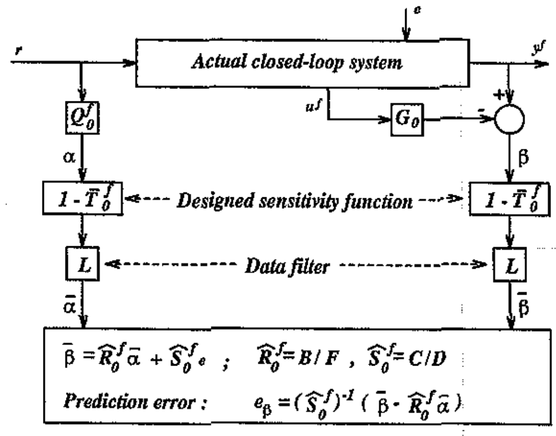

Fig. 2. Identification of R_0^f and S_0^f

where $w = (1 + GK_0^j)^{-1}He$ is the effect of the noise, e, on the actual closed-loop output.

When λ_0^j is increased to a certain value, λ_0^f, $J^f = \|v^f\|_2^2$ (the value of J when $\lambda_0^j = \lambda_0^f$) may become excessively large. At this stage, we like to identify a new model G_1 such that

$$\left\| \left(\frac{GK_0^f}{1 + GK_0^f} - \frac{G_1 K_0^f}{1 + G_1 K_0^f} \right) r \right\|_2^2$$

is minimized. Note that K_0^f is designed on the basis of G_0, when the value of λ_0^j is λ_0^f.

This closed-loop identification problem can be transformed into an open-loop identification problem by employing Hansen's framework of identification (Hansen, 1989). We quote a special case (where the plant and its models are stable) of Theorem 2 proved in Lee *et al.* (1993*a*).

Theorem 1 Let $K_0^f = (1 - G_0 Q_0^f)^{-1} Q_0^f$ stabilize G and G_0, where Q_0^f is a proper stable transfer function. G can be parametrized by a strictly proper stable transfer function R_0^f via

$$G = G_0 + \frac{R_0^f}{1 - R_0^f Q_0^f} \ .$$

Let

$$G_1 = G_0 + \frac{\hat{R}_0^f}{1 - \hat{R}_0^f Q_0^f} \tag{4}$$

be another model stabilized by K_0^f, where $\hat{R}_0^f$ is a strictly proper stable estimate of R_0^f.

Also define

$$\xi_1 = \left(1 - \bar{T}_0^f\right)\left(\beta - \hat{R}_0^f \alpha\right) \tag{5}$$

where $\alpha = Q_0^f r$, $\beta = y^f - G_0 u^f$, and u^f and y^f are, respectively, the input and output of the plant resulting from the application of K_0^f. Then ξ_1 can be expressed as

$$\xi_1 = \left(\frac{GK_0^f}{1 + GK_0^f} - \frac{G_1 K_0^f}{1 + G_1 K_0^f} \right) r + w^f \tag{6}$$

where $w^f = (1 + GK_0^f)^{-1} He$.

Remarks

- If we define $H = (1 - R_0^f Q_0^f)^{-1} S_0^f$, where S_0^f is a proper stable and inversely stable transfer function, then the actual closed-loop system has Hansen's open-loop representation

$$\beta = R_0^f \alpha + S_0^f e \ . \tag{7}$$

- Since the "input" α in equation (7) is independent of the noise disturbance e, identifying R_0^f (and S_0^f) is an *open-loop* identification problem.
- We shall identify R_0^f and S_0^f by the prediction error method (Ljung, 1987) as shown in Fig. 2. Data filters L (typically low-pass) are usually employed to shape the bias-distribution of the estimates (which is due to undermodelling) such that the model error is small in the appropriate frequency range.

3. A FREQUENCY DOMAIN METHOS FOR MODEL VALIDATION

In any system identification problem, it is important to ensure that a model with the right properties is identified. In the following we shall present a model validation method in the frequency domain. It should be emphasized that the model validation procedure is designed with the closed-loop control objective in mind.

Recall that, given the existing model G_0, it is necessary to identify an improved model G_1 when $J^f = \|v^f\|_2^2$ is excessively large. Evidently ξ^f could be large (implying undesirable performance) with one of both of v^f and w^f large. If the former is larger, there is a potential to reduce it by improved model identification. But this will only work (in a particular frequency band) if the signal-to-noise ratio is sufficiently high. Specifically, when only finite durations of input-output measurements are available for identifying R_0^f (which parametrizes G_1), it was shown in Lee *et al.* (1993*b*) that the normalized variance of $\hat{R}_0^f$ will be small only if the signal-to-noise ratio, $\Phi_{v^f}(\omega)/\Phi_{w^f}(\omega)$, associated with $\xi^f = v^f + w^f$ is sufficiently high, where $\Phi_{v^f}(\omega)$ and $\Phi_{w^f}(\omega)$ are, respectively, the power spectra of v^f and w^f. Obviously, then one needs to estimate power spectra for w and v (or more precisely ξ). We shall proceed as follows.

From equations (2) and (3) we observe that when $r = 0$, the sole contributor to ξ is w. Therefore we can compute $\Phi_w(\omega)$ after measuring ξ with $r = 0$. When $r \neq 0$, we have $\xi = v + w$. Assuming that v and w are uncorrelated (which follows if r and e are uncorrelated, a typical situation), then $\Phi_\xi(\omega) = \Phi_v(\omega) + \Phi_w(\omega)$. By visual comparison of $\Phi_\xi(\omega)$ with $\Phi_w(\omega)$, we evaluate the significance of $\Phi_v(\omega)$ with respect to $\Phi_w(\omega)$. If $\Phi_{\xi^f}(\omega)$ is significantly larger than $\Phi_{w^f}(\omega)$ in a frequency band spanning one decade and centered around λ_0^f (when the designed closed-loop bandwidth is λ_0^f), the model G_0 is invalidated for the design of closed-loop systems with bandwidths larger than or equal to λ_0^f.

The method just described can also be used to validate G_1 after it is identified. We simply replace G_0 by G_1, while retaining K_0^f, in the simulation of the designed closed-loop response to the reference input. This allows us to compute ξ_1 and its power spectrum $\Phi_{\xi_1}(\omega)$. By visually comparing $\Phi_{\xi_1}(\omega)$ with $\Phi_w(\omega)$, we have good confidence that G_1 is a reliable model of G (when the designed closed-loop bandwidth is λ_0^f) if $\Phi_{\xi_1}(\omega)$ is comparable to $\Phi_w(\omega)$ up to λ_0^f.

4. A TIME DOMAIN METHOD FOR MODEL VALIDATION

We shall now describe a time domain model validation method. This is useful both for establishing that G_0 should be rejected (that is, as a flag for re-identification) as well as for validating a new model, G_1, replacing G_0.

Referring to Fig. 2 and equation (5), we notice that $e_\beta = L\xi_1$ when $\hat{S}_0^f = 1$, where e_β is the prediction error (also known as the residual). We also observe from equation (4) that $G_1 = G_0$ when $\hat{R}_0^f = 0$. Equation (6) shows that $\xi_1 = \xi^f$ when $G_1 = G_0$. Therefore we have $e_\beta = L\xi^f$ when $\hat{S}_0^f = 1$ and $\hat{R}_0^f = 0$. This suggests that G_0 should be rejected if the cross-correlation of the prediction error e_β with the future values of "input" $\tilde{\alpha}$ exceed its (3σ) confidence limits when $\hat{S}_0^f = 1$ and $\hat{R}_0^f = 0$. This reasoning is independent of the true S_0^f. See Ljung (1987) for more details of model validation by correlation techniques. (Actually it is also easy to apply the same method to validate a pair of newly identified $\hat{R}_0^f$ *and* $\hat{S}_0^f$ before $\hat{R}_0^f$ is used to calculate G_1. We simply check that the auto-correlation function of e_β for non-zero delays as well as the cross-correlation of e_β with the future values of $\tilde{\alpha}$ are within their respective confidence intervals.)

5. COMPARING THE MODEL VALIDATION METHODS

Sections 3 and 4 presented two different methods for model validation. In the following, we shall compare the two methods in two simulation examples. In these examples, the plant is a simulated flexible link robot arm whose transfer function G has poles at $-0.0996 \pm j3.0017$, $-0.3339 \pm j12.131$, $-1.845 \pm j31.481$, zeros at $s = -13.162$, $-10.646 \pm j12.27$, $s = 7.169 \pm j11.54$, and $G(0) = 0.5196$.

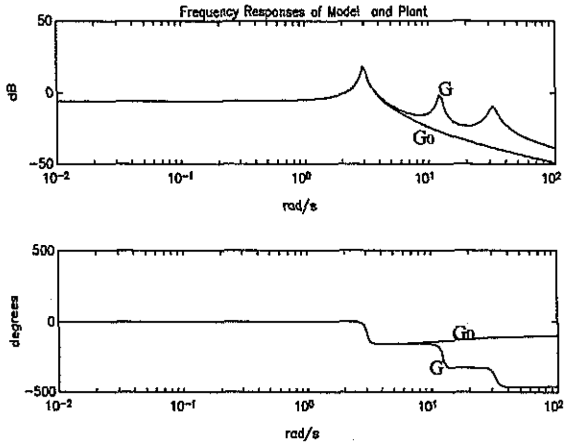

Fig. 3. Frequency response of model G_0

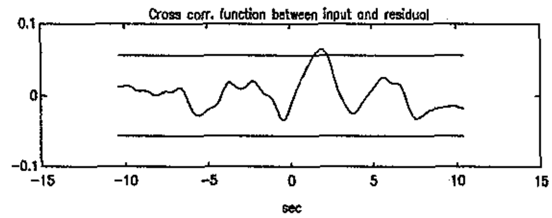

Fig. 4. Validating G_0 ($\lambda_0^f = 1.5\, rad/s$)

5.1. *Example One*

The initial model G_0 is an open-loop description of G up to and including its first resonant frequency (see Fig. 3). G_0 has a pair of poles at $-0.0903 \pm j3.0027$, a zero at $s = -13.31$, and $G_0(0) = 0.5188$.

When the designed closed-loop bandwidth is $1.5 rad/s$, the method of correlations (see Fig. 4) shows that G_0 is not a good model of G whereas the method of power spectra (compare Fig. 5 and 6) shows that the tracking error is still insignificant. In fact, we are unable to identify a better model than G_0 at this stage. When the designed closed-loop bandwidth has increased to $3\ rad/s$, the method of correlations (see Fig. 7) and the method of power spectra (compare Fig. 8 and 9)

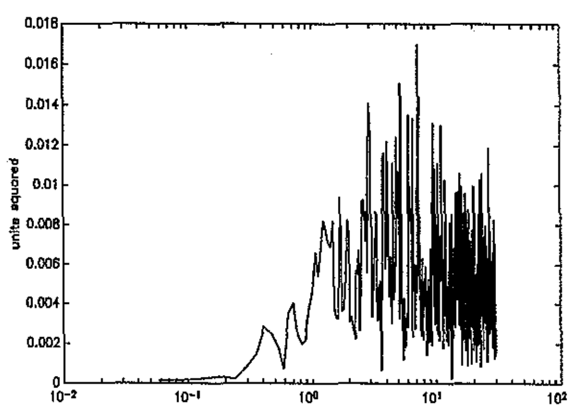

Fig. 5. $\Phi_w(\omega)$ when λ_0^f is $1.5\ rad/s$

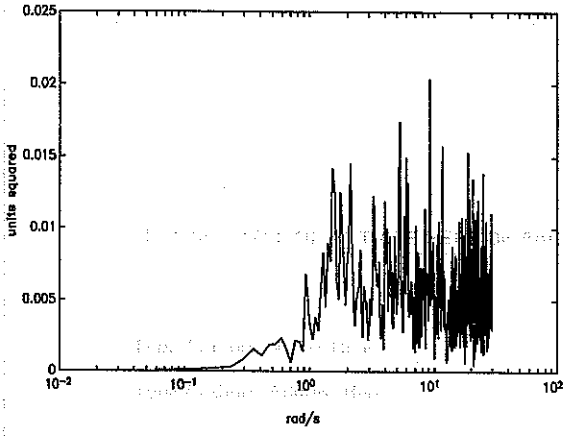

Fig. 6. $\Phi_{\xi}(\omega)$ when λ_0^f is 1.5 rad/s

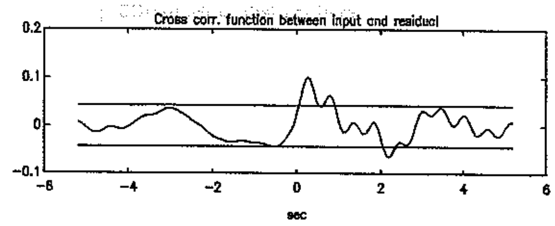

Fig. 7. Validating G_0 ($\lambda_0^f = 3\ rad/s$)

both indicate that G_0 is not a good model. In particular, comparison of Fig. 8 and 9 indicates that the closed-loop output error has a high signal-to-noise ratio at around 12 rad/s. The identified $\hat{R}_0^f$ and $\hat{S}_0^f$ are validated by the method of correlations (see Fig. 10) before $\hat{R}_0^f$ is used to calculate G_1. The resulting G_1 is validated by the method of power spectra (compare Fig. 11 and 8). Fig. 12 shows the frequency response of G_1.

5.2. *Example Two*

By employing the model G_1 of example 1 (which has poles at $s = -0.0895 \pm j3.0026$, $-0.4834 \pm j12.03$, $-2.475 \pm j31.502$, zeros at -12.967, $-7.336 \pm j11.05$, $9.098 \pm j12.07$, and $G_1(0) = 0.5189$), it is possible to increase the designed closed-loop bandwidth to 12 rad/s before it is necessary to identify a better model. At this stage, the method of correla-

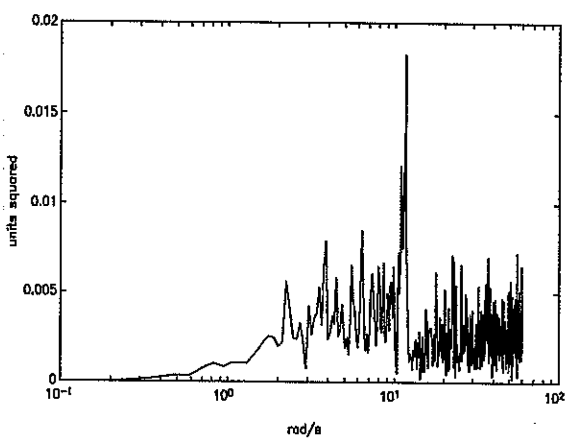

Fig. 8. $\Phi_w(\omega)$ when λ_0^f is 3 rad/s

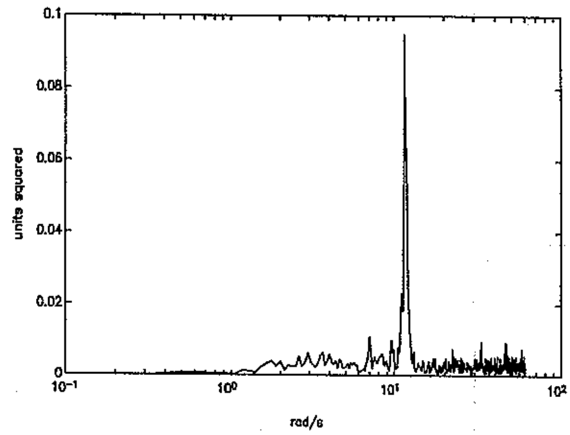

Fig. 9. $\Phi_{\xi}(\omega)$ when λ_0^f is 3 rad/s

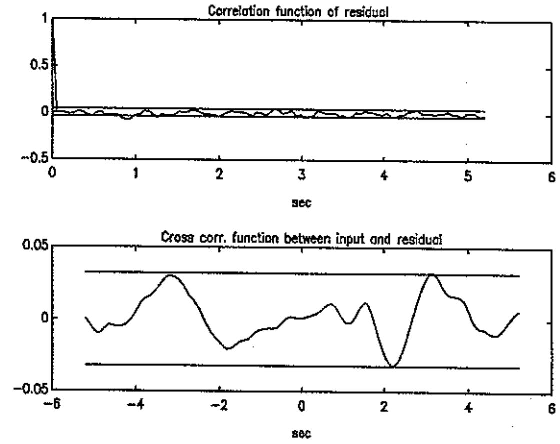

Fig. 10. Validating $\hat{R}_0^f$ and $\hat{S}_0^f$ ($\lambda_0^f = 3\ rad/s$)

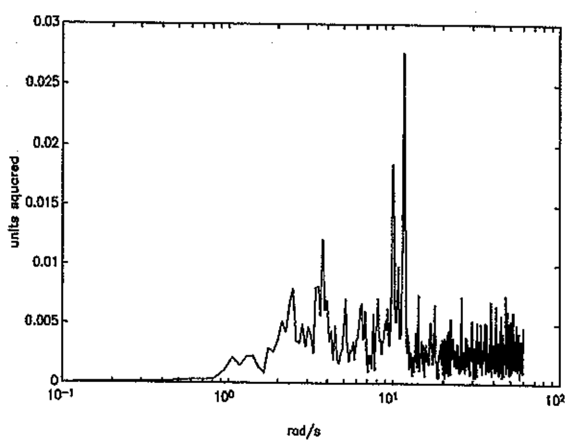

Fig. 11. $\Phi_{\xi_1}(\omega)$ when λ_0^f is 3 rad/s

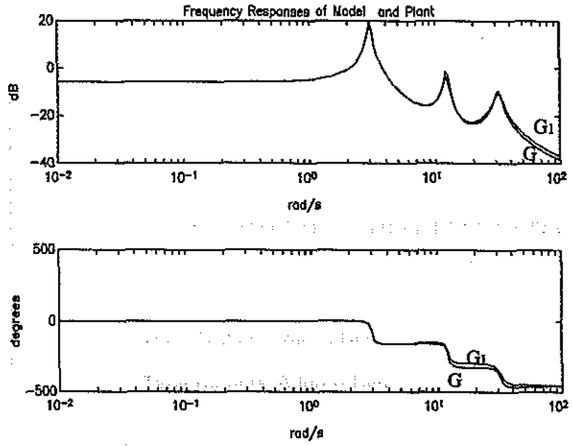

Fig. 12. Frequency response of model G_1

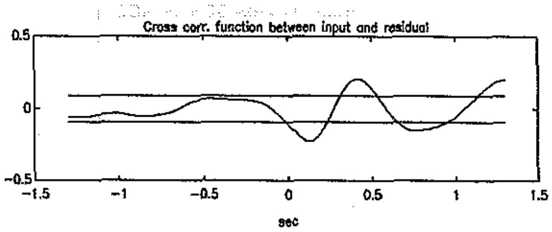

Fig. 13. Validating G_1 ($\lambda_1^f = 12\, rad/s$)

tions (see Fig. 13) and the method of power spectra (compare Fig. 14 and 15) both indicate that G_1 is not a good model. However, we are unable to identify a better model despite considerable efforts and numerous attempts. This phenomenon can be explained as follows. Notice that the designed closed-loop bandwidth (12 rad/s) is close to the critical frequency corresponding to the *unstable zeros* of G_1 (at $s = 9.098 \pm j12.07$). From the view point of control design, it follows from Freudenberg and Looze (1985) that, due to the unstable zeros of G_1, the system has reached its fundamental performance limitations. Furthermore, it can be shown that the tracking error and the effect of noise disturbance at this stage are, respectively, given by

$$v^f = \frac{\left(\frac{G-G_1}{G_1}\right)\bar{T}_1^f}{1+\left(\frac{G-G_1}{G_1}\right)\bar{T}_1^f}\left(1-\bar{T}_1^f\right) r \ ,$$

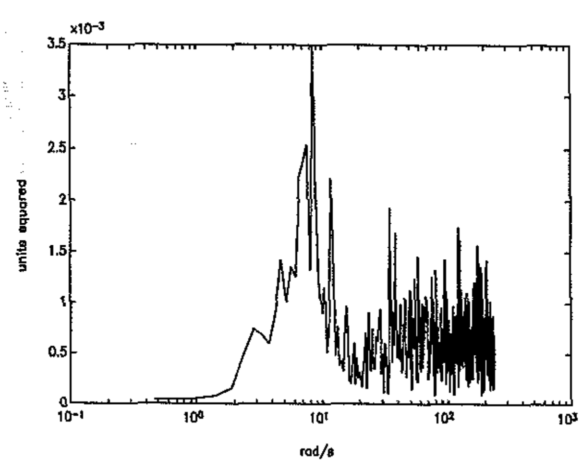

Fig. 14. $\Phi_w(\omega)$ when λ_1^f is 12 rad/s

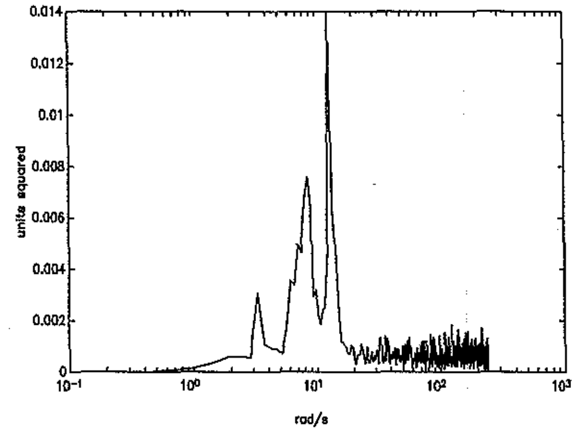

Fig. 15. $\Phi_\xi(\omega)$ when λ_1^f is 12 rad/s

and

$$w^f = \frac{1}{1+\left(\frac{G-G_1}{G_1}\right)\bar{T}_1^f}\left(1-\bar{T}_1^f\right) He \ ,$$

where

$$\bar{T}_1^f = \left(\frac{12}{s+12}\right)[G_1]_a$$

is the designed closed-loop transfer function with

$$[G_1]_a = \text{the all-pass factor associated with } G_1 \ .$$

Note that for the case where G_1 has no unstable zeros, $|1-\bar{T}_1^f(j\omega)| \leq 1$ for all frequencies. However, in the presence of the unstable zeros of G_1 around 12 rad/s and when the designed closed-loop bandwidth is close to 12 rad/s, $|1-\bar{T}_1^f(j\omega)| \approx 2$ at around 12 rad/s. Therefore the tracking error v^f and the effect of noise disturbance w^f are *both amplified by a factor of approximately two at around* 12 rad/s in the latter situation. Hence, for the same magnitude of $[(G-G_1)/G_1]\bar{T}_1^f$ in the neighbourhood of 12 rad/s, the value of $\Phi_{\xi^f}(\omega)$ for the case where G_1 has unstable zeros at around 12 rad/s is about four times of that for the case where G_1 has no unstable zeros. However, since $\Phi_{v^f}(\omega)/\Phi_{w^f}(\omega)$ *does not* depend on $1-\bar{T}_1^f$, the increase in $\Phi_{\xi^f}(\omega)$ that we have just described *does not increase* the signal-to-noise ratio of ξ^f. This implies that we are caught in a situation where it is necessary to improve the model (because $\Phi_{\xi^f}(\omega)$ is excessively large near to 12 rad/s) but we are unable to identify a better model (because $\Phi_{v^f}(\omega)/\Phi_{w^f}(\omega)$ is in not sufficiently high). We therefore conclude that, *owing to the controller designed on the basis of* G_1 *for a closed-loop bandwidth of* 12 rad/s, the closed-loop identification process has reached its natural limits of accuracy constrained by the unstable zeros of G_1 in the region of 12 rad/s.

5.3. *Lessons Learned from The Examples*

From the examples and many other simulation studies, we can make the following conclusions:

- Correlation function estimates and power spectrum estimates are both useful for model validation where the goodness of fit is based on a closed-loop control criterion.

- Correlation function estimates are more sensitive than power spectrum estimates in the sense that the former tend to invalidate a model before identifying a better model is necessary and possible. This *does not imply* that the correlation method is useless. On the contrary, it suggests that the correlation method is useful for detecting incipient model errors.
- Power spectrum estimates not only suggest when a model becomes inadequate but they also indicate the frequency range in which the signal-to-noise ratio is high for identification.
- Prior knowledge of unstable zeros in the existing model (say G_0) is important. As explained in Example two, there is a fundamental limits of accuracy for performing identification on closed-loop systems involving controllers designed on the basis of models with unstable zeros. Let ω_z be the minimum critical frequency corresponding to the unstable zeros of G_0, simulation evidence suggests that it is very difficult, if not imossible, to identify a model better than G_0 if $\lambda_0^f \geq \omega_z/2$. It should be remarked that this is reminiscent of design tradeoffs discussed in Freudenberg and Looze (1985), as opposed to an ill-posed problem.
- In general we should update G_0 if and only if
  1. both methods of model validation suggest that it is necessary to do so, *and*
  2. $\lambda_0^f < \omega_z/2$, where ω_z is the minimum critical frequency corresponding to the unstable zeros of G_0.

6. IDENTIFICATION OF A BETTER MODEL

We have remarked in section 2 that under practical conditions, the accuracy of the identified model can be improved by increasing the signal-to-noise ratio of the closed-loop output error. This is equivalent to having a certain level of deterioration in robust performance relative to the effect of noise disturbance. However, it is undesirable from the control point of view for robust performance to deteriorate too seriously. In this section, we explain how these competing objectives can be resolved.

The signal-to-noise ratio of the closed-loop output error in the frequency range where the current model G_0 has significant error can be increased by increasing the magnitude of the reference input or by increasing the designed closed-loop bandwidth. If practical operation constraints do not allow the magnitude of the reference input to be increased, then the signal-to-noise ratio of the closed-loop output error can only be increased by increasing the designed closed-loop bandwidth. This, however, has the potential danger of causing instability in the actual closed-loop system if the designed closed-loop bandwidth is increased excessively. To avoid this danger, we would like to suggest the following procedure:

1. Reduce the rate of increasing the designed closed-loop bandwidth λ_0^j once the correlation method for model validation has invalidated G_0.
2. Attempt to identify R_0^f (when $\lambda_0^j = \lambda_0^f$) as soon as the power spectrum method for model validation suggests that ξ^f has a sufficiently high signal-to-noise ratio, provided that $\lambda_0^f < \omega_z/2$, where ω_z is the minimum critical frequency corresponding to the unstable zeros of G_0.
   (a) Use the collected data to identify a set of models by experimenting with the likely model structures. Perform model verification on each of these models.
   (b) If an identified model is found to be sufficiently accurate, accept it for the next stage of control design. Otherwise, increase the designed closed-loop bandwidth slightly, collect a new set of measurements and repeat the procedures of model estimation and verification.
   (c) Repeat the last two steps until a sufficiently accurate model is obtained and verified.
3. Terminate the iterative identification and control design procedure if $\lambda_0^f \gtrsim \omega_z/2$ and ξ^f, although unacceptably large, does not facilitate the identification of a better model.

7. ACKNOWLEDGEMENTS

The authors wish to acknowledge the funding of the activities of the Cooperative Research Centre for Robust and Adaptive Systems by the Australian Government under the Cooperative Research Centres Program. W. S. Lee wishes to thank Dr Robert R. Bitmead for his valuable advice.

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