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Filters for Internal Model Control Design for Unstable Plants

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Abstract

In this paper we study the design of a two-degree of freedom filter for the internal model control (IMC) method. The new filter alleviates some disadvantages of the standard IMC filter when the IMC method is applied to unstable plants that do not have nonminimum-phase zeros. We show that by employing the new filter, the resulting system has a flatter frequency response and little overshoot in the step response. Furthermore one of the filter's design parameters can be related directly to the closed-loop bandwidth and the other design parameter can be used to control the recovery time of the step response after an overshoot has occurred. These features are important when the IMC method is employed in a new approach to adaptive robust control. Examples are given in the paper to illustrate the new filter design.

1 Introduction

Recently, it has been shown that the internal model control (IMC) method [1] is a simple and effective technique for designing the underlying control law in a new approach to adaptive robust control when the plant is stable [2, 3]. In such an approach it is desired to design a closed-loop system with a specified bandwidth in the face of model uncertainties. In essence the approach starts with designing a controller such that the nominal closed-loop system has a small bandwidth. The performance of the corresponding actual closed-loop system when the reference input is a step function is monitored. If the performance robustness of the closed-loop system is ascertained while the nominal closed-loop bandwidth is smaller than the specified one, a new controller will be designed such that it results in an increase of the nominal closed-loop bandwidth. It is obvious that, in this connection, it is desirable to have a single design parameter which can be interpreted as the nominal closed-loop bandwidth. In the case of stable plants, the IMC method is found to have the desirable attribute that we have just mentioned. However, if the plant is unstable, the aforementioned single design parameter can no longer be interpreted as the nominal closed-loop bandwidth *even if the plant does not have nonminimum-phase zeros.* This poses a problem if the IMC method is to be used in the new adaptive robust control method when the plant is unstable.

Motivated by the problems discussed above, we study in this paper the design of a new filter for the IMC method.

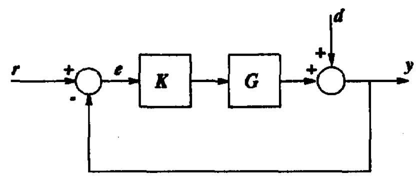

Figure 1: Closed-loop control system

This paper is structured as follows. After briefly reviewing the IMC method in section 2, we introduce the new filter in section 3. In this same section, the frequency response and the time response of the new filter are also studied. This analysis points out the improvement achievable with the new filter, as compared with the standard one, in reducing peak overshoot in the step response of the control system. Finally, we shall conclude the paper in section 4.

2 Standard IMC Filter for Unstable Plants

In this section we shall first briefly outline the IMC method of designing robust control systems as a two stage design method, where the second stage is concerned with the design of a low pass filter (the interested reader is referred to [1] for a comprehensive treatment of the topic). We will then show that, when the method is applied to an unstable plant, the parameter which was interpreted as the nominal closed-loop bandwidth in the stable plant case can no longer be treated simply as such. Furthermore we shall show that the resulting nominal closed-loop frequency response tends to have large resonant peak, which has adverse effects on the transient response as well as the stability robustness of the system.

2.1 IMC design method

We are concerned with the design of a controller K situated within a control loop which involves a partially known plant G as shown in figure 1.

Let $\widehat{G}$ be a model of the plant G. The IMC method paramterizes the controller K in terms of a stable transfer function Q as shown in the IMC structure given in figure 2.

By comparing figure 1 and figure 2, we immediately

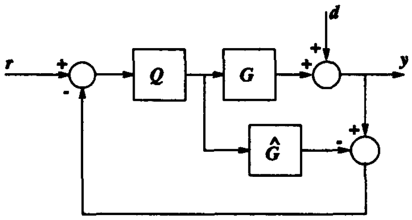

Figure 2: IMC structure

notice that the design of the controller

$$K = \frac{Q}{1-\widehat{G}Q}$$

is equivalent to the design of its parametrization Q. The associated nominal closed-loop transfer function is easily evaluated as $\widehat{G}Q$. Since the plant G is only partially known in terms of its model $\widehat{G}$, any practical design method must take into account the discrepancies between G and $\widehat{G}$. In the IMC design method, the design of Q is carried out in two stages by specializing Q as the product of two transfer functions

$$Q = \widetilde{Q}F \quad ,$$

such that $\widetilde{Q}$ and F can be designed separately. In the first stage, the transfer function $\widetilde{Q}$ is designed with respect to the model $\widehat{G}$ such that, if $G = \widehat{G}$, the objective

$$\|e\|_2^2 = \int_0^\infty e^2 dt$$

is minimized, where e is the error signal resulting from some known reference input r and known disturbance d. At this stage, one is not concerned with the model uncertainties involved in $\widehat{G}$ and/or the properness of the resulting controller as these will be taken care of in the second stage by designing an appropriate IMC filter F. The design of the H_2-optimal controller $\widetilde{Q}$ is summarized in the next theorem, for which a proof is given in [1].

Theorem 2.1 *Suppose that $\widehat{G}$ has no poles on the imaginary axis, except those at the origin, and has no zeros on the imaginary axis. Let $\widehat{G}$ have k poles at $p_1, \ldots, p_k$ in the open right-half-plane and a pole of multiplicity l at the origin. Define*

$$B_{\widehat{G}} = \prod_{i=1}^{k} \frac{-s+p_i}{s+p_i^*} \tag{1}$$

and factor $\widehat{G}$ into an all pass factor $\widehat{G}_a$ (which contains all the zeros of $\widehat{G}$ in the open right-half-plane) and a minimum-phase factor $\widehat{G}_m$ (which includes all the poles of $\widehat{G}$ in the open right-half-plane and at the origin) such that

$$\widehat{G} = \widehat{G}_m \widehat{G}_a \quad .$$

Factor the generalized input $v = r - d$ similarly,

$$v = v_m v_a \quad .$$

Without loss of generality, assume that the open right-half-plane poles of the input v are the first k' poles p_i of the plant in the open right-half-plane[1] *and define accordingly*

$$B_v = \prod_{i=1}^{k'} \frac{-s+p_i}{s+p_i^*} \quad . \tag{2}$$

Assume further that v has at least l poles at the origin.[2] *The controller $\widetilde{Q}$ which minimizes the objective*

$$\|e\|_2^2 = \int_0^\infty e(t)^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |e(j\omega)|^2 d\omega = \left\|(1-\widehat{G}\widetilde{Q})v\right\|_2^2$$

is given by

$$\widetilde{Q} = B_{\widehat{G}}(\widehat{G}_m B_v v_m)^{-1}\left\{(B_{\widehat{G}}\widehat{G}_a)^{-1} B_v v_m\right\}_* \tag{3}$$

where the operator $\{\cdot\}_*$ denotes that after a partial fraction expansion of the operand all terms involving the poles of $\widehat{G}_a^{-1}$ are omitted.

In the second stage of designing the Q controller, the IMC filter F is chosen to be a rational function that satisfies the following requirements:

(i) The controller $Q = \widetilde{Q}F$ is proper.

(ii) The closed-loop system is internally stable.

(iii) The output tracks the reference input asymptotically.

Also, in choosing F, we seek to cope with errors between the true plant G and its estimate $\widehat{G}$ used for design. Assume that $\widehat{G}$ and v have q distinct poles $p_1, \ldots, p_q$ in the open right-half-plane and at the origin (counting poles which appear in both $\widehat{G}$ and v only once). Then, it was shown in [1] that conditions (i), (ii) and (iii) are satisfied if F is given by

$$F(s) = \frac{a_{q-1}s^{q-1} + \cdots + a_1 s + a_0}{(s+\lambda)^{m+q-1}} \; ; \quad \lambda > 0 \text{, arbitrary} \quad , \tag{4}$$

with the q coefficients $a_0, a_1, \ldots, a_{q-1}$ chosen such that

$$F(s) = 1 \quad \text{at } s = p_1, \ldots, p_q \quad , \tag{5}$$

and the integer m is sufficiently large so as $\widetilde{Q}F$ is proper.

The objective of this filter is to secure properness of the controller, by the choice of m, and to detune the H_2-optimal controller, by the choice of a suitable λ, such that the closed-loop system can tolerate the high frequency model uncertainties associated with the model $\widehat{G}$. Typically, small values of λ are associated with small closed-loop bandwidths, and it is in such a situation that high frequency model uncertainties are most easily tolerated.

Remarks

- For the situation where the model $\widehat{G}$ is minimum-phase, if the input v is a step function, it can be shown easily that

$$\widetilde{Q} = G^{-1}$$

[1] As noted in [1], this assumption is needed to make a meaningful problem.

[2] As noted in [1], this assumption is needed in order that the closed-loop system can handle plant input disturbance satisfactorily.

regardless of whether the plant is stable or not. Furthermore, if the plant is stable, a suitable IMC filter is

$$F(s) = \left(\frac{\lambda}{s+\lambda}\right)^m \quad ; \quad m \geq n ,$$

where n is the relative degree of $\widehat{G}$, and $\lambda > 0$ is chosen sufficiently small.

- For the above simple situation, F becomes the nominal closed-loop transfer function. Thus λ can be interpreted as the nominal closed-loop bandwidth with a $-3m\,dB$ attenuation.

- For cases where $\widehat{G}$ is nonminimum-phase but stable, if the input v is a step function, it can be shown that the nominal closed-loop transfer function is $F\widehat{G}_a$ where F is still given by

$$F(s) = \left(\frac{\lambda}{s+\lambda}\right)^m .$$

  Since the all-pass factor $\widehat{G}_a$ does not affect the magnitude of the nominal closed-loop frequency response, the interpretation of λ as the nominal closed-loop bandwidth still applies in this case.

- We will show in section 2.2 that λ can no longer be simply related to the nominal closed-loop bandwidth if $\widehat{G}$ is unstable, *even if it does not have nonminimum-phase zeros.*

2.2 Problems with the standard IMC filter

In the following our main objective is to show that in the case of an unstable plant, the design parameter λ in the standard IMC filter can no longer enjoy the simple interpretation as the nominal closed-loop bandwidth even if the plant does not have nonminimum-phase zeros and the reference input is a step function. A simple example will suffice to illustrate the problem.

Consider an unstable plant G which is described by the simple minimum-phase unstable model

$$\widehat{G}(s) = \frac{a}{s-p} \quad ; \quad p > 0 .$$

For a unit step reference input, we can use theorem 2.1 to obtain

$$\widetilde{Q}(s) = \left(\frac{a}{s-p}\right)^{-1} .$$

In order that the controller $Q = \widetilde{Q}F$ is proper, we would require the standard IMC filter F to have a relative degree of one. Furthermore, it is necessary that the conditions specified by equation (5) be satisfied; that is, the two conditions

$$F(0) = 1,$$

and

$$F(p) = 1.$$

Therefore, the filter must have two free parameters in the numerator. Hence F has the following structure

$$F(s) = \frac{\alpha_1 s + \alpha_0}{(s+\lambda)^2} .$$

Simple calculations show that

$$F(s) = \frac{(p+2\lambda)s + \lambda^2}{(s+\lambda)^2} .$$

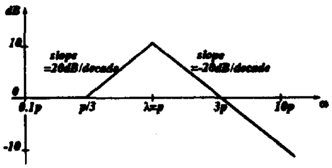

Figure 3: Asymptotic plot of $20\log|F|$ for $\lambda = p$

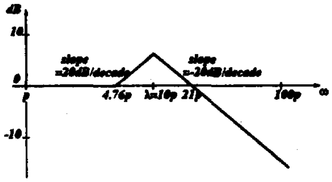

Figure 4: Asymptotic plot of $20\log|F|$ for $\lambda = 10p$

Since the nominal closed-loop transfer function is $\bar{T} = Q\widehat{G}$, where $Q = \widehat{G}^{-1}F$, it is evident that F is the nominal closed-loop transfer function with a double pole at $-\lambda$ and a simple zero at $-\lambda^2/(p+2\lambda)$.

Remark

- Notice that, since $p > 0$, the zero of F is always between the origin and one half the location of its double pole.

The asymptotic log-magnitude diagrams for the cases where $\lambda = p$ and $\lambda = 10p$ are shown, respectively, in figure 3 and figure 4.

Remarks

- The frequency responses of F always involve a resonant peak at $\omega = \lambda$.

- For the case where $\lambda = p$, the nominal closed-loop bandwidth is approximately 3λ.

- For $\lambda \gg p$, the nominal closed-loop bandwidth is approximately 2λ.

- The nominal closed-loop bandwidth in general depends on both p and λ. Therefore λ of the standard IMC filter cannot be related directly to the nominal closed-loop bandwidth. This may cause difficulty in its use for the controller design stages of an adaptive controller that increases the nominal closed-loop bandwidth progressively.

It is well known from classical control theory [4] that a large resonant peak in the frequency response of a transfer function is correlated with large peak overshoot in its time response. It can be shown that the time response of a closed-loop system which involves an unstable plant always has overshoot [6]. The use of an inappropriate filter in the IMC design method, in this case the standard IMC filter, can aggravate the situation.

Furthermore, robustness considerations allow us to conclude that the sufficient condition for robust stability,

$$\left\| \frac{G - \widehat{G}}{\widehat{G}} \bar{T} \right\|_{\infty} < 1 \ ,$$

may be violated in the range of frequency where the resonant peak occurs. Therefore the use of the standard IMC filter in the case of unstable plant may worsen the stability robustness of the closed-loop system.

The above discussions naturally prompt the need to consider a new IMC filter design that can possibly alleviate the above mentioned problems.

3 Improved IMC Filter Design for Simple Unstable Plants

Due to space limitations we shall consider in this section the simple case where the plant is minimum-phase with simple unstable pole. Extensions to more general cases will be reported elsewhere [5].

For a plant described by a minimum-phase model $\widehat{G}$ with a simple unstable pole located at p, we consider a filter of the form

$$F(s) = \frac{\mu(s+\alpha)}{(s+\gamma)(s+\lambda)(s+10\lambda)^m} \ ; \quad \gamma > 0, \ \lambda > 0 \ . \tag{6}$$

In equation (6), m has to be selected in order to meet the properness requirement (i) of section 2.1 and the constants μ and α are chosen by imposing the two interpolation constraints $F(0) = 1$ and $F(p) = 1$. The filter poles λ and γ are free parameters which can be selected without any concern for nominal stability or asymptotic tracking of the reference input. It is worth noticing that the presence of the pair of parameters λ and γ, as opposed to the single parameter λ in the standard IMC filter, constitutes the main new feature of the proposed filter (6). As will be shown later, by suitably selecting the values of λ and γ (using a systematic procedure), one can improve both time response and robust stability as compared to the standard IMC filter. The key is the following:

Theorem 3.1 *Consider the filter $F(s)$ of equation (6), parametrized by $\gamma > 0$ and $\lambda > 0$, with μ and α to be chosen so that $F(0) = F(p) = 1$. If γ and λ are chosen so that*

$$\gamma \ll p \ll \lambda \ , \tag{7}$$

then $\alpha \simeq \gamma$, and

$$F(s) \simeq \frac{\mu}{(s+\lambda)(s+10\lambda)^m} \ . \tag{8}$$

Furthermore, for $\omega \in [\gamma, \lambda]$, there holds approximately,

$$F(j\omega) \simeq \frac{1+(p/\lambda)}{1+(j\omega/\lambda)} . \tag{9}$$

Proof

The proof is not given due to space limitations. ∎

Equation (8) points out two main properties enjoyed by filter (6) under the conditions $\gamma \ll p$ and $\lambda \gg p$:

1. Parameter λ is the filter bandwidth.
2. The frequency response is almost flat and does not present a large resonant peak at any frequency.

Remarks

- In the case of minimum-phase plants, application of theorem 2.1 gives $\widetilde{Q} = \widehat{G}^{-1}$ and hence, the closed-loop transfer function of the nominal closed-loop system, $\bar{T} = \widetilde{Q}F\widehat{G}$, coincides with that of the filter. The flatness of the filter frequency response (property 2) is then particularly significant for good transient response. Furthermore it can be shown [5] that systems with flatter frequency responses would have better robust stability.

- As is well known [7], in the case of unstable plants, the requirement that the nominal closed-loop transfer function $\bar{T}$ is stable translates into integral constraints on the sensitivity function $1 - \bar{T}$. For instance, provided that the open-loop transfer function has a relative degree greater than or equal to two, the sensitivity function must satisfy the following equation

$$\int_0^{\infty} \log_{10} |1 - \bar{T}(j\omega)| d\omega = \pi \left(\log_{10} e\right) \sum_i \mathrm{Re}\{p_i\} \ , \tag{10}$$

where the sum in the right-hand-side is computed over the set of unstable poles p_i of the open-loop transfer function. If the sensitivity function is kept small over a large range in the low frequency region, then the contribution due to this frequency region to the integral in the left-hand-side of equation (10) is negative with large absolute value. Consequently, in order to satisfy equation (10) $1 - \bar{T}$, and in turn $\bar{T}$ itself, must be large in some other frequency range. This produces an undesirable resonant peak in the frequency response of $\bar{T}$. Roughly speaking, the zero at $-\alpha$ and the pole at $-\gamma$ in filter (6) form a dipole that shifts the magnitude of the sensitivity function to a value slightly greater than zero. As the dipole is located near the origin, this effect starts at very low frequency and is maintained up to a frequency near λ. This allows the negative contribution to the integral in the left-hand-side of Equation (10) to be kept small, and hence prevents the resonant peak from occurring in the closed-loop frequency response.

- It cannot be overemphasized that the improvement of filter (6) over the standard IMC filter can only be achieved if $\lambda \gg p$.

- Notwithstanding the last remark, it should be realized that there is no limit to the achievable flatness in the frequency response of the new filter once the condition $\lambda \gg p$ is satisfied. This is in contrast with the standard IMC filter where it is impossible for the resonant peak in its frequency response to go below a certain threshold no matter how large λ is.

- Property 1 is particularly important for the use of the filter in the design of an adaptive robust controller via the approach discussed in [3]. As a matter of fact, in such a design procedure, it is most convenient to relate the bandwidth of the closed-loop system to a single design parameter of the controller. This allows one to easily design, step by step, a controller with progressively increasing bandwidth.

For easy comparison the frequency responses of the standard IMC filter and the new filter, associated with the design of controllers for a minimum-phase unstable plant

$$G(s) = \frac{1}{-s+2} ,$$

are shown in figure 5 for different values of bandwidth. For the same filters, the step responses are displayed in figure 6.

We shall next give some guidelines for the selection of parameters λ and γ. In the case of unstable plants it is well known that transient response considerations lead to the conclusion that the bandwidth of the closed-loop system cannot be made arbitrarily small [6]. In particular, to keep the overshoot in the step response below 40%, such a bandwidth has to be about five times the value of the real unstable pole. This translates to the following lower bound on λ:

$$\lambda \geq 5p .$$

Recall that in theorem 3.1, we have shown that

$$F(j\omega) \simeq \frac{1+(p/\lambda)}{1+(j\omega/\lambda)}$$

in the range of frequency $\omega \in [\gamma, \lambda]$. As a consequence, the initial part of the step response of the filter is mainly governed by the following approximate transfer function of F:

$$\frac{1+(p/\lambda)}{1+(s/\lambda)} ;$$

that is, the initial step response tends to the value $1 + (p/\lambda)$ with the time constant $1/\lambda$. Then, in the long run, the step response recovers to the value of one due to the fact that $F(0) = 1$. From the above considerations, the conclusion can be drawn that

$$\text{overshoot in the unit step response} \simeq \frac{p}{\lambda} .$$

This result can be seen as a "rule of thumb" in the selection of the value of λ.

As far as the parameter γ is concerned, its value should be selected close to zero to hold the frequency response of the filter flat. Since the dominant mode of the transient response is $\exp(-\gamma t)$ when $\gamma \ll p$ (while $p \ll \lambda$) and the step response of the closed-loop system involving an unstable plant always has an overshoot, it is clear that $1/\gamma$ is the dominant time constant in the recovery to the steady-state value after the overshoot has occurred in the step response. Therefore it is important to emphasize that the requirement of a fast recovery after the overshoot has occurred in the step response may be in conflict with the condition that $\gamma \ll p$ and a tradeoff could be necessary.

4 Conclusions

The internal model control (IMC) approach to the design of controllers for linear time-invariant unstable plants has been critically reviewed in this paper. It has been shown that, if the standard filter proposed in [1] is used in the IMC design, the corresponding closed-loop frequency response always involves a resonant peak. This reflects into overshoot in the step response of the control system. We have shown that these disadvantages are not inherent in the control problem and may be alleviated by introducing a new two-degree of freedom filter.

The basic idea behind the new filter can be summarized as follows. If λ is the desired closed-loop bandwidth, a pole of the filter is placed at $-\lambda$. Then a number of filter poles equal to the number of unstable poles of the plant is placed at $-\gamma$, where $-\gamma$, when compared to the unstable poles, is very close to the origin. It turns out that the *zeros* selected in the IMC design procedure so as to take care of nominal stability and reference tracking constraints "almost cancel" these poles. As a consequence, they do not affect significantly the filter transfer function. Finally a suitable number of filter poles are located far into the negative real axis in order to meet the properness requirement of the controller. In this way, a filter and a closed-loop system characterized by a flat frequency response is obtained. Another significant advantage of the new filter is that the parameter λ can be interpreted as its bandwidth. These results are particularly meaningful for minimum-phase plants because, in this case, the transfer function of the closed-loop system coincides with that of the filter.

In this paper, the theory has been developed in detail for minimum-phase plants with a single unstable pole. However, the idea has also been found to work well in the case of more unstable poles. Simulation examples have shown that, in the latter case, the improvement achievable with respect to the standard IMC filter is even larger than in the single pole case.

Some lines of further research are the following:

- When the plant is nonminimum-phase, the filter transfer function can no longer be interpreted as the closed-loop transfer function, and the theory developed in this paper cannot be directly applied. Further work should be done to handle this case as well, particularly with a view to identifying the filter parameter that corresponds to closed-loop bandwidth.
- The characteristics of the new filter can be exploited in the adaptive approach to robust control discussed in [3]. However, the use of the new filter in such a context requires some more work and is currently underway.

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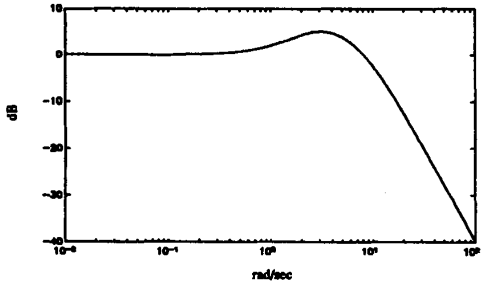
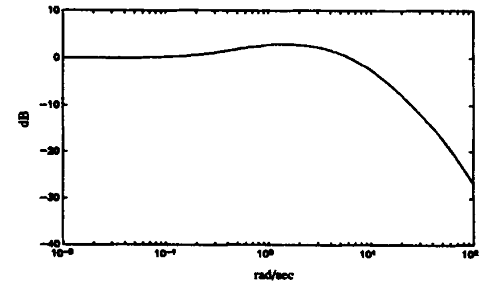
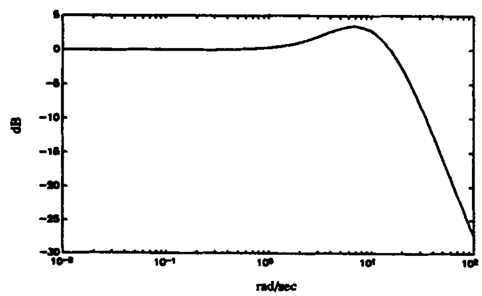


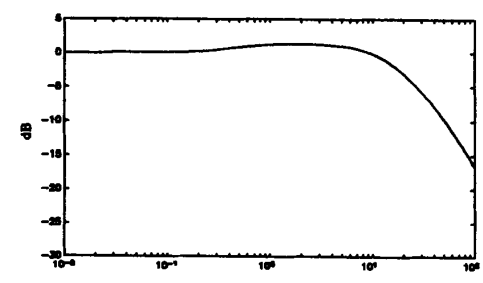
(a) Standard IMC Filter with Bandwidth=10 rad/sec



(b) New Filter with Bandwidth=10 rad/sec

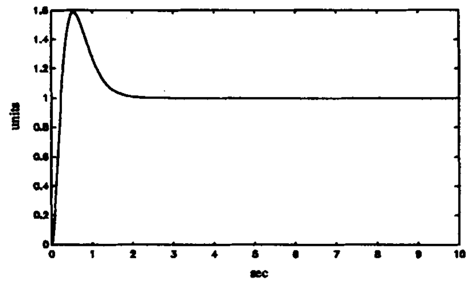


(c) Standard IMC Filter with Bandwidth=20 rad/sec

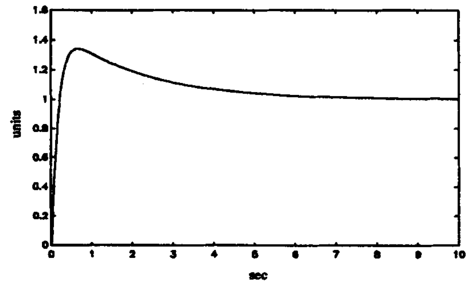


(d) New Filter with Bandwidth=20 rad/sec

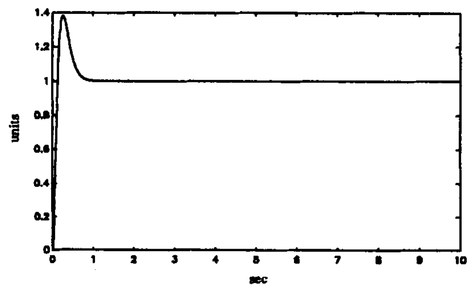
Figure 5: Frequency responses of standard IMC filter and new filter with $\gamma = 0.5$



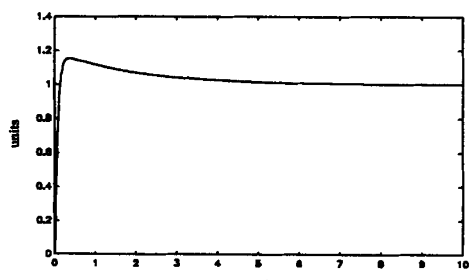
(a) Standard IMC Filter with Bandwidth=10 rad/sec



(b) New Filter with Bandwidth=10 rad/sec



(c) Standard IMC Filter with Bandwidth=20 rad/sec



(d) New Filter with Bandwidth=20 rad/sec

Figure 6: Step responses of standard IMC filter and new filter with $\gamma = 0.5$