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On Robust Performance Improvement through The Windsurfer Approach to Adaptive Robust Control

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Abstract

In this paper it is shown that, given a stable strictly proper model of a stable strictly proper plant, we can improve the performance robustness of the closed-loop system through the windsurfer approach to adaptive robust control if the deterioration in performance robustness caused by increasing the closed-loop bandwidth resulted in a sufficiently high signal-to-noise ratio for a certain closed-loop output error. This is the key conclusion arising from an examination of a series of questions about the procedure that focus on what one would like to do, and what one can do. Situations that may cause the iterative identification and control design process in the windsurfer approach to terminate prematurely are also indicated.

1 Introduction

A new adaptive control design paradigm known as *the windsurfer approach* was first introduced in [1]. The objective of this approach is to increase the bandwidth of a closed-loop system, if possible, to a specified value through *an iterative identification and control design procedure*, given that the initial model of the plant may involve significant error in the high frequency region. Furthermore, as the closed-loop bandwidth is being increased, the closed-loop frequency response is to be kept approximately flat in the passband so that the closed-loop transient response is not too oscillatory or having excessive peak overshoot.

A practical scheme for the windsurfer approach was presented in [4]. It was demonstrated by simulations that the bandwidth of a closed-loop system can be increased by iterative applications of the internal model control (IMC) method [8] and a closed-loop system identification procedure pioneered by Hansen [3].

In this paper, we examine a number of crucial questions which arise in this approach for the case of stable plants with no zeros on the imaginary axis. Among the issues considered are the following: When can one redesign the controller and expand the closed-loop bandwidth, without re-identifying? When should one re-identify? What does one want to identify in the re-identification procedure? What can one identify in the re-identification process? Will re-identification always lead to improved closed-loop performance? Attention is restricted to open-loop stable plants with no finite zeros on the imaginary axis. Extensions to more general situations are currently under investigation [2].

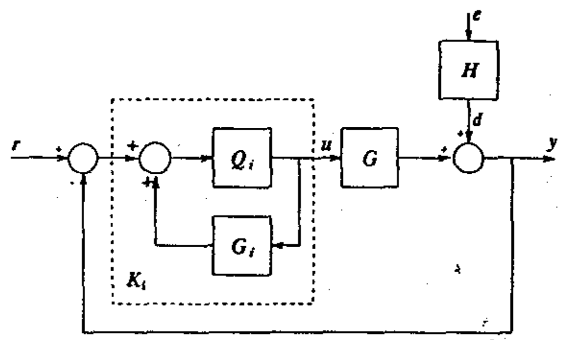

Figure 1: Internal model control structure

This paper is structured as follows. In section 2 we shall describe the IMC design stage of the windsurfer approach and introduce some key concepts and main notations used in the paper. In section 3 we shall establish properties of good models for the windsurfer approach. In section 4 we shall describe the control-relevant system identification method employed by the windsurfer approach. In section 5 we shall study mechanisms that may influence the iterative identification and control design process of the windsurfer approach. In section 6 it will be demonstrated that the performance robustness of a closed-loop system can be improved through the windsurfer approach. We shall conclude the paper in section 7.

2 Preliminaries

In the following we shall outline the IMC method as applied in the control design step of the windsurfer approach when the reference input is a step function. We shall restrict our study in this paper to the case where the plant and the models are stable.

We are concerned with a closed-loop system as shown in figure 1 where G is the transfer function of a stable plant. A sequence of models (identified from data obtained under closed-loop condition) will usually be involved in the windsurfer approach. We therefore use G_i to denote the i^{th} member in the sequence of stable models $\{G_0, G_1, G_2, \ldots\}$. On the basis of G_i a finite sequence of controllers $\{K_i^0, K_i^1, \ldots, K_i^f\}$ is designed with the objective of increasing the bandwidth of the closed-loop system while keeping the frequency responses approximately flat within the pass band. Note that we shall in general use K_i to denote one of the controllers in the sequence

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$\{K_i^0, K_i^1, \ldots, K_i^f\}$ when it is immaterial to the discussion which particular controller is involved. Figure 1 shows that

$$K_i = \frac{Q_i}{1 - G_i Q_i} \quad (1)$$

with Q_i defined in the IMC method by

$$Q_i = [G_i]_m^{-1} F_i \quad (2)$$

where $[G_i]_m$ is *the minimum phase factor* of G_i and

$$F_i = \left(\frac{\lambda_i}{s + \lambda_i}\right)^n; \quad \lambda_i > 0 \quad (3)$$

is the suitable IMC filter when the model is stable and when the reference input is a step function. The integer n of the IMC filter is chosen such that Q_i is proper. The design parameter λ_i must be chosen such that *the actual closed-loop transfer function*

$$T_i = \frac{GK_i}{1 + GK_i} \quad (4)$$

is stable.

Remark

- Note that the system becomes open-loop when λ_i approaches zero. Since G is stable, it is always possible to make T_i stable by choosing a sufficiently small λ_i.

It can be shown easily that *the designed closed-loop transfer function*

$$\tilde{T}_i = \frac{G_i K_i}{1 + G_i K_i}$$

can also be written as

$$\tilde{T}_i = G_i Q_i \quad (5)$$

or

$$\tilde{T}_i = F_i [G_i]_a \quad (6)$$

where $[G_i]_a$ is the all-pass factor associated with G_i.

Remark

- Since $[G_i]_a(j\omega)$ does not affect the magnitude of $\tilde{T}_i(j\omega)$, it is clear that $|\tilde{T}_i(j\omega)|$ is flat in its passband and λ_i is *the designed closed-loop bandwidth* with an attenuation of $-3n\,dB$.

Although the nominal closed-loop transfer function $\tilde{T}_i$ (or the pair (G_i, K_i)) is always well behaved, the actual closed-loop transfer function T_i (or the pair (G, K_i)) could become unstable when λ_i is too large. To facilitate further discussions we shall introduce the following definitions.

Definition 2.1 *We say that the closed-loop system involving the controller K_i, designed on the basis of the model G_i, has* robust stability *if the stability of the pair (G_i, K_i) implies the stability of the pair (G, K_i). In this case we also say that K_i* robustly stabilizes *G_i.*

Definition 2.2 *For any two closed-loop systems designed by the method described in this subsection, we say that the one with a* larger *value of λ_i has a* better nominal performance.

Definition 2.3 *We say that, with respect to the given reference input r, the closed-loop system has* good robust performance *with designed closed-loop bandwidth λ_i if*

$$\|\bar{T}_i\|_r \stackrel{\text{def}}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} |\bar{T}_i(j\omega)|^2 \Phi_r(\omega) d\omega$$

is small, where $\bar{T}_i = T_i - \tilde{T}_i$ is the error in the closed-loop transfer function induced by the error in the model G_i, and $\Phi_r(\omega)$ is the power spectral density of the known reference input signal.

Remark

- We noted in [4] that if T_i is stable, then there exists a strictly- proper transfer function R_i such that

$$G = G_i + \frac{R_i}{1 - Q_i R_i}.$$

It can then easily be shown that

$$\bar{T}_i = R_i Q_i \left(1 - T_i\right). \quad (7)$$

Since the objective of the windsurfer approach is to increase the closed-loop bandwidth to a specified value, we would naturally ask the following question:

When can the closed-loop bandwidth be increased with safety; *that is,* without losing stability, *while retaining the use of the model G_i?*

Due to space limitations, it suffice to say that by using equation (7) we can arrive at the following conclusion:

C1 *We can increase the designed closed-loop bandwidth* cautiously *if the existing closed-loop system has robust stability.*

Remark

- When λ_i is increased, a stage can be reached (before the occurrence of instability) where, because of the modelling errors associated with G_i making a significant contribution to $\|\bar{T}_i\|_r$, the performance robustness has deteriorated beyond an acceptable level. At this stage the nominal closed-loop bandwidth is $\lambda_i = \lambda_i^f$ and it cannot be increased further before a more accurate model G_{i+1} is identified.

3 Properties of Good Models

In section 2 we have concluded that when the performance robustness of the closed-loop system has deteriorated beyond an acceptable level, it is necessary to identify a model better than the existing one before λ_i can be increased further. This prompts us to ask the following question:

What WOULD WE LIKE *to identify, in order that, with the new model, the robust performance of the closed-loop system can be improved through controller redesign?*

Before we proceed to answer the last question, we shall observe that each stage of the windsurfer approach involves an existing model G_i and an updated model G_{i+1}. Since every stage of the iteration proceeds in a similar fashion we shall discuss only the stage for $i = 0$ to simplify the notations. Therefore we shall denote the current model by G_0 and the updated or newly identified model by G_1. This system of notations will carry over to all transfer functions involved in the following discussions.

Suppose that G_1 is identified when λ_0 has reached λ_0^f. A new controller K_1^0 will then be designed on the basis of G_1 such that λ_1^0 has the same value as λ_0^f. Obviously we would like $\left\|\tilde{T}_1^0\right\|_r$ to be small. By using equations (1) to (5), with appropriate adjustments made to the notations, we can write $\tilde{T}_1^0 = T_1^0 - \bar{T}_1^0$ as

$$\tilde{T}_1^0 = \frac{\left(\frac{G-G_1}{G_1}\right)\bar{T}_1^0}{1+\left(\frac{G-G_1}{G_1}\right)\bar{T}_1^0}\left(1-\bar{T}_1^0\right). \tag{8}$$

Clearly it is necessary that $\tilde{T}_1^0$ is stable. Since $G - G_1$ is unknown, therefore we conclude that

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C2 *We* WOULD LIKE *to identify G via a G_1 of sufficient accuracy such that the model G_1 satisfies the sufficient condition of robust stability:*

$$\left\|\left(\frac{G-G_1}{G_1}\right)\bar{T}_1^0\right\|_\infty < 1.$$

Furthermore we observe that the magnitude of the designed sensitivity function $1 - \bar{T}_1^0$ in the right hand side of equation (8) could approach a magnitude of two if G_1 has right-half plane zeros within the passband of $\bar{T}_1^0 = F_1^0[G_1]_a$. In order that $\tilde{T}_1^0$ has a small magnitude, we require in addition to the above robust stability condition that

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C3 *We* WOULD LIKE *to identify G via a G_1 of sufficient accuracy such that*

$$\left|\left[\frac{G(j\omega)-G_1(j\omega)}{G_1(j\omega)}\right]\bar{T}_1^0(j\omega)\right|$$

is sufficiently small for all frequencies above the lesser of the passband of $\bar{T}_1^0$ and the frequency where right-half plane zeros of $\bar{T}_1^0$ appear.

Note that right-half plane zeros of $\bar{T}_1^0$ are those of G_1 which (in a situation with good identification) will be those of the plant G.

If G_1 has right-half plane zeros located within the passband of $\bar{T}_1^0$, it is likely that there is a range of frequency within the passband of $\bar{T}_1^0$ where the magnitude of the designed sensitivity function $1 - \bar{T}_1^0$ is greater than one. This has the following consequences:

1. There is a range of frequency within the passband of $\bar{T}_1^0$ where the designed system has poor output disturbance rejection and the measurement noise is not well attenuated.

2. Since the magnitude of the designed sensitivity function is the inverse of the distance of the open-loop frequency response curve from the critical point of stability at $-1 + j0$, the designed system may have poor stability margins and transient response if the magnitude of the designed sensitivity function is excessively large near the edge of the system passband.

For these reasons, we may not want to increase the designed closed-loop bandwidth beyond $\lambda_1^0 = \lambda_0^f$ if G_1 is found to have right-half plane zeros within the passband of λ_1^0.

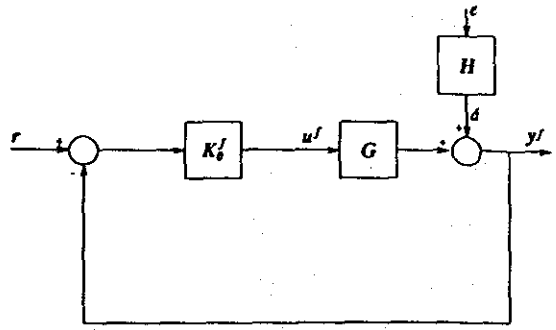

Figure 2: Closed-loop system just before identification

4 System Identification in the Windsurfer Approach

Notwithstanding the fact that we have established in the last section desirable properties of a good model for the windsurfer approach, it is important to ask the following question:

What CAN WE *identify by using the system identification procedure embedded in the windsurfer approach?*

In this section we shall answer the last question by:

1. Describe a control-relevant system identification problem.

2. Deduce conditions necessary for good identification.

4.1 Control-relevant system identification

It was indicated at the end of section 2 that when the designed closed-loop bandwidth has reached a certain value denoted by λ_0^f, the robust performance measure

$$\left\|\tilde{T}_0^f\right\|_r = \left\|\frac{GK_0^f}{1+GK_0^f} - \frac{G_0K_0^f}{1+G_0K_0^f}\right\|_r$$

associated with the controller K_0^f designed on the basis of G_0 has become excessively large. It was shown in section 3 that, at this stage, we would like to identify a new model G_1 such that

$$\left|\left[\frac{G(j\omega)-G_1(j\omega)}{G_1(j\omega)}\right]\bar{T}_1^0(j\omega)\right|$$

is sufficiently small in an appropriate frequency range. Unfortunately, it is not clear how to process input-output measurements to determine G_1 so that this condition is naturally or automatically satisfied. To overcome this difficulty we shall use input-output measurements and possibly the reference input of the stable closed-loop system as shown in figure 2 to identify G_1 such that

$$\left\|\frac{GK_0^f}{1+GK_0^f} - \frac{G_1K_0^f}{1+G_1K_0^f}\right\|_r$$

is minimized. This closed-loop identification problem can be transformed into an open-loop identification problem by employing Hansen's framework of identification [3]. We shall state this result in the following theorem. It is a special case of Theorem 2 in [4] when the plant G and the model G_0 are stable.

Theorem 4.1 *Let K_0^f stabilize G and G_0 and let*

$$K_0^f = \frac{Q_0^f}{1 - G_0 Q_0^f}$$

where Q_0^f is a proper stable transfer function.

Let G_1 be another model of G, also stabilized by K_0^f and therefore having a description

$$G_1 = G_0 + \frac{\hat{R}_0^f}{1 - \hat{R}_0^f Q_0^f}$$

where $\hat{R}_0^f$ is a strictly proper stable transfer function. Also define

$$\xi = \left(1 - G_0 Q_0^f\right)\left(\beta - \hat{R}_0^f \alpha\right) \tag{9}$$

where

$$\alpha = Q_0^f r, \quad \beta = y^f - G_0 u^f,$$

and u^f and y^f are, respectively, the input and output of the plant resulting from the application of the controller K_0^f. Thus ξ is a frequency weighted error arising in the (open-loop) identification of R_0^f that parametrizes G via

$$G = G_0 + \frac{R_0^f}{1 - R_0^f Q_0^f}.$$

Then ξ can be expressed as

$$\xi = \left(\frac{GK_0^f}{1 + GK_0^f} - \frac{G_1 K_0^f}{1 + G_1 K_0^f}\right) r + \frac{1}{1 + GK_0^f} He.$$

Remarks

- From theorem 4.1 it is clear that minimizing

$$\left\| \frac{GK_0^f}{1 + GK_0^f} - \frac{G_1 K_0^f}{1 + G_1 K_0^f} \right\|_r$$

  with respect to G_1 is equivalent to minimizing

$$\left\| \left(1 - G_0 Q_0^f\right)\left(\beta - \hat{R}_0^f \alpha\right) \right\|_{rms}$$

  with respect to $\hat{R}_0^f$, provided that G_1 is updated according to

$$G_1 = G_0 + \frac{\hat{R}_0^f}{1 - \hat{R}_0^f Q_0^f}. \tag{10}$$

  (The notation $\|\cdot\|_{rms}$ denotes the average power of the argument involved.)

- Since the "input" α is independent of the measurement noise which affects the "output" β, identification of the stable transfer function

$$R_0^f = \frac{G - G_0}{1 + Q_0^f (G - G_0)}$$

  via the stable transfer function

$$\hat{R}_0^f = \arg\min_{\rho} \left\| \left(1 - G_0 Q_0^f\right)(\beta - \rho\alpha) \right\|_{rms} \tag{11}$$

  is an *open-loop* identification problem.

We can summarise the above discussions as follows:

C4 *We can transform the problem of identifying G in closed-loop into a problem of identifying*

$$R_0^f = \frac{G - G_0}{1 + Q_0^f (G - G_0)}$$

in open-loop.

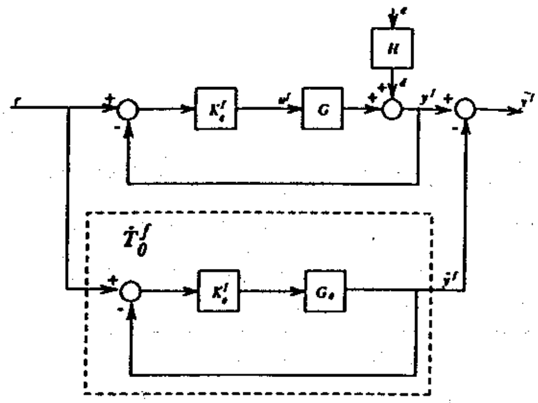

Figure 3: Closed-loop output error

4.2 Accurate identification of R_0^f

In the following we shall show that R_0^f can be identified effectively (using a finite duration of input-output measurements) only if the signal-to-noise ratio of a certain closed-loop output error (to be defined immediately) is sufficiently high.

We begin with the definition of the closed-loop output error $\tilde{y}^f$ as shown in figure 3, where

$$\tilde{y}^f = y^f - \tilde{T}_0^f r.$$

By substituting the expressions for α and β into equation (9) and noting that

$$u^f = \frac{Q_0^f}{1 - G_0 Q_0^f}(r - y),$$

we can obtain

$$\hat{R}_0^f = \arg\min_{\rho} \left\| \tilde{y}^f - \rho Q_0^f \left(1 - \tilde{T}_0^f\right) r \right\|_{rms}. \tag{12}$$

Now we can use the fact that $y^f = T_0^f r + \left(1 - T_0^f\right) He$ to write $\tilde{y}^f$ as

$$\tilde{y}^f = v + w \tag{13}$$

where

$$v = \tilde{T}_0^f r \tag{14}$$

and

$$w = \left(1 - T_0^f\right) He. \tag{15}$$

It is apparent that v is the signal component in $\tilde{y}^f$ that carries the useful information about the existing modelling error under closed-loop condition and w is the noise component in $\tilde{y}^f$ that will be a hindrance to the determination of $\hat{R}_0^f$. Therefore we can draw an immediate conclusion:

C5 *We can identify R_0^f only if the signal-to-noise ratio associated with the closed-loop output error resulting from the existing controller K_0^f is high.*

We shall next show that the normalized variance for an unbiased estimate of R_0^f will be asymptotically small in the frequency range where the signal-to-noise ratio associated with the closed-loop output error is sufficiently high.

By substituting equations (13) and (14) into equation (12) and noting from equation (7) that $\tilde{T}_0^f = Q_0^f \left(1 - \tilde{T}_0^f\right) R_0^f$, we can write

$$\hat{R}_0^f = \arg\min_{\rho} \left\| Q_0^f \left(1 - \tilde{T}_0^f\right) \left(R_0^f - \rho\right) r + w \right\|_{rms}.$$

In practice we use sampled input-output data to estimate a discrete time model for $\hat{R}_0^f$ before converting it to a continuous time transfer function. We shall assume that the errors involved in this conversion are negligible. Following [6] we can write the variance of an unbiased estimate of R_0^f approximately as

$$\mathrm{E}\left(\left|\hat{R}_0^f(j\omega) - R_0^f(j\omega)\right|^2\right) \sim$$

$$\frac{m}{M} \frac{\Phi_w(\omega) z}{\left|Q_0^f(j\omega)\left(1 - \tilde{T}_0^f(j\omega)\right)\right|^2 \Phi_r(\omega)},$$

where $\Phi_w(j\omega)$ is the power spectral density of w, under the condition that the order of the discrete time model for $\hat{R}_0^f$ (denoted by m) and the number of data (denoted by M) are large and the ratio m/M is small.

Since $\Phi_v(\omega) = \left|R_0^f(j\omega) Q_0^f(j\omega)\left[1 - \tilde{T}_0^f(j\omega)\right]\right|^2 \Phi_r(\omega)$ is the power spectral density of v, we can write the normalized variance of $\hat{R}_0^f$ as

$$\mathrm{E}\left(\left|\frac{\hat{R}_0^f(j\omega) - R_0^f(j\omega)}{R_0^f(j\omega)}\right|^2\right) \sim \frac{m}{M} \frac{\Phi_w(\omega)}{\Phi_v(\omega)}$$

for the frequencies where $R_0^f(j\omega) \neq 0$.

Remark

- For a finite number of data, the normalized variance of $\hat{R}_0^f$ can only be small in the frequency range where the signal-to-noise ratio associated with the closed-loop output error is sufficiently high.

We now summarise the above discussion as follows:

C6 *We can obtain an unbiased estimate of R_0^f with a small asymptotic normalized variance in a certain frequency range $\omega_1 \le \omega \le \omega_2$ only if*

1. *the structure of the model set used in the estimation of R_0^f is sufficiently general,*
2. *the condition*

$$\frac{\Phi_v(\omega)}{\Phi_w(\omega)} \ge \mu, \quad \text{for } \omega_1 \le \omega \le \omega_2$$

is satisfied for a sufficiently large $\mu > 0$.

It is clear that nothing comes for free and it is prudent to ask the following question:

What is the price that we have to pay, in terms of system performance, before a sufficiently high signal-to-noise ratio of the closed-loop output error can be achieved?

We shall next show that it is necessary to have a certain level of deterioration in robust performance of the system relative to the effect of noise disturbance before the closed-loop output error can achieve a sufficiently high signal-to-noise ratio.

By using equations (14) and (15) we can deduce that

$$\frac{\Phi_v(j\omega)}{\Phi_w(j\omega)} \ge \mu, \quad \text{for } \omega_1 \le \omega \le \omega_2$$

if and only if

$$\left|\tilde{T}_0^f(j\omega)\right|^2 > \mu \left|\left[1 - T_0^f(j\omega)\right] H(j\omega)\right|^2 \Phi_e(\omega), \qquad (16)$$

$$\text{for } \omega_1 \le \omega \le \omega_2$$

where $\Phi_e(\omega)$ is the power spectral density of the disturbance noise e. Upon integration we get

$$\frac{1}{\pi}\int_{\omega_1}^{\omega_2} \left|\tilde{T}_0^f(j\omega)\right|^2 \Phi_r(\omega) d\omega >$$

$$\frac{\mu}{\pi}\int_{\omega_1}^{\omega_2} \left|\left[1 - T_0^f(j\omega)\right] H(j\omega)\right|^2 \Phi_e(\omega) d\omega$$

Since

$$\left\|\tilde{T}_0^f\right\|_r > \frac{1}{\pi}\int_{\omega_1}^{\omega_2} \left|\tilde{T}_0^f(j\omega)\right|^2 \Phi_r(\omega) d\omega,$$

therefore

$$\left\|\tilde{T}_0^f\right\|_r > \frac{\mu}{\pi}\int_{\omega_1}^{\omega_2} \left|\left[1 - T_0^f(j\omega)\right] H(j\omega)\right|^2 \Phi_e(\omega) d\omega.$$

We can therefore restate the conditions necessary for the estimation of R_0^f as follows:

C7 *We can obtain an unbiased estimate of R_0^f with a small asymptotic normalized variance in a certain frequency range $\omega_1 \le \omega \le \omega_2$ only if*

1. *the structure of the model set used in the estimate of R_0^f is sufficiently general,*
2. *there is a certain level of deterioration in robust performance bounded from below by*

$$\frac{\mu}{\pi}\int_{\omega_1}^{\omega_2} \left|\left[1 - T_0^f(j\omega)\right] H(j\omega)\right|^2 \Phi_e(\omega) d\omega$$

for a sufficiently large $\mu > 0$.

Remark

- Due to space limitations it will be reported elsewhere [5] that we can verify experimentally that $\hat{R}_0^f$ is a practically unbiased estimate of R_0^f.

5 Mechanisms that influence performance robustness and identification

We have deduced in section 4.2 that deterioration in robust performance is necessary before we can attempt to find a good estimate of R_0^f. However we should ask:

Does it means that, irrespective of the causes, deterioration in robust performance is always helpful to the identification of R_0^f?

The answer is obviously no. We shall next show that there are three mechanisms that may lead to deterioration in robust performance. However only one of them will contribute to the high signal-to-noise ratio needed for a successful estimation of R_0^f.

With appropriate substitutions into equations (14) and (15) respectively, we can obtain

$$v = \frac{\left(\frac{G-G_0}{G_0}\right)\tilde{T}_0^f}{1+\left(\frac{G-G_0}{G_0}\right)\tilde{T}_0^f}\left(1-\tilde{T}_0^f\right)r \qquad (17)$$

and

$$w = \frac{1}{1+\left(\frac{G-G_0}{G_0}\right)\tilde{T}_0^f}\left(1-\tilde{T}_0^f\right)He \qquad (18)$$

Since $v = \tilde{T}_0^f r$ we observe that, disregarding changes in disturbance suppression ability, the deterioration in robust performances is determined by

$$\left\|\tilde{T}_0^f\right\|_r = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_v(\omega)d\omega.$$

We therefore conclude from the right-hand side of equation (17) that, for a given reference input, there are three factors that influence $\|\tilde{T}_0^f\|_r$ through $\Phi_v(\omega)$:

1. The effect of the factor

$$\left(\frac{G-G_0}{G_0}\right)\tilde{T}_0^f$$

on $\Phi_v(\omega)$ is independent of the phase angle of

$$\left(\frac{G-G_0}{G_0}\right)\tilde{T}_0^f.$$

We shall call this the *phase insensitive factor*.

2. The effect of the factor

$$\left[1+\left(\frac{G-G_0}{G_0}\right)\tilde{T}_0^f\right]^{-1}$$

on $\Phi_v(\omega)$ depends on the gain and phase margins of

$$\left(\frac{G-G_0}{G_0}\right)\tilde{T}_0^f.$$

We shall call this the *stability margin factor*.

3. The effect of the factor $1-\tilde{T}_0^f$ on $\Phi_v(\omega)$ depends on the existence of right-half plane zeros of G_0 within the passband of $\tilde{T}_0^f = F_0^f[G_0]a$. We shall call this the *non-minimum phase zeros dependent factor*.

However by using equations (17) and (18) we can write the signal-to-noise ratio of the closed-loop output error as

$$\frac{\Phi_v(\omega)}{\Phi_w(\omega)} = \frac{\left|\left[\frac{G(j\omega)-G_0(j\omega)}{G_0(j\omega)}\right]\tilde{T}_0^f(j\omega)\right|^2}{|H(j\omega)|^2}\frac{\Phi_r(\omega)}{\Phi_e(\omega)}$$

which indicates that for a given situation of reference input and noise disturbance, only an increase in the magnitude of the phase insensitive factor can increase the signal-to-noise ratio of the closed-loop output error.

We shall now summarize the above discussions as follows:

C8

1. *There are three factors that can cause the performance robustness to deteriorate. They are namely, the phase insensitive factor, the stability margin factor, and the non-minimum phase dependent factor. Among there factors, only the response of the phase insensitive factor to the reference input alone can improve the signal-to-noise ratio of the closed-loop output error.*

2. *When the non-minimum phase zeros dependent factor or the stability margin factor are the main causes of deterioration in robust performance, the signal-to-noise ratio of the closed-loop output error can be poor and it may be difficult to obtain a practically unbiased estimate of R_0^f with a small asymptotic normalized variance. This may cause difficulties in continuing the iterative identification and control design process.*

6 Robust Performance Improvement through The Windsurfer Approach

Now we know what can be identified, and previously we have indicated what we would like to identify. It is therefore logical to ask:

How does the object which we CAN *identify relate to the object which we* WOULD LIKE *to identify?*

The answer is that the objects are virtually the same, although it is not obvious. What we can identify is couched in terms of R_0^f, and what we would like to identify is couched in terms of G_1. We need to connect these characterizations. In this section we shall show that provided that certain conditions are satisfied, the controller designed on the basis of the model G_1 updated through an estimate of $\hat{R}_0^f$ can improve the performance robustness of the system.

Recall from equation (16) that just before the model G_0 is updated through a good estimate of R_0^f, it is necessary that the signal-to-noise ratio of the closed-loop output error signal is high and

$$|\tilde{T}_0^f(j\omega)|^2\Phi_r(\omega) > \mu|[1-T_0^f(j\omega)]H(j\omega)|^2\Phi_e(\omega)$$

$$\text{for } \omega_1 \le \omega \le \omega_2.$$

These requirements imply that $|\tilde{T}_0^f(j\omega)|$ is no longer small in the frequency range $\omega_1 \le \omega \le \omega_2$ and particularly because of the contribution of the phase insensitive factor.

In order that the robust performance could be improved through re-identification and re-design, it is necessary that the complementary-sensitivity weighted multiplicative modelling error associated with the updated model G_1 after controller re-design (while keeping $\lambda_1^0 = \lambda_0^f$) be small in the frequency range $\omega_1 \le \omega \le \omega_2$.

It is then relevant to consider the magnitude of the ratio

$$\frac{\frac{G-G_1}{G_1}\tilde{T}_1^0}{\frac{G-G_0}{G_0}\tilde{T}_0^f}$$

in the frequency range $\omega_1 \le \omega \le \omega_2$.

We can establish the following theorem:

Theorem 6.1 *Let G_0 be a stable strictly proper model of G. Suppose that G is stabilized by the controller K_0^f designed according to the IMC method described by equations (1) and (2). If*

1. *G_1 is estimated according to the scheme described by equations (10) and (11) with $\hat{R}_0^f$ having the same relative degree as G_0, and G_1 satisfies the constraint*

$$\left|\left(\frac{G_1 - G_0}{G_0}\right)\tilde{T}_0^f\right| < \eta \quad for\ \omega_1 \le \omega \le \omega_2,$$

2. *K_1^0 is designed by the IMC method with $\lambda_1^0 = \lambda_0^f$,*

then, for each $\omega_1 \le \omega \le \omega_2$ such that $R_0^f(j\omega) \neq 0$,

$$\frac{\left|\frac{\Delta_{G_1}}{G_1}\tilde{T}_1^0\right|^2}{\left|\frac{\Delta_{G_0}}{G_0}\tilde{T}_0^f\right|^2} < \delta^2(1+\eta)^2\left|\frac{\hat{R}_0^f - R_0^f}{R_0^f}\right|^2, \tag{19}$$

where

$$\delta = \sup_{\omega_1 \le \omega \le \omega_2}\left|\frac{[G_0]_m}{[G_1]_m}\right|.$$

Proof
The proof is not given due to space limitations. □

We noted immediately from theorem 6.1 that if

$$\left|\frac{\hat{R}_0^f - R_0^f}{R_0^f}\right|^2 \ll \frac{1}{\delta^2(1+\eta)^2}$$

in the frequency range $\omega_1 \le \omega \le \omega_2$, then the phase insensitive factor associated with G_1 and $\tilde{T}_1^0$ will be smaller in magnitude than that associated with G_0 and $\tilde{T}_0^f$ in the same frequency range. We therefore have the following conclusion, which should be read in conjunction with conclusions C7 and C8:

C9 *If a practically unbiased estimate of R_0^f with a sufficiently small normalized variance can be obtained over the frequency range $\omega_1 \le \omega \le \omega_2$, with $\hat{R}_0^f$ and G_1 satisfying the constraints stated in theorem 6.1, then it is possible to achieve robust performance improvement through controller re-design if the non-minimum phase zeros of G_1 are outside the nominal closed-loop passband.*

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7 Conclusions

We have shown mainly in this paper that, given a strictly-proper stable model of a strictly-proper stable plant, it is possible to improve the performance-robustness of a closed-loop system through the windsurfer approach to adaptive robust control if the deterioration in performance robustness caused by increasing the closed-loop bandwidth resulted in a sufficiently high signal-to-noise for the closed-loop output error.

References

[1] B.D.O.Anderson, and R.L.Kosut, Adaptive Robust Control: On-Line Learning, *Proc CDC'91*, 297-298, Brighton, England, Dec 1991.

[2] M.Campi, W.S.Lee, B.D.O.Anderson, New Filters for Internal Model Control Design, to appear in *Int. J. Robust and Nonlinear Control.*

[3] F.R.Hansen, A Fractional Representation Approach to Closed-loop System Identification and Experiment Design, *PhD dissertation*, Stanford University, March 1989.

[4] W.S.Lee, B.D.O.Anderson, R.L.Kosut, and I.M.Y.Mareels, A New Approach to Adaptive Robust Control, to appear in *Int. J. Adaptive Control and Signal Processing.*

[5] W.S.Lee, B.D.O.Anderson, R.L.Kosut, and I.M.Y.Mareels, On Some Practical Issues in System Identification for The Windsurfer Approach to Adaptive Robust Control, to be submitted to 10_{th} *IFAC Symposium on System Identification*, Copenhagen, Denmark, Jul 1994.

[6] L.Ljung, *System Identification: Theory for the user*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

[7] L.Ljung, B.Wahlberg, and H.Hjalmarsson, Model Quality: The Role of Prior Knowledge and Data Information, *Report LiTH-ISY-I-1254, Department of Electrical Engineering*, Linköping University, Linköping, Sweden, Sep 1991.

[8] M.Morari, and E.Zafiriou, *Robust Process Control*, Prentice Hall, Englewood Cliffs, NJ, 1989.