1992 ACC/FA8

On Adaptive Robust Control and Control-Relevant System Identification

Wee Sit Lee[1] Brian D. O. Anderson[1] Robert L. Kosut[2] Iven M. Y. Mareels[1]

[1]Department of Systems Engineering
Australian National University
G.P.O. Box 4, Canberra
ACT 2601, Australia

[2]Integrated Systems Inc.
2500 Mission College Blvd.
Santa Clara, CA 95054

Abstract A new approach is given for the design of adaptive robust control in the frequency domain. Starting with an initial model and a robust stabilizing controller, the new (windsurfer) approach allows the bandwidth of the closed-loop system to be increased progressively through an iterative control-relevant system identification and control design procedure. Encouraging results are obtained in the case studies that serve as a benchmark test for the new idea.

1 Introduction

It has long been understood that a key problem in control systems design is to handle the *uncertainties* associated with the plant [12]. Two main techniques for the analysis and design of systems with significant uncertainties are adaptive control [8] and robust control [6, 15].

In the traditional approach to analysis and design of an adaptive control system [8], it is assumed that the unknown plant can be represented by a model in which everything is known except for the values of a finite number of parameters. Once the parameters are estimated (and even during the estimation process), the principle of certainty equivalence is normally invoked to update the controller. Normally the unstructured uncertainties of the model are *ignored* in this approach. Therefore it is not surprising, as pointed out in [18], that these adaptive controllers are often not robust. Further, the extensions of the traditional approach to adaptive control which purportedly cope with unstructured (and other) uncertainties involve conditions which are often hard to apply or to grasp intuitively, see for example [1, 3, 13]. A further problem with the traditional approach is that extreme transient excursions are possible even when global convergence and asymptotic performance are guaranteed [21].

To be more specific, we consider an adaptive control system as shown in figure 1, where G is the unknown transfer function of the plant. The time axis is divided into intervals such that during the i^{th} interval, the control input applied to the plant is obtained from K_i, where K_i is the transfer function of the controller designed using the model G_{i-1} obtained at the end of the $(i-1)^{th}$ time interval.

In an adaptive control problem, the ulterior objective for finding G_i, an estimate of G updated from G_{i-1}, is to redesign a better controller K_{i+1} than K_i, such that certain control objectives are improved. For example if T^d represents the desired complementary sensitivity function, then we may like to have

$$\left\| \frac{GK_i}{1+GK_i} - T^d \right\|_\infty \leq \left\| \frac{GK_{i-1}}{1+GK_{i-1}} - T^d \right\|_\infty, \quad \forall i.$$

Implicitly, this means we would like to minimize

$$\left\| \frac{GK_i}{1+GK_i} - T^d \right\|_\infty, \quad \forall i.$$

Since G, the transfer function of the plant, is unknown, we could only base our design of K_i on G_{i-1} such that

$$K_i = \arg\min_{\gamma} \left\| \frac{G_{i-1}\gamma}{1+G_{i-1}\gamma} - T^d \right\|_\infty, \quad \forall i.$$

Note that, as usual, we have invoked the principle of certainty equivalence. However, it is important to realize that

$$\left\| \frac{GK_i}{1+GK_i} - T^d \right\|_\infty$$

is not necessarily small, even though

$$\left\| \frac{G_{i-1}K_i}{1+G_{i-1}K_i} - T^d \right\|_\infty$$

is a minimum. This partly explains why traditional adaptive control systems, which invariably invoked the principle of certainty equivalence, have unsatisfactory robustness property.

In the robust control approach [6, 15], a controller is designed based on a nominal model of the plant with the associated parametric and unstructured model uncertainties explicitly taken into account. Therefore stability robustness is guaranteed and performance robustness is achieved sometimes. The *weakness* of this approach is that it considers only the a priori information on the model, and *neglects* the fact that characteristics of the plant could be learnt while it is being controlled. Therefore, the robust control approach tends to result in a conservative design in terms of performance. It is likely that a posteriori knowledge about the plant could be used to reduce the conservatism in a robust control design.

2 The Windsurfer Approach to Adaptive Control

By considering how humans learn windsurfing, Anderson and Kosut [2] have made the following observations:

1. The human first learns to control over a limited bandwidth, and learning pushes out the bandwidth over which an accurate model of the plant is known.

2. The human first implements a low gain controller, and learning allows the loop to be tightened.

Based on these observations an adaptive robust control design philosophy, *the windsurfer approach*, is proposed in [2]. It recognizes that, at the outset, the plant characteristics can differ greatly from the estimated model at any one time, particularly during the initial learning stage. In the new design approach, a low gain controller will first be implemented; and the control bandwidth will be small. Based on learning a frequency domain description of the plant *in closed- loop*, with the learning process progressively *increasing the bandwidth over which the plant is accurately known*, the controller gain can be increased appropriately over an increasing frequency band. For details, refer to [2]. Importantly, in the method suggested, the necessary closed-loop system identification task is simplified into an open-loop system identification problem through the use of coprime fractional representations as discussed in [9, 10].

It was shown recently in [19] that the best model for control design cannot be derived from open-loop experiments alone. The controller to be implemented should be taken into account by the system identification experiments. However, this controller is not yet available, as its determination rests on the results of the system identification to be carried out. Hence, a general solution to the combination of system identification and control design is necessarily iterative. It was also shown in [22] that an iterative approach for model refinement and control robustness enhancement can be developed for a H_2 control problem. Although the emphasis of [19] is on the problem of modeling for control design, its approach is very similar to that of [2]. In the next section, we would like to illustrate the windsurfer approach by considering a model matching problem in the context of adaptive control.

3 Adaptive Model Matching

Let G be the unknown transfer function of the plant, and let T^d represent a desired complementary sensitivity function. We wish to achieve, through iterative system identification and control design, the minimization of the cost function

$$\left\|\frac{GK}{1+GK}-T^d\right\|_\infty,$$

where K is the transfer function of a controller to be design.

We begin by designing a controller $K_{1,0}$ to stabilize a known initial model G_0, which may be obtained from an open-loop system identification exercise. If $K_{1,0}$ also stabilizes the unknown transfer function G, then we say that $K_{1,0}$ robustly stabilizes G_0. Notice that we use $K_{j,i}$ to denote the j^{th} controller designed using the i^{th} model which has a transfer function G_i. In general, we attach the subscript j,i to a transfer function to denote that it is either spicified or derived on the basis of the i^{th} model for the plant at the j^{th} iteration of control design. Since G_0 may involve significant uncertainties, the resulting controller $K_{1,0}$ may not be able to achieve a small value for

$$\left\|\frac{G_0K_{1,0}}{1+G_0K_{1,0}}-T^d\right\|_\infty$$

while robustly stabilizing G_0. In general, we need to consider how to handle the question of securing robust stabilization of G_i by $K_{j,i}$. This is bound up with the question of selection of T^d. It is in fact to be expected that a sequence of T^d will be selected in such a way that the end control objective can be approached in stages. We shall therefore proceed as follows.

Associated with each of the models G_i, a sequence of controllers $K_{j,i}$ is to be designed such that

$$K_{j,i}=\arg\min_{\gamma}\left\|\frac{G_i\gamma}{1+G_i\gamma}-T^d{}_{j,i}\right\|_\infty,\quad\forall j,\tag{3.1}$$

where the sequence of functions $T^d{}_{j,i}$ is specified with $T^d{}_{j+1,i}$ normally of wider bandwidth than $T^d{}_{j,i}$, and with $T^d{}_{1,i}$ resulting in a controller $K_{1,i}$ that robustly stabilizes G_i. A stage will be reached (say when $j=N$) where the bandwidth of the nominal closed-loop transfer function,

$$\tilde{T}_{N,i}=\frac{G_iK_{N,i}}{1+G_iK_{N,i}},\tag{3.2}$$

cannot be increased further without causing the effects of model uncertainties in G_i to be too significant. This occurs when the value of

$$\|T_{N,i}-\tilde{T}_{N,i}\|_\infty$$

is no longer small, where

$$T_{N,i}=\frac{GK_{N,i}}{1+GK_{N,i}}\tag{3.3}$$

is the actual closed-loop transfer function of the system.

At this stage it is necessary to improve the accuracy of the model in such a way that is relevant to the control objective. This means that we should try to find an updated model G_{i+1} such that

$$G_{i+1}=\arg\min_{\theta}\left\|\frac{GK_{N,i}}{1+GK_{N,i}}-\frac{\theta K_{N,i}}{1+\theta K_{N,i}}\right\|_\infty.\tag{3.4}$$

Equation 3.4 would be the formulation of a standard rational function approximation problem, provided that G were known. In the simulation (section 6), we shall take this approach by using a known transfer function for G. This serves as a benchmark test of the windsurfer approach as it corresponds to performing system identification with an infinite number of noiseless measurements. It is a topic of further research to deal with this problem in a realistic system identification setting when only a finite number of (possibly noisy) input-output measurements are available.

Once G_{i+1} is found, we can continue to increase the closed-loop bandwidth by repeating the procedure described for G_i previously. However G_{i+1} should be used instead of G_i, and we specify a new sequence of functions $T^d{}_{j,i+1}$ with $T^d{}_{1,i+1}=T^d{}_{N,i}$. The iterative process is continued until the end control objective is achieved or it is prematurely terminated because of one or more of the following constraints:

1. fundamental performance limitations due to right half plane poles and zeros of the plant and/or models [7].
2. unstable model is obtained. (This is a consequence of our simplified control design method. Appropriate extensions of the control design method [15] allow us to deal with this restriction.)
3. finite control energy.

4 Closed-loop System Identification

We first review a method for closed-loop system identification developed by Hansen [10]. Subsequently, in theorem 4.2, we demonstrate that with appropriate signal filtering, Hansen's method provides a suitable framework to carry out the *control-relevant* system identification formulated in section 3. For the sake of expository simplicity, we shall consider only scalar plants. We begin with the following theorem [20]:

Theorem 4.1 *If $K=\frac{X}{Y}$ is a controller, where X and Y are stable proper transfer functions, and if N and D are stable proper transfer functions that satisfy the Bezout identity*

$$NX+DY=1,$$

then the set of all plants stabilized by the controller K is precisely the set of elements in

$$\mathcal{G}=\{\frac{N+RY}{D-RX}:R\text{ is a stable proper transfer function}\}.$$

Consider the feedback system shown in figure 4, where y and u are the measured output and the control input, respectively, e is an unpredictable white disturbance, and r_1 and r_2 are user applied inputs. It is assumed that $K_{j,i}$ is a known stabilizing controller, G is unexactly known and possibly unstable, and, as is standard [14], H is imperfectly known, stable and inversely stable. The system identification problem is to obtain improved estimates of G and H from a finite interval of measured and known data $\{y,u,r_1,r_2:0\le t\le T\}$.

Following Hansen [10], we introduce the stable proper transfer functions $X_{j,i}$, $Y_{j,i}$, $N_{j,i}$, and $D_{j,i}$ which satisfy

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}},$$

$$G_i = \frac{N_i}{D_i},$$

and

$$N_i X_{j,i} + D_i Y_{j,i} = 1.$$

The interpretation is that G_i is a known but imperfect model of the plant which is also stabilized by $K_{j,i}$. Applying theorem 4.1 as shown in [10], there exist stable proper transfer functions $R_{i,j}$ and $S_{i,j}$, with $S_{i,j}$ also inversely stable, such that

$$G = \frac{N_i + R_{i,j} Y_{j,i}}{D_i - R_{i,j} X_{j,i}}, \tag{4.1}$$

$$H = \frac{S_{i,j}}{D_i - R_{i,j} X_{j,i}}, \tag{4.2}$$

where $R_{i,j}$ denotes the parametrization of G using the i^{th} model and its associated j^{th} controller $K_{j,i}$.

As a result, system identification of G and H in closed-loop is equivalent to system identification of the stable proper transfer functions $R_{i,j}$ and $S_{i,j}$. Using equations 4.1 and 4.2, we can represent the feedback system as shown in figure 4.

From figure 4, we can write

$$\beta = R_{i,j}\alpha + S_{i,j}e, \tag{4.3}$$

where

$$\alpha = X_{j,i} y + Y_{j,i} u, \tag{4.4}$$

and

$$\beta = D_i y - N_i u. \tag{4.5}$$

However, as

$$u = K_{j,i}(r_i - y) + r_2$$

and

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}},$$

equation 4.4 can be re-written as

$$\alpha = X_{j,i} r_1 + Y_{j,i} r_2. \tag{4.6}$$

It is important to observe from equations 4.3, 4.5 and 4.6 that α depends on the applied signals r_1 and r_2 operated on by known stable proper transfer functions $X_{j,i}$ and $Y_{j,i}$ respectively, and β depends on measured signals y and u operated by known stable proper transfer functions D_i and N_i respectively. Moreover, α is independent of the transfer functions G and H and the disturbance e. Hence the system identification of G and H in closed-loop has been recast into the system identification of $R_{i,j}$ and $S_{i,j}$ in open-loop. We shall next state a result which is highly relevant to the system identification step of the windsurfer approach to adaptive control.

Theorem 4.2 *Let the controller $K_{j,i}$ stabilize the plant transfer function G and the model transfer function*

$$G_i = \frac{N_i}{D_i},$$

where N_i and D_i are stable proper transfer functions, and let

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}},$$

where $X_{j,i}$ and $Y_{j,i}$ are stable proper transfer functions satisfying the Bezout identity

$$N_i X_{j,i} + D_i Y_{j,i} = 1.$$

Let G_{i+1} be another model of G, also stabilized by $K_{j,i}$ and therefore having a description

$$G_{i+1} = \frac{N_i + r_{i,j} Y_{j,i}}{D_i - r_{i,j} X_{j,i}} \tag{4.7}$$

where $r_{i,j}$ is a stable proper transfer function. Also define the filtered output error

$$\xi = Y_{j,i}(\beta - r_{i,j}\alpha),$$

where, with $r_2 = 0$,

$$\alpha = X_{j,i} r_1,$$

$$\beta = D_i y - N_i u,$$

$$r_1 = \textit{reference signal},$$

$$y = \textit{plant output},$$

$$u = \textit{control input}.$$

Thus ξ is an error arising in the (open-loop) identification of $R_{i,j}$ through an estimate $r_{i,j}$. Then the filtered output error can be expressed as

$$\xi = \left(\frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_{i+1}K_{j,i}}{1+G_{i+1}K_{j,i}}\right) r_1 + \frac{H}{1+GK_{j,i}} e$$

The proof is not given due to space limitations.

Suppose that the value of

$$\left\| \frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_i K_{j,i}}{1+G_i K_{j,i}} \right\|_\infty \tag{4.8}$$

has become large. As it was described in section 3, we want a new identification of G via G_{i+1} for which

$$\left\| \frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_{i+1} K_{j,i}}{1+G_{i+1} K_{j,i}} \right\|_\infty \tag{4.9}$$

is small. We are going to use the $r_{i,j}$ parametrization of G_{i+1}. By substituting equations 4.1 and 4.7 into expression 4.9, and noting that

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}},$$

we can, after simplification, conclude that

$$\left\| \frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_{i+1} K_{j,i}}{1+G_{i+1} K_{j,i}} \right\|_\infty = \|Y_{j,i} X_{j,i}(R_{i,j} - r_{i,j})\|_\infty \tag{4.10}$$

should be small.

Remarks

- Note that

$$T_{j,i} = \frac{GK_{j,i}}{1+GK_{j,i}}$$

is the actual closed-loop transfer function of the system, and

$$\tilde{T}_{j,i} = \frac{G_i K_{j,i}}{1+G_i K_{j,i}}$$

is the nominal closed-loop transfer function of the system. Therefore, using similar substitutions that resulted in equation 4.10, we can obtain

$$T_{j,i} - \tilde{T}_{j,i} = Y_{j,i} X_{j,i}(R_{i,j} - \tilde{R}_{j,i}). \tag{4.11}$$

However, since

$$\tilde{R}_{j,i} \equiv 0, \quad \forall j, \forall i,$$

we therefore have

$$T_{j,i} - \tilde{T}_{j,i} = Y_{j,i} X_{j,i} R_{i,j}. \tag{4.12}$$

By comparing the argument of the H_∞ norm given in expression 4.8 with the left hand side of equation 4.12, we see immediately that when the value of

$$\left\| \frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_iK_{j,i}}{1+G_iK_{j,i}} \right\|_\infty$$

has became large; that is, when the closed-loop property of the actual system ($T_{j,i}$) is significantly different from the closed-loop property of the nominal system ($\bar{T}_{j,i}$), the value of

$$\|Y_{j,i}X_{j,i}R_{i,j}\|_\infty$$

will be large.

- From the signals defined in theorem 4.2, we observed that $R_{i,j}$, the transfer function to be identified, is excited by the signal α, where
$$\alpha = X_{j,i}r_1,$$
and
$$X_{j,i} = \frac{K_{j,i}}{1+G_iK_{j,i}}.$$
Since the nominal closed-loop transfer function of the system is
$$\bar{T}_{j,i} = \frac{G_iK_{j,i}}{1+G_iK_{j,i}},$$
we can write
$$X_{j,i} = \frac{\bar{T}_{j,i}}{G_i}.$$
Therefore, $X_{j,i}$ will have large magnitude when we try to push the nominal closed-loop bandwidth beyond the nominal open-loop bandwidth. Since a model usually has its uncertainties become significant for frequencies beyond its bandwidth, from figure 4, we see that if the spectrum of r_1 is white, we automatically get the right weighting for the input to $R_{i,j}$ for the system identification scheme.

- It is shown in theorem 4.2 that the effect of e on ξ is g given by $\frac{He}{1+GK_{j,i}}$. Note that this is the effect of e on y attenuated by the sensitivity function of the actual closed-loop system.

5 Approximate Identification of the $R_{i,j}$ Transfer Function for IMC Controller Design

In section 4, we have shown that the closed-loop system identification of the plant transfer function G can be reformulated into an open-loop system identification of the stable proper transfer function $R_{i,j}$ that parametrized the transfer function G via the equation

$$G = \frac{N_i + R_{i,j}Y_{j,i}}{D_i - R_{i,j}X_{j,i}}.$$

In this and the following sections, we shall, for simplicity, study the case where the plant is stable and has no zeros on the imaginary axis of the s-plane, and where the IMC method [15] is used to design the controller $K_{j,i}$. We shall also assume that all estimates G_i of the plant are stable.

If the model

$$G_i = \frac{N_i}{D_i}$$

is also stable, we can let $N_i = G_i$ and $D_i = 1$ so that

$$G = G_i + \frac{R_{i,j}}{1 - R_{i,j}Q_{j,i}}, \tag{5.1}$$

where $Q_{j,i}$ is a stable proper transfer function that parametrized the controller

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}},$$

and

$$Q_{j,i} \stackrel{\text{def}}{=} \frac{K_{j,i}}{1+G_iK_{j,i}}. \tag{5.2}$$

Note that

$$X_{j,i} = Q_{j,i},$$

and

$$Y_{j,i} = 1 - Q_{j,i}G_i.$$

Since the parametrization of G by $R_{i,j}$ depends intimately on $Q_{j,i}$, we shall briefly explain how $Q_{j,i}$ is obtained in the design of the controller $K_{j,i}$. We will use the notations n_H and d_H to denote the numerator polynomial and the denominator polynomial of a rational transfer function H.

Given a stable model,

$$G_i = \frac{n_{G_i}}{d_{G_i}},$$

where d_{G_i} has no zeros in the closed right half s-plane, if n_{G_i} has no zeros on the imaginary axis of the s-plane, we can write

$$G_i = \frac{\tilde{n}_{G_i}\prod_i(z_i - s)}{d_{G_i}},$$

where all z_i have positive real parts, and $\tilde{n}_{G_i}$ has no zeros in the closed right half s-plane. By writing G_i as

$$G_i = [G_i]_m[G_i]_a,$$

where

$$[G_i]_m = \frac{\tilde{n}_{G_i}\prod_i(z_i^* + s)}{d_{G_i}}; \quad z_i^* \text{ is the complex-conjugate of } z_i,$$

and

$$[G_i]_a = \frac{\prod_i(z_i - s)}{\prod_i(z_i^* + s)},$$

we have factored G_i as a product of its minimum-phase factor $[G_i]_m$, and the associated all-pass factor $[G_i]_a$. We can design a controller, using the internal model control (IMC) approach [15], by setting

$$Q_{j,i} = [G_i]_m^{-1}F_{j,i}, \tag{5.3}$$

where $F_{j,i}$ is a low pass filter of the form

$$F_{j,i} = \left(\frac{\lambda_{j,i}}{s+\lambda_{j,i}}\right)^n,$$

with n chosen large enough so that $Q_{j,i}$ is proper, and $\lambda_{j,i}$ selected (possibly on-line) small enough so that $K_{j,i}$ robustly stabilizes G_i.

In the ideal situation where $G_i = G$ is stable and minimum-phase, it follows that the nominal and the actual closed-loop transfer functions of the system are equal and are given by the transfer function $F_{j,i}$. Therefore $\lambda_{j,i}$ is both the nominal and actual closed-loop system bandwidth with a $-3n\,dB$ attenuation. In general, $G_i \neq G$ and $\lambda_{j,i}$ serves only as an approximate bandwidth of the actual closed-loop system.

With the controller designed using the above procedure, we shall now show that the transfer function to be identified, $R_{i,j}$, is the product of a known stable proper transfer function and an unknown stable strictly-proper transfer function. An analysis of the form of the unknown factor in $R_{i,j}$ indicates how it can be sensibly approximated by a low-order transfer function. We shall first rewrite equation 5.1 as

$$R_{i,j} = \frac{G - G_i}{1 + Q_{j,i}(G - G_i)}. \tag{5.4}$$

Then we can obtain, after substituting equations 5.2 and 5.3 into equation 5.4, and performing some algebraic manipulations,

$$R_{i,j} = \{[G_i]_m d_{F_{j,i}}\}\left\{\frac{d_{G_i}n_G - d_G n_{G_i}}{d_{K_{j,i}}d_G + n_{K_{j,i}}n_G}\right\}. \tag{5.5}$$

Note that equation 5.5 can also be written as

$$R_{i,j} = \tilde{R}_{i,j}\hat{R}_{i,j}, \tag{5.6}$$

where

$$\widetilde{R}_{i,j} = [G_i]_m d_{F_{j,i}} \quad (5.7)$$

is a *known* stable proper transfer function, and

$$\widehat{R}_{i,j} = \frac{d_{G_i} n_G - d_G n_{G_i}}{d_{K_{j,i}} d_G + n_{K_{j,i}} n_G}. \quad (5.8)$$

is an *unknown* stable strictly proper transfer function that depends on the unknown transfer function G. Therefore the problem of identifying $R_{i,j}$ has become one of identifying its unknown factor $\widehat{R}_{i,j}$. We shall summarize this important result in the following theorem.

Theorem 5.1 *Consider a plant which has an unknown stable proper transfer function G, and a model with a known stable proper transfer function G_i. If G and G_i have no zeros along the imaginary axis of the s-plane, and*

$$G_i = [G_i]_m [G_i]_a,$$

where $[G_i]_m$ is the minimum-phase factor of G_i, and $[G_i]_a$ is the all-pass factor of G_i, then with

$$Q_{j,i} = [G_i]_m^{-1} F_{j,i}$$

and

$$F_{j,i} = \left(\frac{\lambda_{j,i}}{s + \lambda_{j,i}}\right)^n,$$

where n is chosen such that $Q_{j,i}$ is a stable proper transfer function, the controller

$$K_{j,i} = \frac{Q_{j,i}}{1 - Q_{j,i} G_i}$$

will robustly stabilize G_i for all sufficiently small values of $\lambda_{j,i} \geq 0$. Furthermore, the unknown stable strictly proper transfer function to be identified,

$$R_{i,j} = \frac{G - G_i}{1 + Q_{j,i}(G - G_i)},$$

can be factorized as

$$R_{i,j} = \widetilde{R}_{i,j} \widehat{R}_{i,j},$$

where $\widehat{R}_{i,j}$ is an unknown stable proper transfer function to be identified, and $\widetilde{R}_{i,j}$ is a known stable proper transfer function given by

$$\widetilde{R}_{i,j} = [G_i]_m d_{F_{j,i}},$$

where $d_{F_{j,i}}$ is the denominator polynomial of the filter $F_{j,i}$.

Remarks

- Note that the factorization of $R_{i,j}$ given in theorem 5.1 is naturally induced by the IMC [15] controller design procedure that we have adopted.
- The poles of $\widehat{R}_{i,j}$ are the poles of $T_{j,i}$, the actual closed-loop transfer function of the system.
- It is important to note that $\widehat{R}_{i,j} = 0$ if and only if $G = G_i$.
- The order of $\widehat{R}_{i,j}$ is constraint by the degree of the polynomial $d_{K_{j,i}} d_G$, which is an unknown.

As we do not know the order of $\widehat{R}_{i,j}$ a priori, and since only step response information is available, it is reasonable to employ a low-order transfer function for the approximate identification of $\widehat{R}_{i,j}$. Since we are going to identify $\widehat{R}_{i,j}$ (actually $R_{i,j}$) and update G_i to G_{i+1} when the step response of the actual closed-loop system exhibits unacceptable oscillations and/or overshoots, we expect $\widehat{R}_{i,j}$ to have complex-conjugate poles. Therefore, the lowest possible order that we can assume for the transfer function which serve as an approximation of $\widehat{R}_{i,j}$ is two.

It was shown in equation 4.10 that the system identification problem is to find

$$r_{i,j} = \arg\min_{\sigma} \|X_{j,i} Y_{j,i} (R_{i,j} - \sigma)\|_\infty. \quad (5.9)$$

If we define

$$r_{i,j} = \widetilde{R}_{i,j} \widehat{r}_{i,j}, \quad (5.10)$$

where $\widehat{r}_{i,j}$ is an unknown second-order stable strictly proper transfer function, then by substituting equations 5.3, 5.6, and 5.10 into equation 5.9, we can show that the system identification problem becomes one of finding

$$\widehat{r}_{i,j} = \arg\min_{\phi} \|\lambda_{j,i}{}^n Y_{j,i} (\widehat{R}_{i,j} - \phi)\|_\infty. \quad (5.11)$$

Remark

- Since $Y_{j,i}$ is the nominal sensitivity function of the closed-loop system, we immediately see that the frequency shaping in the identification criterion given by equation 5.11 will force the updated model to have small modelling error in the range of frequencies where the nominal sensitivity function cannot be made small by the controller $K_{j,i}$.
- When updating the model using the equation

$$G_{i+1} = G_i + \frac{r_{i,j}}{1 - r_{i,j} Q_{j,i}},$$

the order of the model may increase. To prevent the model order from increasing indefinitely, we use a frequency weighted balanced truncation scheme to reduce the order of G_{i+1}. Specifically, we find

$$\widehat{G}_{i+1} = \arg\min_{\eta} \left\| \frac{G_{i+1} K_{j,i}}{1 + G_{i+1} K_{j,i}} - \frac{\eta K_{j,i}}{1 + \eta K_{j,i}} \right\|_\infty,$$

where $\widehat{G}_{i+1}$ is the reduced order model. If the model order is restricted to m, the controller will be at most of order $2m$ (see controller design equations given in theorem 5.1). In this way the controller complexity will be limited.

6 Simulation Results

We shall present some simulation results of applying the windsurfer approach to the control of a plant with the transfer function

$$G(s) = \frac{9}{(s+1)(s^2 + 0.06s + 9)}.$$

We first summarize the procedure in the following algorithm:

Step 1:

Set $G_i = G_0$, where G_0 is the transfer function of an initial model of the plant.

Step 2:

Factorize G_i as

$$G_i = [G_i]_m [G_i]_a,$$

where $[G_i]_m$ is the minimum-phase factor of G_i, and $[G_i]_a$ is the associated all-pass factor of G_i.

Step 3:

For $j = 1$, find

$$K_{j,i} = \frac{Q_{j,i}}{1 + Q_{j,i} G_i},$$

with

$$Q_{j,i} = [G_i]_m^{-1} F_{j,i},$$

where the positive integer n and the parameter $\lambda_{j,i}$ in the transfer function

$$F_{j,i} = \left(\frac{\lambda_{j,i}}{s + \lambda_{j,i}}\right)^n$$

are chosen such that $Q_{j,i}$ is a stable proper transfer function, and $K_{j,i}$ robustly stabilizes G_i in the sense that the step response of the actual closed-loop system has, at most, little oscillations and/or overshoots. Stop here if such a robust stabilizing controller cannot be found. Also stop here if the robust stabilizing controller results in a closed-loop system which meets the specified bandwidth. Otherwise, proceed to the next step.

Step 4:

Let $j = j+1$ and set $\lambda_{j,i} = \lambda_{j-1,i} + \epsilon$ for small $\epsilon > 0$, and redesign the controller $K_{j,i}$ using the equations given in Step3. Stop here if the design produces a robust stabilizing controller with the closed-loop system satisfying the specified bandwidth. Otherwise, repeat this step if $K_{j,i}$ robustly stabilizes G_i; else proceed to the next step.

Step 5:

Perform rational function approximation to obtain

$$\hat{r}_{i,j} = \arg\min_{\phi} \|\lambda_{j,i}{}^{n} Y_{j,i}(\hat{R}_{i,j} - \phi)\|_{\infty}.$$

Then update the model using the following set of equations:

$$\tilde{R}_{i,j} = [G_i]_m d_{F_{j,i}},$$

$$r_{i,j} = \tilde{R}_{i,j}\hat{r}_{i,j},$$

and

$$G_{i+1} = G_i + \frac{r_{i,j}}{1 - r_{i,j}Q_{j,i}}.$$

Step 6:

If G_{i+1} is stable, find the reduced order model

$$\hat{G}_{i+1} = \arg\min_{\eta} \left\| \frac{G_{i+1}K_{j,i}}{1+G_{i+1}K_{j,i}} - \frac{\eta K_{j,i}}{1+\eta K_{j,i}} \right\|_{\infty}.$$

Otherwise, stop here.

Step 7:

Set $G_i = \hat{G}_{i+1}$ and return to Step 2.

Remarks

- In the algorithm, rational function approximation has to be carried out when $\|T_{N,i} - \tilde{T}_{N,i}\|_{\infty}$ is no longer small. Broadly speaking, this will correspond to a significant difference between the designed nominal performance (depending on G_i and $K_{N,i}$) and the actual performance (depending on G and $K_{N,i}$). In particular, the observed step response may exhibit much more oscillations and/or overshoots than the designed values. This is not of course the same thing as guaranteeing that the H_{∞} error above has became large, but neither is it unrelated.

- To be more precise, we define the peak gain of a system, whose transfer function is T, by

$$\|T\|_1 = \sup_{\|w\|_{\infty} \neq 0} \frac{\|Tw\|_{\infty}}{\|w\|_{\infty}}.$$

This is also equal to the *total variation* of the system's unit step response [4] defined as the sum of all consecutive peak-to-valley differences in the unit step response. It can be shown [5] that, if T is a stable strictly proper transfer function,

$$\|T\|_{\infty} \leq \|T\|_1 \leq 2n\|T\|_{\infty},$$

where n is the order of the transfer function T. Now we consider the peak error

$$\|T_{N,i} - \tilde{T}_{N,i}\|_1.$$

Since

$$\|T_{N,i} - \tilde{T}_{N,i}\|_1 \geq \|T_{N,i}\|_1 - \|\tilde{T}_{N,i}\|_1,$$

therefore, if the observed step response of $T_{N,i}$ exhibits much more oscillations and/or overshoots than the designed step response of $\tilde{T}_{N,i}$, we would expect

$$\|T_{N,i}\|_1 \gg \|\tilde{T}_{N,i}\|_1,$$

and hence,

$$\|T_{N,i} - \tilde{T}_{N,i}\|_1 \gg \epsilon, \quad \epsilon > 0.$$

Since the peak gain also provides a *loose* lower bound for the H_{∞} gain, it is likely that

$$\|T_{N,i} - \tilde{T}_{N,i}\|_{\infty}$$

becomes large when the observed actual step response exhibits much more oscillations and/or overshoots than the desired one.

- This explains why, in the simulation, the models are updated whenever the actual step response exhibits *unacceptable* oscillations and/or overshoots.

The simulation results are presented in figure 5 and figure 6. These figures correspond respectively to the following case studies:

- Case 1: the initial model is $G_0(s) = \frac{1.2}{s+1.2}$,
- Case 2: the initial model is $G_0(s) = \frac{0.8}{s+0.8}$,

We present unit step responses at various steps in the system identification/control design iteration, and frequency responses achieved just before the iteration process is stopped.

In the first case study, see figure 5, the bandwidth of the closed-loop system cannot be increased beyond $10\,rad/sec$ because we have stopped the iterative system identification and control design process when an unstable model is obtained. Note that only two model updates, G_1 and G_2, are required in the process, and the results are sufficiently good for most practical purposes.

The results for the second case study are given in figure 6. These results show that the closed-loop bandwidth can easily be pushed to $10\,rad/sec$ with very good step responses. Note that in this case, the model has to be updated only once.

Remark

- We must emphasize that in these simulations, instead of performing a system identification using input-output measurements, we actually perform the model approximation

$$\hat{r}_{i,j} = \arg\min_{\phi} \|\lambda_{j,i}{}^{n} Y_{j,i}(\hat{R}_{i,j} - \phi)\|_{\infty},$$

where $\hat{R}_{i,j}$ is obtained from the known G. The reasons for doing this are:

1. Our results, although preliminary, serve as a benchmark in the sense that using the transfer function G corresponds to performing system identification with an infinite number of noiseless measurements.
2. We like to know how serious the problems may be due to employing a low-order approximation for $\hat{r}_{i,j}$. This is important for later system identification studies.
3. We are, at this stage, more concerned with the concept of iterative system identification and control design as applied to adaptive robust control, rather than the details.
4. Efficient algorithms for performing H_{∞} system identification are still lacking, and the corresponding theory is still not well understood [11, 16, 17].

7 Discussions and Conclusions

We have reviewed in section 1 the strength and weakness of both the traditional adaptive control and the robust control design methods. These methods should be able to complement each other and there should be natural ways in which they could be blended harmoniously. We proposed that one of the possible ways is by the windsurfer approach, which was first mentioned in [2]. We have shown, by simulation, that by starting with a (crude) initial model of the plant and a (small bandwidth) robustly stabilizing controller, the bandwidth of the closed-loop system can be increased progressively through an iterative control-relevant system identification and control design procedure. We shall highlight the following points which we believe are reasons for the success of the approach:

- The use of control-relevant frequency weighting in the system identification criterion.
- Updating of the model when its effects is no longer small in the closed-loop response. This will ensure that model uncertainties are emphasized in the correct range of frequencies.
- The controller designed by using the IMC method always has integral action. Therefore it is insensitive to model uncertainties at low frequencies, provided the gain of the model at low frequencies is of the right sign.
- The controller designed by using the IMC method induces a natural factorization in the parametrization of the unknown transfer function of the plant. This enable the system identification problem to be solved effectively.

In conclusion, we would like to emphasize that only the case of stable plant and model is considered in this preliminary study. We will like to address the following problems in the near future:

- The extension of the method to deal with unstable plant and model.
- Use of orthogonalized exponentials in the system identification procedure such that it becomes a convex optimization problem.
- To prove that the algorithm actually converges in some sense.
- To study other control design methods in the context of the windsurfer approach.

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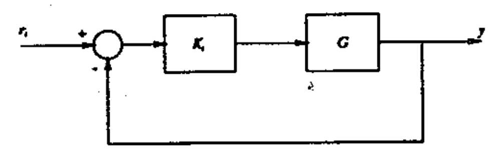

Figure 1: Adaptive control system

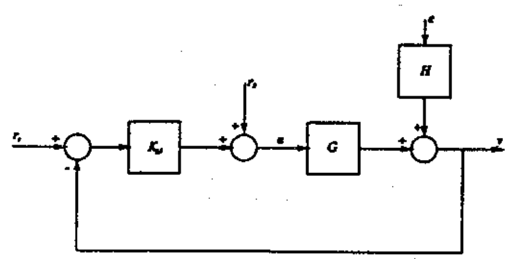

Figure 2: Closed-loop system

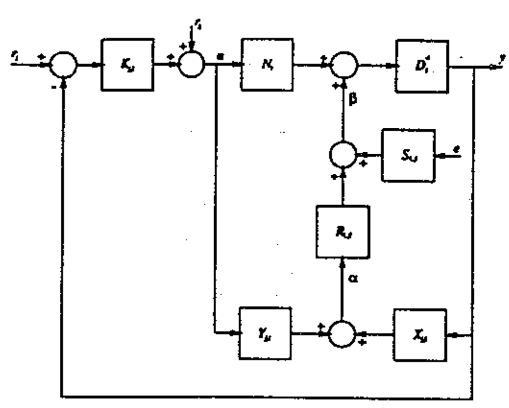

Figure 3: Closed-loop system identification

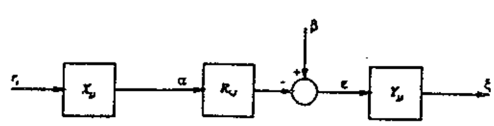

Figure 4: Excitation of R_{ij}

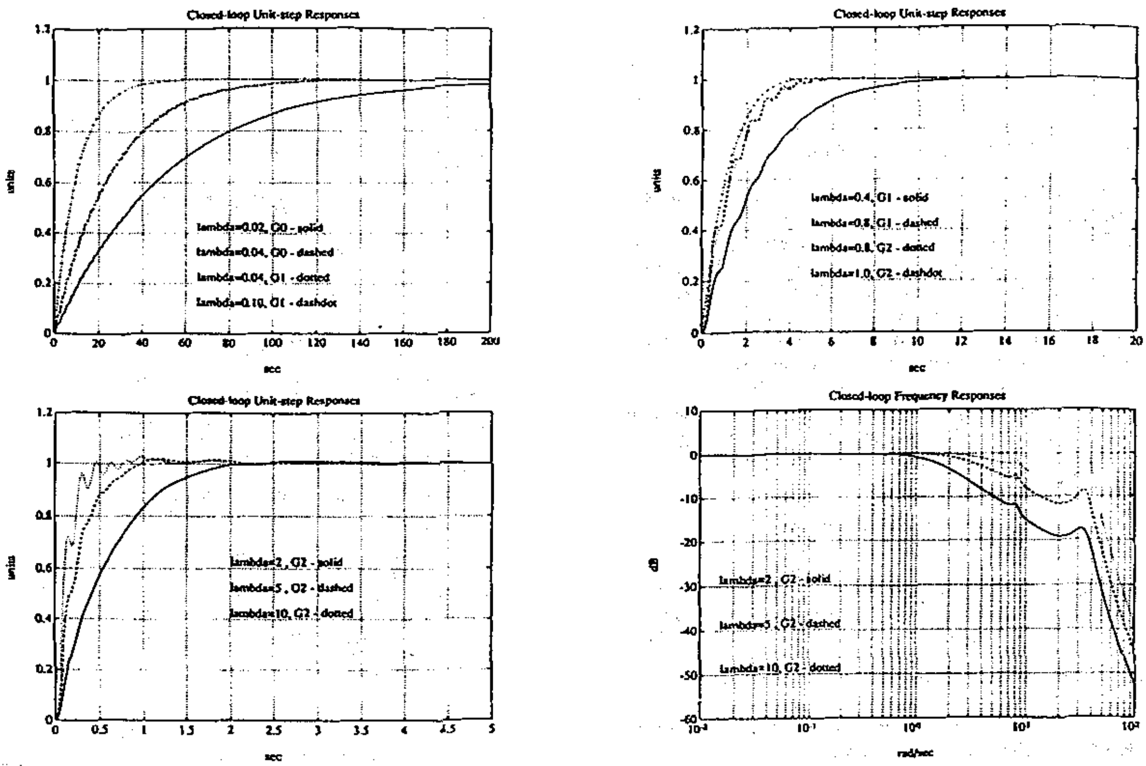

Figure 5: Closed-loop responses with $G_0 = \frac{1.2}{s+1.2}$

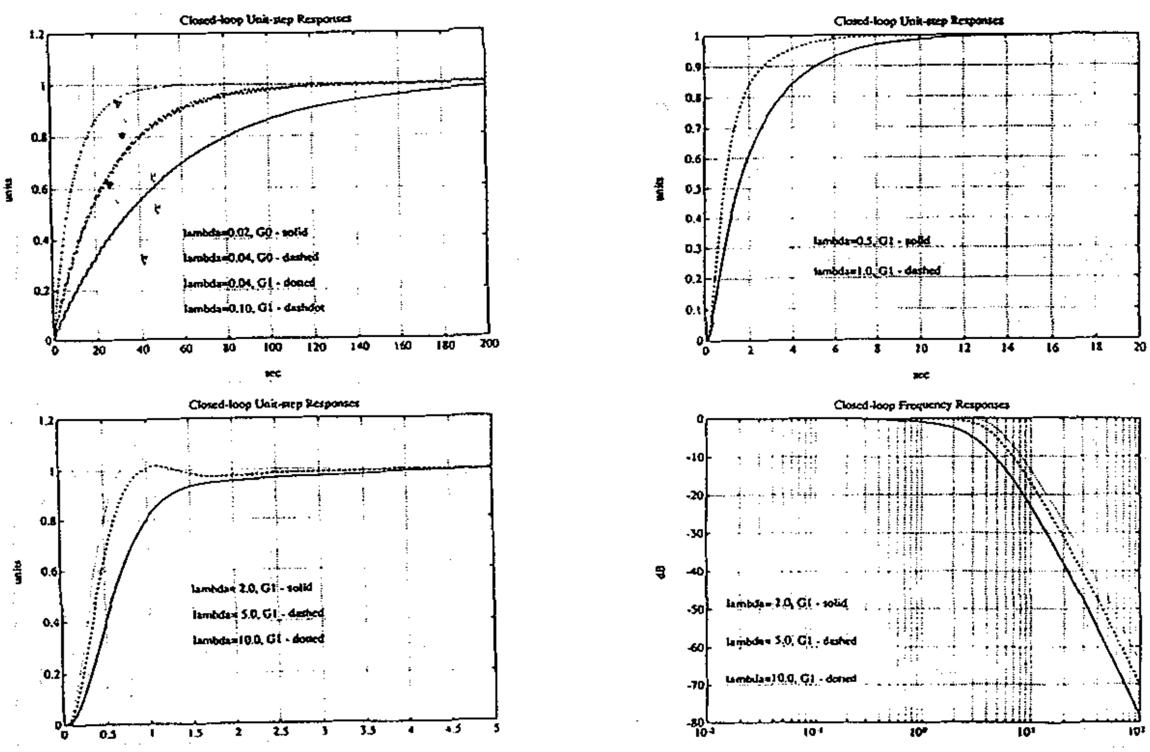

Figure 6: Closed-loop responses with $G_0 = \frac{0.8}{s+0.8}$