LEAST SQUARES IDENTIFICATION AND THE ROBUST STRICT POSITIVE REAL PROPERTY

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Abstract.

Certain problems of adaptive systems revolve round finding a transfer function $q(s)$ (which is not proper) such that a set of transfer functions $\frac{q(s)}{b(s)} - 1/2$ is strictly positive real for all polynomial $b(s)$ in a certain set. This paper treats this and some related problems.

1 Introduction

In a number of adaptive linear systems problems, a positive real property of certain transfer functions is frequently sought, in order to assure convergence of the adaptive algorithm, [1,2]. Output error identification with a gradient algorithm is a typical example. In this problem, the plant is assumed to be of the form $c(s)/b(s)$, with $b(s) = s^n + b_1 s^{n-1} + \cdots + b_n$ or $c(z^{-1})/b(z^{-1})$ with $b(z^{-1}) = 1 + b_1 z^{-1} + \cdots + b_n z^{-n}$, and one seeks a polynomial $q(s) = s^n + q_1 s^{n-1} + \cdots + q_n$ or $q(z^{-1}) = 1 + q_1 z^{-1} + \cdots + q_n z^{-n}$ such that $q(s)/b(s)$ or $q(z^{-1})/b(z^{-1})$ is strictly positive real, i.e. the transfer function in question is stable, and has strictly positive real part on the $j\omega$-axis or unit circle boundary, as appropriate.

Because the plant denominator is unknown, the question immediately arises as to how q should be chosen. This question is treated in [3], where it is argued that if the set of allowed denominators $b(\cdot)$ is defined (in coefficient space) by a convex polytope, and every denominator corresponding to a point in this polytope is stable, then a form of $q(\cdot)$ can be constructed using the (finite number of) corners of the polytype, which guarantees the SPR property via this single q for all b in the polytope. We say "a form" of q because in the discrete time case $q(\cdot)$ may have degree (as a polynomial in z^{-1}) higher than n, and in the continuous-time case, one finds a $q(s)$ of the form (polynomial of degree $n + M)/(s+\alpha)^M$ for some positive α and integer M, rather than simply a polynomial. (These variations on the form of q usually used in the adaptive algorithm can be accommodated).

In this paper, we treat a related but more difficult problem, motivated by output error identification with a least squares algorithm. This is treated for the discrete-time case in [1] and the continuous-time case in [4]. Again, $b(s)$ or $b(z^{-1})$ corresponds to the denominator of the unknown plant, and one seeks $q(s) = s^n + q_1 s^{n-1} + \cdots + q_n$ or $q(z^{-1}) = 1 + q_1 z^{-1} + \cdots + q_n z^{-n}$ with the property that $q/b - 1/2$ is strictly positive real for this one q and all b inside a given set, such as a convex polytope determined by the coefficients of b. We shall call this a strengthened robust SPR problem.

Three more points should be noted at this stage:

(a) Once again, it is satisfactory to replace a search for a monic polynomial q of degree n in s or z^{-1} by a search for a transfer function of the form (monic polynomial of degree $n + M)/(s + \alpha)^M$ for some $\alpha > 0$ and integer M, or a search for a monic polynomial in z^{-1} of arbitrary degree.

(b) Least-squares algorithms with a forgetting factor can be contemplated, leading to a requirement that $q/b - \frac{1}{2}\gamma$ be SPR for all b, some q, and some $\gamma > 0$.

(c) the monicity condition on q and b provides a normalisation for q/b: either $\lim_{s\to\infty} q/b = 1$ or $\lim_{z\to\infty} q/b = 1$. Note that without any normalisation being imposed at all, the problem of interest would be a trivial variation on the problem already treated in [3].

Let us comment further on this third point. In [3], it was shown that the continuous and discrete-time cases were more or less treatable simultaneously, using a bilinear transformation $s = \frac{1-z^{-1}}{1+z^{-1}}$ or $z^{-1} = \frac{1-s}{1+s}$. This transformation does not map $s = \infty$ to $z = \infty$ and inversely, and so a simple connection is not possible between the two problems with a strengthened SPR requirement. Another viewpoint of the same difficulty comes from observing that for the continuous-time problem, the normalisation condition arises at a point on the locus of points used in defining the SPR property, i.e. the point ∞ on the $j\omega$-axis; on the other hand, for the discrete-time problem, the normalisation condition does not arise at such a point, since $z = \infty$ is not located on the boundary of the unit circle.

In view of this comment, we see that the continuous-time and discrete-time strengthened robust SPR problems are of different character. So far, we can only treat the continuous-time version.

An outline of the paper is as follows. In Section 2, by way of motivation, we present a stability result applicable in adaptive systems involving a strengthened SPR condition in conjunction with a least squares algorithm (Appendix A shows how the result is relevant in output error identification). In Section 3, we show how the search for q can be narrowed, in the sense that it suffices to find a q such that q/b_j has the strengthened SPR property for each corner b_j of the convex polytope defining the polynomials b of interest. In Section 4, we explain how q can be constructed; the key is to find first a q satisfying the SPR condition (in nonstrengthened form) for the set of b_j, and then to modify it. Section 5 contains concluding remarks.

Once again, we stress that the main result (in contrast to some preliminary results) is restricted to continuous-time. While some might argue that there is limited interest in any continuous-time adaptive algorithm, recent work on robot control, see e.g. [5] would appear to refute this view.

2 A stability result

Let $\tilde\theta(t)$ denote a parameter error vector, i.e. a quantity $\theta_0 - \hat\theta(t)$, where θ_0 denotes the true parameter and $\hat\theta(t)$ its estimate at time t. Let $\phi(t)$ be a regression vector used in an output error algorithm. Then the form of the equations governing the algorithm is:

$$\dot{\tilde\theta} = -R^{-1}(t)\phi(t)Z(s)[\phi'(t)\tilde\theta(t)] \quad (2.1a)$$

$$\dot R = \phi(t)\phi'(t) \quad (2.1b)$$

where $Z(s)$ is a certain transfer function [$q(s)/b(s)$ in fact, where $q(\cdot)$ is designer-determined, and $b(s)$ is the (unknown) denominator of the plant transfer function, see Appendix A]. Equation (2.1a) is illustrated in Figure 2.1. The operator in the feedback path is passive if (2.1b) is replaced by $\dot R = 0$ (which corresponds to the gradient algorithm case); however, when (2.1b) holds, this is no longer in general true.

To analyze the stability of (2.1), let us consider the arrangement of Figure 2.2, which is equivalent from the stability point of view. If $Z(s) - 1/2$ is strictly positive real, and if the mapping from v to w is defined by a passive operator, then a form of input-output stability for the arrangement is guaranteed, [1,6]. Persistence of excitation may then allow conclusions such as $\tilde\theta(t) \to 0$ (if the algorithm includes a forgetting factor, then convergence is exponentially fast, see e.g. [2]); we leave this latter point to more detailed treatments such as [2] and [7], and in this paper, we shall merely demonstrate the passivity property of the operator linking v to w in Figure 2.2. The reason for this is, in turn, simply to motivate the desire to have the SPR property for $q(s)/b(s) - 1/2$.

Lemma 2.1 *Consider the system defined by*

$$\dot{\tilde{\theta}} = R^{-1}\phi(v - \frac{1}{2}\phi'\tilde{\theta}) \quad (2.2a)$$

$$w = \phi'\tilde{\theta} \quad (2.2b)$$

$$\dot{R} = \phi\phi', \quad R(0) = R_0 > 0 \quad (2.2c)$$

(see Figure 2.2.) Suppose that $\tilde{\theta}(0) = 0$. *Then the following passivity property holds*

$$\int_0^T w(s)v(s)ds \geq 0 \quad (2.3)$$

$\forall v(\cdot)$.

Proof Observe that

$$\begin{aligned} w(s)v(s) &= \tilde{\theta}'(s)\phi(s)v(s) \\ &= \tilde{\theta}'(s)R(s)\dot{\tilde{\theta}}(s) + \frac{1}{2}\tilde{\theta}'(s)\phi(s)\phi'(s)\tilde{\theta}(s) \\ &= \frac{d}{ds}[\frac{1}{2}\tilde{\theta}'(s)R(s)\tilde{\theta}(s)] \end{aligned}$$

Hence

$$\int_0^T w(s)v(s) = \frac{1}{2}\tilde{\theta}'(T)R(T)\tilde{\theta}(T) \geq 0$$

■

It is also important to contemplate the use of exponential forgetting. In fact, suppose that in lieu of (2.1b), equivalently (2.2c), one uses

$$\dot{R} = -\beta R + \gamma\phi\phi', \quad \beta > 0, \gamma \leq 1 \quad (2.4)$$

(With $\gamma = 1$, this is usually termed exponential forgetting. Here, we allow a slightly more general forgetting profile.) Then one has passivity for the lower (feedback) block of the system defined as in Figure 2.2 except the negative feedback of 1/2 within the block is changed to $\gamma/2$: (2.2a) is replaced by

$$\dot{\tilde{\theta}} = R^{-1}\phi(m - \frac{1}{2}\gamma\phi'\tilde{\theta}) \quad (2.5)$$

and then, copying the calculation of the proof of Lemma 2.1, there results

$$\int_0^T w(s)v(s)ds = \frac{1}{2}\tilde{\theta}'(T)R(T)\tilde{\theta}(T) + \int_0^T \frac{1}{2}\beta\tilde{\theta}'(s)R(s)\tilde{\theta}(s)ds \quad (2.6)$$

3 Use of polytope corners to simplify the strengthened SPR problem

Suppose that the set $b(s)$ of denominators, denoted B, is a convex polytope with corners $b_j(s)$. Each $b(s) \in B$ is stable and monic. Perhaps somewhat surprisingly, we have:

Lemma 3.1 *Let $q(s)$ be a stable rational transfer function such that*

$$\lim_{s\to\infty} \frac{q(s)}{b_j(s)} = 1 \quad (3.1)$$

$$\frac{q(s)}{b_j(s)} - 1/2 \quad \text{is SPR } \forall j \quad (3.2)$$

Then $\frac{q(s)}{b(s)} - 1/2$ *is SPR* $\forall b \in B$.

Proof The stability aspect of the SPR property is immediate from the assumptions. We must check the $j\omega$-axis positivity. Notice that no b, and thus no b_j, can ever be zero at a point on the imaginary axis. Omitting now mention of the argument $j\omega$, we have for all ω

$$\begin{aligned} \text{Re}\left[\frac{q}{b_j} - 1/2\right] > 0 \quad &\Leftrightarrow \quad \frac{q}{b_j} + \frac{q^*}{b_j^*} - 1 > 0 \\ &\Leftrightarrow \quad qb_j^* + q^*b_j - b_jb_j^* > 0 \\ &\Leftrightarrow \quad -qq^* + qb_j^* + q^*b_j - b_jb_j^* > -qq^* \\ &\Leftrightarrow \quad (q - b_j)(q^* - b_j^*) < qq^* \\ &\Leftrightarrow \quad (1 - \frac{b_j}{q})(1 - \frac{b_j^*}{q}) < 1 \\ &\Leftrightarrow \quad \left|\frac{b_j}{q} - 1\right| < 1 \end{aligned}$$

Thus, see Figure 3.1, each $\frac{b_j}{q}$ must lie in a circle of centre 1 and radius 1. It follows that if $b = \sum_{j=1}^{m} \mu_j b_j$ (with $\sum \mu_j = 1, \mu_j \geq 0$), i.e. b is a convex combination of the b_j, then b/q is the same convex combination of the b_j/q, and so it must lie in the same circle. Thus

$$\left|\frac{b}{q} - 1\right| < 1 \text{ which implies that } \text{Re}[q/b - 1/2] > 0$$

■

Notice that this argument applies equally well in the discrete-time case.

4 Main result

Let us first recall the main result of [3].

Theorem 4.1 *Let $\{b_j(s)\}$ be a finite set of stable monic polynomials of degree n, and suppose that all convex combinations of the $b_j(s)$ are stable. Then there exists $q(s)$ of the form (polynomial of degree $n+M$)$/(s+\alpha)^M$ for some positive α and integer M such that $q(s)/b_j(s)$ is SPR for all j.*

The main result of the paper is the following:

Theorem 4.2 *Assume the same hypotheses as Theorem 4.1. Then there exists $q(s)$ of the form (monic polynomial of degree $n+M$)$/(s+\alpha)^M$ for some positive α and integer M such that, given $\beta \in (0,1)$, the transfer functions $q(s)/b_j(s) - \beta$ are SPR for all j.*

The proof of this theorem will depend on Theorem 4.1 and the following Lemma, which guarantees the construction of a stable, minimum phase, biproper transfer function with certain properties.

Lemma 4.1 *Let $\epsilon > 0$ be small, and let $\omega_0 > 0, \gamma \in (0,1)$, then there exists a stable, minimum phase, biproper transfer function $r(s)$ with the following properties.*

$$|\arg r(j\omega)| < \epsilon \; \forall\omega \quad (4.1a)$$

$$|r(j\omega) - 1| < \epsilon \; \forall\omega \in [0, \omega_0] \quad (4.1b)$$

$$\text{Re}\, r(j\omega) \text{ is monotone decreasing} \quad (4.1c)$$

$$\lim_{\omega\to\infty} r(j\omega) = \gamma \quad (4.1d)$$

Remark Roughly speaking, this says we can find a stable, minimum phase transfer function for which the imaginary part is very small at all frequencies, and for which the real part is approximately 1 over $[0, \omega_0]$, and decreases monotonically to $\gamma < 1$ at $\omega = \infty$.

Proof Consider the transfer function

$$r_1(j\omega) = \frac{\omega_1}{j\omega + \omega_1} \cdot \frac{j\omega + \omega_2}{\omega_2} \quad (4.2)$$

with $\omega_1 < \omega_2$. The Bode diagrams for $r_1(j\omega)$ are sketched in Figure 4.1. If $\frac{\omega_2 - \omega_1}{\omega_1}$ is small enough, the phase shift at all frequencies can be kept arbitrarily small, and it is essentially zero a decade away from ω_1 or ω_2. As ω moves from 0 to ∞, the gain decreases from 1 to ω_1/ω_2.

A simple calculation reveals that

$$\mathrm{Re}\, r_1(j\omega) = \frac{\omega_1^2\omega_2 + \omega_1\omega^2}{\omega_1^2\omega_2 + \omega_2\omega^2}$$

so that $\mathrm{Re}\, r_1(j\omega)$ is monotone decreasing from 1 to ω_1/ω_2.

Now select a series of frequencies $\omega_1, \omega_2, \cdots, \omega_{2m+2}$ with $(\omega_{2i} - \omega_{2i-1})/\omega_{2i-1}$ very small, and with $\omega_0 \ll \omega_1 \ll \omega_3 \ll \cdots$.

By forming $r_1(s)r_2(s)\cdots r_m(s)$, one evidently secures a transfer function where the phase shift at any frequency can be kept arbitrarily small (by keeping $(\omega_{2i} - \omega_{2i-1})/(\omega_{2i-1})$ arbitrarily small and $\omega_{2i-1} \ll \omega_{2i+1}$). The gain properties are clear: the gain is monotonic decreasing from 1 to $\prod_{i=1}^{m} \frac{\omega_{2i-1}}{\omega_{2i}}$, with the decreases occurring *in small steps placed effectively in* the intervals (ω_1, ω_2), (ω_3, ω_4); the first step is accordingly well above the frequency ω_0. The real part tracks very closely the gain, in view of the phase shift being kept uniformly small.

By choosing m large enough, and the spacing of the breakpoints wide enough, it is evident that $r(s)$, as defined in the Lemma, can be found.

Proof of Theorem 4.2 By Theorem 4.1, there exists a $q_1(s)$ such that $q_1(s)$ is (polynomial of degree $n + M$)$/(s+\alpha)^M$ for some positive α and integer M, with $q_1(s)/b_j(s)$ SPR for all j. Notice that $\lim_{s\to\infty} q_1(s)/b_j(s) > 0$ for all j. This means that there exists some $\eta > 0$ such that

$$\mathrm{Re}\left(\frac{q_1(j\omega)}{b_j(j\omega)}\right) > \eta \ \forall j, \omega \qquad (4.3)$$

Let $K = \beta/\eta$ and $q_2 = Kq_1$. Then we will have

$$\mathrm{Re}\left(\frac{q_2(j\omega)}{b_j(j\omega)}\right) > \beta \ \forall j > \omega \qquad (4.4)$$

i.e. $q_2/b_j - \beta$ is SPR for all j. Should q_2 have the form (monic polynomial of degree $n + M$)$/(s+\alpha)^M$, *we have an immediate solution.* But in general, this cannot happen. Suppose that

$$\lim_{s\to\infty} \frac{q_2}{b_j} = \rho \qquad (4.5)$$

(Note that, because the b_j are all monic, the limit is independent of j). If $\rho < 1$, it suffices to choose $q = \rho^{-1} q_2$ to solve the problem. So suppose $\rho > 1$. Since (4.5) implies $\mathrm{Re}\, q_2/b_j \to \rho$ as $\omega \to \infty$, it follows that there exists a frequency ω_0 and suitably small ϵ such that for $\omega > \omega_0$, $\mathrm{Re}\, q_2/b_j > \rho - \epsilon > 1 - \epsilon > \beta$. Choose $r(s)$ according to the procedure of Lemma 4.1, with $\rho^{-1} = \gamma$ and set $q = q_2 r$. Since $r(j\omega)$ is effectively real at all frequencies, and decreases monotonically from 1 to ρ^{-1} starting at some frequency above ω_0, we see that for $\omega < \omega_0$

$$\mathrm{Re}\frac{q}{b_j} \simeq \mathrm{Re}\frac{q_2}{b_j} > \beta \qquad (4.6)$$

and for $\omega > \omega_0$,

$$\mathrm{Re}\frac{q}{b_j} = \mathrm{Re}\frac{q_2}{b_j} r > (\rho - \epsilon)\cdot\frac{1}{\rho} > \beta \qquad (4.7)$$

and for $\omega \to \infty$,

$$\mathrm{Re}\frac{q}{b_j} = \rho \cdot \rho^{-1} = 1 \qquad (4.8)$$

Of course, (4.8) guarantees that $q(s)$ has the form

$$q(s) = \frac{(\text{monic polynomial of degree } n + M)}{(s+\alpha)^M} \times (\text{improper stable minimum phase transfer function})$$

with $\alpha > 0$. It is clear that we can approximate the second term uniformly and with arbitrary accuracy by a term of the form (polynomial)$/(s+\alpha)^P$ for some P, with exact interpolation at $\omega = \infty$. Such an adjustment produces a new q which evidently retain the properties (4.6) through (4.8), thus establishing the Theorem. ∎

5 Two Special Cases

In [3], we considered sets of polynomial denominators with each coefficients lying in an interval — i.e., the denominator formed a Kharitonov set. For the case of polynomials of degree 2,3 and 4 we showed that a single numerator polynomial of the same order could be *found guaranteeing the SPR property for the ratio.*

Here, we shall show that for polynomials of degree 2 and 3 we can find a single numerator polynomial of the same order guaranteeing the strengthened SPR property [$z(s) - 1/2$ is SPR].

Second-Order Polynomials Consider the transfer function set

$$z(s) = \frac{s^2 + as + b}{s^2 + cs + d} - \frac{1}{2} = \frac{1}{2}\frac{s^2 + (2a - c)s + (2b - d)}{s^2 + cs + d} \qquad (5.1)$$

where $c \in [c_1, c_2]$, $d \in [d_1, d_2]$, $c_1 > 0$ and $d_1 > 0$. Evidently,

$$\mathrm{Re}\, z(j\omega) = \frac{1}{2}\frac{\omega^4 + (2ac - c^2 - 2b)\omega^2 + 2bd - d^2}{(-\omega^2 + d)^2 + c^2\omega^2} \qquad (5.2)$$

and this is positive for all ω if we have a, b such that

$$2ac - c^2 - 2b > 0 \qquad (5.3a)$$

$$2bd - d^2 > 0 \qquad (5.3b)$$

for all allowed c and d. By choosing

$$2b > d_2 \qquad (5.4a)$$

$$2a > c_1^{-1}(c_2^2 + 2b) \qquad (5.4b)$$

it is clear that we secure the desired property.

Third-order Polynomials Consider the transfer function set

$$\begin{aligned} z(s) &= \frac{s^3 + as^2 + bs + c}{s^3 + ds^2 + es + f} - \frac{1}{2} \\ &= \frac{1}{2}\frac{s^3 + (2a - d)s^2 + (2b - e)s + (2c - f)}{s^3 + ds^2 + es + f} \end{aligned} \qquad (5.5)$$

where $d \in [d_1, d_2]$, $e \in [e_1, e_2]$ and $f \in [f_1, f_2]$; further, to ensure stability of the denominator for all allowed values of d, e and f we have

$$d_1 > 0, \ e_1 > 0, \ f_1 > 0, \text{ and } d_1e_1 - f_2 > 0 \qquad (5.6)$$

Now

$$\mathrm{Re}\, z(j\omega) = \frac{1}{2}\frac{N(j\omega)}{|(j\omega)^3 + d(j\omega)^2 + e(j\omega) + f|^2}$$

with

$$\begin{aligned} N(j\omega) = \ & \omega^6 + \omega^4[d(2a - d) - (2b - e) - e] + \omega^2[(2b - e)e \\ & -(2c - f)d - (2a - d)f] + f(2c - f) \end{aligned}$$

A sufficient condition for positivity is that for suitably chosen a, b and c and all allowed d, e, f the following inequalities holds

$$d(2a - d) - 2b > 0 \qquad (5.7a)$$

$$(2b - e)e + (2c - f)d - (2a - d)f > 0 \qquad (5.7b)$$

$$2c - f > 0 \qquad (5.7c)$$

We shall explain how a, b and c can be chosen. First of all, consider the region in $a - b$ space defined by

$$a - \frac{b}{d_1} > \frac{d_2^2}{2d_1} \qquad (5.8a)$$

$$a - b\left(\frac{e_1}{f_2}\right) < \frac{-e_2^2 + 2f_1 d_1}{2f_2} - d_2 \qquad (5.8b)$$

Notice that because of $d_1 e_1 > f_2$, see (5.6), we have $d_1^{-1} < e_1 f_2^{-1}$. The two inequalities separately define shaded regions as illustrated in Figure 5.1. (The intercept with the vertical axis must be negative in the second figure.) Because of the relation between the slopes of the region boundaries, it is evident that there exist *simultaneous* solutions of the two inequalities; choose such a solution.

Also, c is chosen to satisfy

$$\frac{f_2}{2} < c < f_2 \qquad (5.9)$$

Now let us verify that the three inequalities (5.7) hold.

$$\begin{aligned} d(2a-d) - 2b &= 2ad - d^2 - 2b \\ &> 2ad_1 - d_2^2 - 2b \\ &= 2d_1\left[a - \frac{b}{d_1} - \frac{d_2^2}{2d_1}\right] \\ &> 0 \text{ by (5.8a) and the fact that } d_1 > 0 \end{aligned}$$

$$(2b-c)e - (2c-f)d - (2a-d)f$$

$$\begin{aligned} &= -2af + 2be - e^2 + 2df - 2cd \\ &> -2af_2 + 2be_1 - e_2^2 + 2d_1 f_1 - 2cd_2 \\ &= 2f_2\left[-a + \frac{be_1}{f_2} + \frac{-e_2^2 + 2d_1 f_1}{2f_2} - d_2\right] + 2d_2[f_2 - c] \\ &> 0 \text{ by (5.8b) and (5.9)} \end{aligned}$$

Lastly,

$$\begin{aligned} 2c - f &> 2c - f_2 \\ &> 0 \text{ by (5.9)} \end{aligned}$$

These two cases of course confirm the main result of the paper; they also show that in some cases, the construction of the numerator transfer function can be very simple.

6 Conclusions

In this paper, we have examined a robustness question relating to a strengthened SPR condition. We have shown that in the case of continuous-time systems, the existence question has the same answer as for the case of an unstrengthened SPR condition, while the construction procedure is more complicated, and can be expected to yield an answer containing more parameters. The robustness question for a strengthened discrete-time SPR condition remains however unresolved. Unpublished work however has revealed a counterexample to the conjecture that the answer with the strengthened SPR condition is identical to that for the unstrengthened condition.

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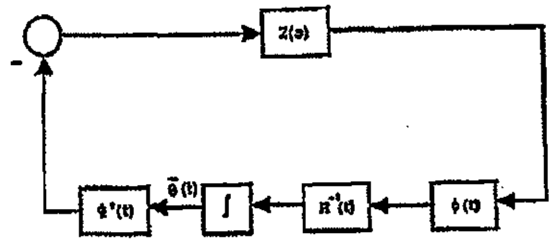

Figure 1.1 Adaptive error system

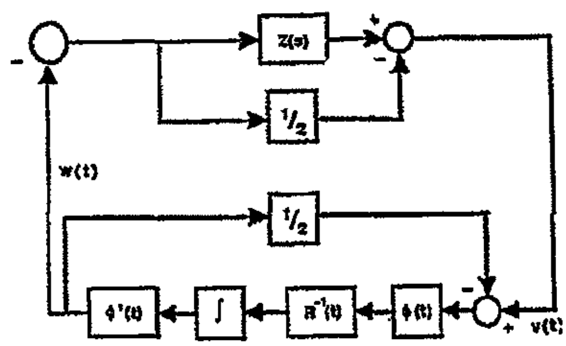

Figure 2.1 Redrawing of adaptive error system

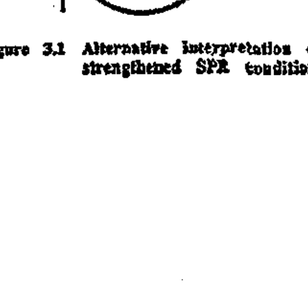

Figure 3.1 Alternative interpretation of strengthened SPR condition

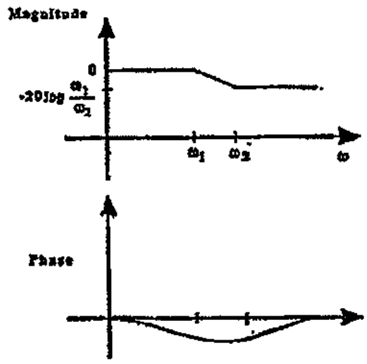

Figure 4.1 Bode diagrams for $r_1(s) = \frac{\omega_1}{s+\omega_1} \frac{s+\omega_2}{\omega_2}$

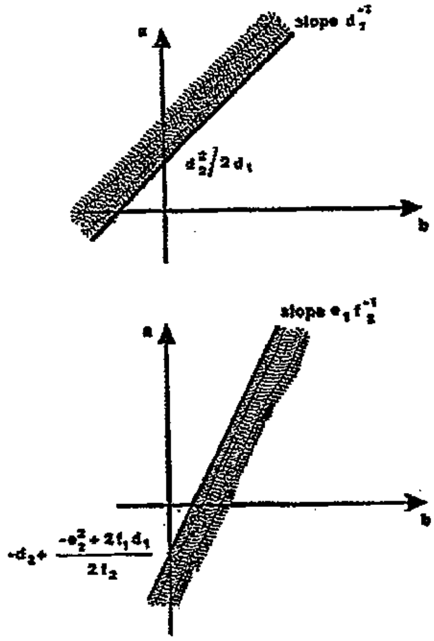

Figure 5.1 Regions defined by inequalities (5.8)