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New Algorithms for Adaptive Control

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Summary : In this paper, we present the windsurfer approach for the design of adaptive robust control systems. In section 1 we review the weakness of traditional adaptive control systems. In section 2 we illustrate the idea of the windsurfer approach by considering an adaptive model matching problem. In section 3, we show how a closed-loop system identification problem associated with the windsurfer approach can be transformed into an open-loop system identification procedure. In section 4 we show that the system identification procedure can be carried out effectively for the case where the controller is designed by the internal model control (IMC) method. It is shown in section 5 that the method is applicable to a class of plants that occur in many industrial systems - those which, except for a pole at the origin, are strictly stable. Encouraging simulation results are presented in section 6 and we conclude in section 7 that the windsurfer approach is an effective method for designing adaptive robust control systems.

1 Introduction

In the traditional approach to analysis and design of an adaptive control system [1], it is assumed that the unknown plant can be represented by a model in which everything is known except for the values of a finite number of parameters. Once the parameters are estimated (and even during the estimation process), the principle of certainty equivalence is almost invariably invoked to update the controller. Normally the unstructured uncertainties of the model are *ignored* in this approach. Therefore it is not surprising, as pointed out in [2], that these adaptive controllers are often not robust.

A new class of algorithms (the windsurfer approach) for adaptive control was first proposed in [3]. A preliminary study for an algorithm in this class under noiseless conditions was reported in [4]. Under noisy conditions, encouraging results were obtained for the case where the plant is strictly stable [5].

It is well known that a large class of practical systems involves plants which, except for a pole at the origin, are strictly stable. These plants include electromechanical actuators for position control systems and those in industrial processes like sugar cane crushing mills. Therefore, it is desirable to show that the new approach is also applicable to these plants. In the sequel, we shall call this class of plants *type 1 stable plants.* This paper shows that, under very mild conditions, the new class of algorithms is applicable to type 1 stable plants, and contains additional results on more general unstable plants.

We shall first illustrate the idea of the windsurfer approach by applying it to an adaptive model matching problem.

2 Adaptive Model Matching

Consider a system as shown in figure 1. Let G be the unknown transfer function of the plant, and let T^d represent a desired complementary sensitivity function. We wish to minimize the cost function

$$\left\| \frac{GK}{1+GK} - T^d \right\|_\infty ,$$

where K is the transfer function of a controller to be designed.

We begin by designing a controller $K_{1,0}$ to stabilize a known initial model G_0, which may be obtained from an open-loop system identification exercise. If $K_{1,0}$ also stabilizes the unknown transfer function G, then we say that $K_{1,0}$ robustly stabilizes G_0. In general, we attach the subscript j,i to a transfer function to denote that it is either specified or derived on the basis of the i^{th} model for the plant at the j^{th} iteration of control design. Since G_0

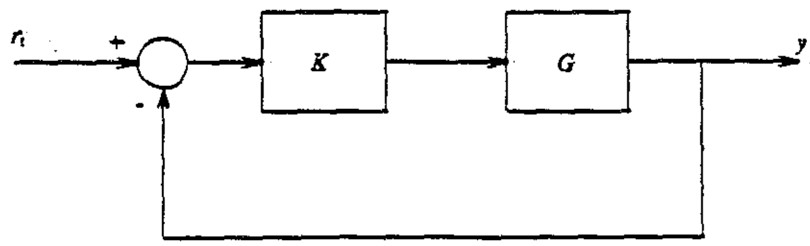

Figure 1: Closed-loop control system

may involve significant uncertainties, the resulting controller $K_{1,0}$ may not be able to achieve a small value for

$$\left\| \frac{G_0 K_{1,0}}{1 + G_0 K_{1,0}} - T^d \right\|_\infty$$

while robustly stabilizing G_0. In general, we need to consider how to handle the question of securing robust stabilization of G_i by $K_{j,i}$. We shall therefore proceed as follows so that the end control objective may be approached in stages.

Associated with each of the models G_i, a sequence of controllers $K_{j,i}$ is designed such that

$$K_{j,i} = \arg\min_{\gamma} \left\| \frac{G_i \gamma}{1 + G_i \gamma} - T^d_{j,i} \right\|_\infty, \quad \forall j, \tag{1}$$

where the sequence of functions $T^d_{j,i}$ is specified with $T^d_{j+1,i}$ normally of wider bandwidth than $T^d_{j,i}$. It is also necessary that $T^d_{1,i}$ results in a controller $K_{1,i}$ that robustly stabilizes G_i. A stage will be reached (say when $j = N$) where the bandwidth of the nominal closed-loop transfer function,

$$\bar{T}_{N,i} = \frac{G_i K_{N,i}}{1 + G_i K_{N,i}}, \tag{2}$$

cannot be increased further without causing the effects of model uncertainties in G_i to be too significant. This occurs when the value of

$$\|T_{N,i} - \bar{T}_{N,i}\|_\infty$$

is no longer small, where

$$T_{N,i} = \frac{G K_{N,i}}{1 + G K_{N,i}} \tag{3}$$

is the actual closed-loop transfer function of the system.

At this stage it is necessary to improve the accuracy of the model in such a way that is relevant to the control objective. This means that we should try to find an updated model G_{i+1} such that

$$G_{i+1} = \arg\min_{\theta} \left\| \frac{G K_{N,i}}{1 + G K_{N,i}} - \frac{\theta K_{N,i}}{1 + \theta K_{N,i}} \right\|_\infty. \tag{4}$$

Once G_{i+1} is found, we can continue to increase the closed-loop bandwidth by repeating the procedure described for G_i previously. However G_{i+1} should be used instead of G_i, and we specify a new sequence of functions $T^d_{j,i+1}$ with $T^d_{1,i+1} = T^d_{N,i}$. The iterative process is continued until the end control objective is achieved or it is prematurely terminated because of one or more of the following constraints:

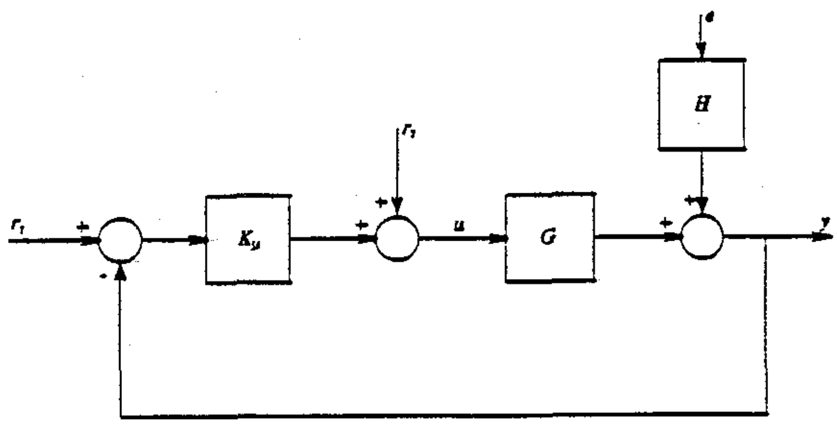

Figure 2: Adaptive control system

1. fundamental performance limitations due to right half plane poles and zeros of the plant and/or models [6].
2. finite control energy.
3. no further improvements in the identified model can be made for a reasonably large set of input output measurements.

3 Closed-loop System Identification

Consider the feedback system shown in figure 2, where y and u are the measured output and the control input, respectively, e is an unpredictable white disturbance, and r_1 and r_2 are user applied inputs. It is assumed that $K_{j,i}$ is a known stabilizing controller, G is inexactly known and possibly unstable, and, as is standard [7], H is imperfectly known, stable and inversely stable. The system identification problem is to obtain improved estimates of G and H from a finite interval of measured and known data $\{y, u, r_1, r_2 : 0 \leq t \leq T\}$.

It was shown in Hansen [8] that, if $X_{j,i}$, $Y_{j,i}$, $N_{j,i}$, and $D_{j,i}$ are stable proper transfer functions such that

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}}, \quad G_i = \frac{N_i}{D_i},$$

and

$$N_i X_{j,i} + D_i Y_{j,i} = 1,$$

then

$$X_{j,i} = \frac{K_{j,i}}{D_i + N_i K_{j,i}}, \tag{5}$$

and

$$Y_{j,i} = \frac{1}{D_i + N_i K_{j,i}}. \tag{6}$$

Furthermore, there exist stable proper transfer functions $R_{j,i}$ and $S_{j,i}$, with $S_{j,i}$ also inversely stable, such that

$$G = \frac{N_i + R_{j,i} Y_{j,i}}{D_i - R_{j,i} X_{j,i}}, \tag{7}$$

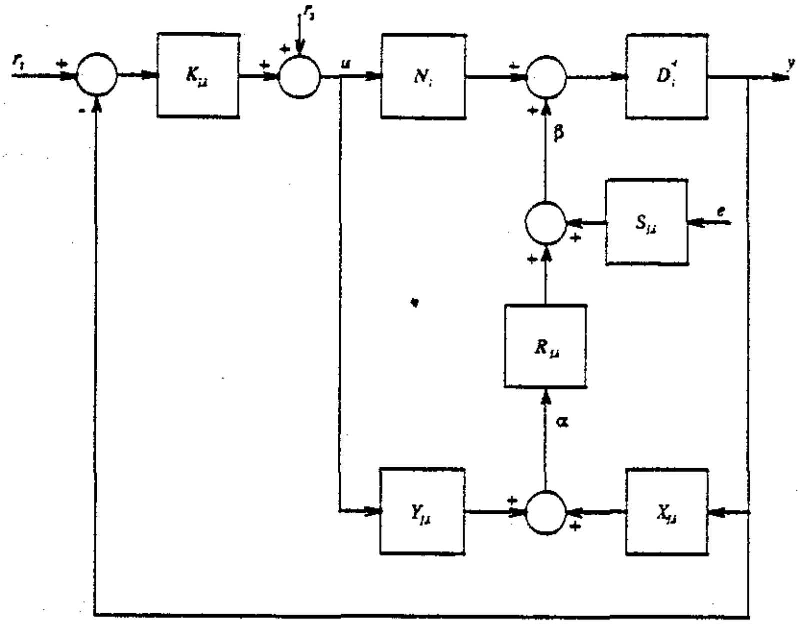

Figure 3: Closed-loop system identification

$$H = \frac{S_{j,i}}{D_i - R_{j,i}X_{j,i}}. \tag{8}$$

Using equations (7) and (8), we can represent the feedback system as shown in figure 3. From figure 3, we can write

$$\beta = R_{j,i}\alpha + S_{j,i}e, \tag{9}$$

where

$$\alpha = X_{j,i}r_1 + Y_{j,i}r_2, \tag{10}$$

and

$$\beta = D_i y - N_i u. \tag{11}$$

It is important to observe from equations (10) and (11) that α depends only on the applied signals r_1 and r_2 operated on by known stable proper transfer functions $X_{j,i}$ and $Y_{j,i}$ respectively, and β depends on measured signals y and u operated by known stable proper transfer functions D_i and N_i respectively. Moreover, α is independent of the transfer functions G and H and the disturbance e. Hence equations (7) to (11) show that the system identification of G and H in closed-loop has been recast into the system identification of $R_{j,i}$ and $S_{j,i}$ in open-loop.

In [5], we have obtained the following result which is highly relevant to the system identification step of the windsurfer approach to adaptive control.

Theorem 3.1 *Let the controller $K_{j,i}$ stabilize the plant G and the model*

$$G_i = \frac{N_i}{D_i},$$

where N_i and D_i are stable proper transfer functions, and let

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}}, \tag{12}$$

where $X_{j,i}$ and $Y_{j,i}$ are stable proper transfer functions satisfying the Bezout identity

$$N_i X_{j,i} + D_i Y_{j,i} = 1.$$

Let G_{i+1} be another model of G, also stabilized by $K_{j,i}$ and therefore having a description

$$G_{i+1} = \frac{N_i + r_{j,i} Y_{j,i}}{D_i - r_{j,i} X_{j,i}} \tag{13}$$

where $r_{j,i}$ is a stable proper transfer function. Also define the filtered output error

$$\xi = Y_{j,i}(\beta - r_{j,i}\alpha),$$

where, with $r_2 = 0$,

$$\alpha = X_{j,i} r_1,$$

$$\beta = D_i y - N_i u,$$

$$r_1 = \textit{reference signal},$$

$$y = \textit{plant output},$$

$$u = \textit{control input}.$$

Then the filtered output error can be expressed as

$$\xi = \left(\frac{GK_{j,i}}{1 + GK_{j,i}} - \frac{G_{i+1}K_{j,i}}{1 + G_{i+1}K_{j,i}}\right) r_1 + \frac{1}{1 + GK_{j,i}} He$$

4 Approximate Identification of the $R_{j,i}$ Transfer Function for IMC Controller Design

In this and the following sections, we shall assume that the plant transfer function G is strictly proper, has no zeros on the imaginary axis of the s-plane, and no poles on the imaginary axis except at the origin. In the sequel, we will use the notations n_H and d_H to denote the numerator polynomial and the denominator polynomial of a rational transfer function H.

For a model

$$G_i = \frac{n_{G_i}}{d_{G_i}}$$

with k strictly unstable poles $p_1, p_2, \ldots, p_k$ and a pole of multiplicity l at the origin, we can write

$$n_{G_i} = \tilde{n}_{G_i} \prod_i (-s + z_i), \quad \text{and} \quad d_{G_i} = s^l \tilde{d}_{G_i} \prod_{i=1}^{k} (-s + p_i),$$

where the polynomials $\tilde{n}_{G_i}$ and $\tilde{d}_{G_i}$ are strictly stable, and all of z_i and p_i have positive real parts.

We can then write

$$G_i = [G_i]_m [G_i]_a,$$

where

$$[G_i]_m = \frac{\tilde{n}_{G_i} \prod_i (s + z_i^*)}{d_{G_i}}; \quad z_i^* \text{ is the complex-conjugate of } z_i,$$

and

$$[G_i]_a = \frac{\prod_i (-s + z_i)}{\prod_i (s + z_i^*)}.$$

Furthermore, we can use coprime fractional representations to write

$$G_i = \frac{N_i}{D_i},$$

with the transfer functions D_i and N_i defined as

$$D_i = \frac{d_{G_i}}{q}, \quad \text{and} \quad N_i = \frac{n_{G_i}}{q},$$

where q is a strictly stable polynomial of the same degree as d_{G_i}. Then we have

$$N_i = [N_i]_m [N_i]_a,$$

with

$$[N_i]_m = \frac{\tilde{n}_{G_i} \prod_i (s + z_i^*)}{q}, \quad [N_i]_a = [G_i]_a,$$

and

$$D_i = [D_i]_m [D_i]_a,$$

with

$$[D_i]_m = \frac{s^l \tilde{n}_{G_i} \prod_{i=1}^{k} (s + p_i^*)}{q}, \quad [D_i]_a = \frac{\prod_{i=1}^{k} (-s + p_i)}{\prod_{i=1}^{k} (s + p_i^*)}.$$

Using these notations, we can state the following theorem [9]:

Theorem 4.1 *Let G_i have k strictly unstable poles at $p_1, p_2, \ldots, p_k$ and a pole of multiplicity l at the origin. For a unit step input, the (detuned H_2-optimal) IMC controller is given by*

$$K_{j,i} = \frac{Q_{j,i}}{1 - Q_{j,i} G_i},$$

where $Q_{j,i}$ is a strictly proper transfer function given by

$$Q_{j,i} = s[D_i]_a [G_i]_m^{-1} \left\{ [D_i]_a^{-1} [G_i]_a^{-1} \frac{1}{s} \right\}_* F_{j,i},$$

(The operator $\{\}_*$ denotes that after a partial fraction expansion of the operand, all terms involving the poles of $[G_i]_a^{-1}$ are omitted.)

The IMC filter $F_{j,i}$ is given by the transfer function

$$F_{j,i} = \frac{\lambda_{j,i}^{k+l+n} \left(\sum_{i=0}^{k+l-1} a_i s^i \right)}{(s + \lambda_{j,i})^{k+l+n}},$$

where n is the relative order of G_i. The filter constants $a_0, a_1, \ldots, a_{k+l-1}$ are determined from the constraints:

$$F_{j,i} = 1 \quad \text{at the } k \text{ strictly unstable poles } s = p_1, p_2, \ldots, p_k,$$

and, if there is a pole of multiplicity l at the origin,

$$F_{j,i} = 1 \quad \text{at } s = 0,$$

and

$$\frac{d^m F_{j,i}}{ds^m} = 0 \quad \text{for } m = 1, 2, \ldots, l-1.$$

Furthermore, if the plant G is strictly stable, the controller will robustly stabilize G_i for all sufficiently small values of the tuning parameter $\lambda_{j,i} \geq 0$.

If the controller $K_{j,i}$ robustly stabilizes G_i, then using equations (5), (6), and (7), we can obtain

$$R_{j,i} = \frac{D_i^2(G - G_i)}{1 + D_i X_{j,i}(G - G_i)}.$$

The next theorem shows that this transfer function to be identified is the product of a known stable proper transfer function and an unknown stable strictly proper transfer function.

Theorem 4.2 *Let the controller be designed according to the conditions stated in theorem 4.1; then the unknown stable strictly proper transfer function to be identified,*

$$R_{j,i} = \frac{D_i^2(G - G_i)}{1 + D_i X_{j,i}(G - G_i)},$$

can be factorized as

$$R_{j,i} = \tilde{R}_{j,i}\hat{R}_{j,i},$$

where

$$\hat{R}_{j,i} = (s + \lambda_{j,i})^{k+l}\left[\prod_{i=1}^{k}(s + p_i^*)\right]\frac{d_{G_i}n_G - d_G n_{G_i}}{d_G d_{K_{j,i}} + n_G n_{K_{j,i}}}$$

is an unknown stable strictly proper transfer function to be identified, and

$$\tilde{R}_{i,j} = D_i[N_i]_m(s + \lambda_{j,i})^n$$

is a known stable proper transfer function.

Proof

The proof is not given due to space limitations. □

Since $\tilde{R}_{j,i}$ is a *known* stable proper transfer function and $\hat{R}_{j,i}$ is an *unknown* stable strictly proper transfer function that depends on the unknown transfer function G, the problem of identifying $R_{j,i}$ has become one of identifying its unknown, lower order factor $\hat{R}_{j,i}$. This involves a trivial adjustment to the scheme described in section 3.

As we do not know the order of $\hat{R}_{j,i}$ a priori, and since we are going to identify $\hat{R}_{j,i}$ (actually $R_{j,i}$) and update G_i to G_{i+1} when the step response of the actual closed-loop system exhibits unacceptable oscillations and/or overshoots (associated with model uncertainties), we expect $\hat{R}_{j,i}$ to have complex-conjugate poles. Therefore, the transfer function which serves as an approximation of $\hat{R}_{j,i}$ has to have an order of at least two. Moreover, since the smallest possible relative degree of a strictly proper transfer function is one and the relative degree of G is unknown, we have to assume that the relative degree of $\hat{R}_{j,i}$ could be one.

Remarks

- If the model G_i has a pole at the origin, then D_i and hence $\tilde{R}_{j,i}$ will have a zero at the origin. From figure 4, we notice that, under this condition, $\hat{R}_{j,i}$ will not be properly excited in the low frequency range. Therefore, if G_i has a pole at the origin, we have to replace the D_i in $\tilde{R}_{j,i}$ by a biproper transfer function similar to D_i but with the zero at the origin removed.

- When updating the model using the equation

$$G_{i+1} = \frac{N_i + r_{j,i}Y_{j,i}}{D_i - r_{j,i}X_{j,i}},$$

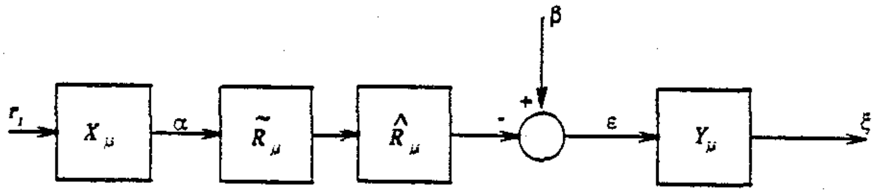

Figure 4: Excitation of $\hat{R}_{j,i}$

the order of the model may increase. To prevent the model order from increasing indefinitely, we use a frequency weighted balanced truncation scheme [10] to reduce the order of G_{i+1}. Specifically, we would like, ideally, to find

$$\hat{G}_{i+1} = \arg\min_{\eta} \left\| \frac{G_{i+1}K_{j,i}}{1+G_{i+1}K_{j,i}} - \frac{\eta K_{j,i}}{1+\eta K_{j,i}} \right\|_{\infty},$$

where $\hat{G}_{i+1}$ is the reduced order model. In practice, we obtain approximately, under frequency weighted balanced truncation,

$$\hat{G}_{i+1} = \arg\min_{\eta} \left\| \frac{K_{j,i}(G_{i+1}-\eta)}{(1+G_{i+1}K_{j,i})^2} \right\|_{\infty}.$$

5 Stabilization of Type 1 Stable Plants

We shall show that the new algorithm is applicable to systems with type 1 stable plants. In particular, we shall show that, under very mild conditions, it is possible to initialize the algorithm with low gain robust stabilizing controllers. Due to space limitations, the proofs of these results will not be given.

Using simple algebraic arguments, we can establish the following lemma:

Lemma 5.1 *Let a plant be*

$$G(s) = \frac{n(s)}{sd(s)},$$

where $d(s)$ is strictly stable and $n(0) \neq 0$. Then a controller

$$K(s) = \varepsilon \left\{ sgn \left[\frac{n(0)}{d(0)} \right] \right\}, \quad \varepsilon > 0,$$

stabilizes $G(s)$ for sufficiently small ε.

Using lemma 5.1, the theory of Youla-parametrization of all stabilizing controllers, and Routh-Hurwitz criterion, we can prove the following theorems. If it is known apriori that the plant has a pole at the origin, then we have:

Theorem 5.1 *Let a model of the plant given in lemma 5.1 be*

$$G_0(s) = \frac{\hat{n}(0)}{s\hat{d}(0)}.$$

Then a controller, designed using the IMC method,

$$K_{1,0}(s) = \frac{\lambda_{1,0}^2 \hat{d}(0)}{\hat{n}(0)(s + 2\lambda_{1,0})}, \quad \lambda_{1,0} > 0,$$

robustly stabilizes $G_0(s)$ for sufficiently small $\lambda_{1,0}$ if

$$\frac{\hat{n}(0)}{\hat{d}(0)} \quad \text{and} \quad \frac{n(0)}{d(0)}$$

have the same sign.

Sometimes it may not be known apriori that the plant has a pole at the origin, and the plant is inaccurately modelled as a first order lag. In this case we have:

Theorem 5.2 *Let a model of the plant given in lemma 5.1 be*

$$G_0(s) = \frac{\hat{n}(0)}{\hat{d}(0)(\tau s + 1)}.$$

Then a controller, designed using the IMC method,

$$K_{1,0}(s) = \frac{\lambda_{1,0}^2 \hat{d}(0)(\tau s + 1)}{\hat{n}(0)s(s + 2\lambda_{1,0})}, \quad \lambda_{1,0} > 0,$$

robustly stabilizes $G_0(s)$ if

$$\frac{\hat{n}(0)}{\hat{d}(0)} \quad \text{and} \quad \frac{n(0)}{d(0)}$$

have the same sign, and if

$$\lambda_{1,0} > \frac{1}{2\tau}$$

is suitably small.

6 Simulation Results

With reference to figure 2, we shall present some simulation results of applying the windsurfer approach to the control of a system with

$$G(s) = \frac{9(-s+2)}{s(s^2 + 0.06s + 9)},$$

$$H(s) = 1,$$

and e is zero mean disturbance which a constant energy density of 0.025 from 0 to 100 Hz. Note that the plant $G(s)$ is nonminimum phase.

The simulation results are presented in figures 5, 6, and 7. We start with an initial model which has the transfer function

$$G_0 = \frac{1.6}{s}.$$

In all of these figures, the graphs on the left show the noisy unit step responses of the actual closed-loop systems and those on the right show the corresponding low-pass filtered signals. Graphs (a) and (b) of figure 5 show the responses of the actual closed-loop system with a nominal bandwidth of 0.1 rad/s. Note that overshoots and oscillations are absent

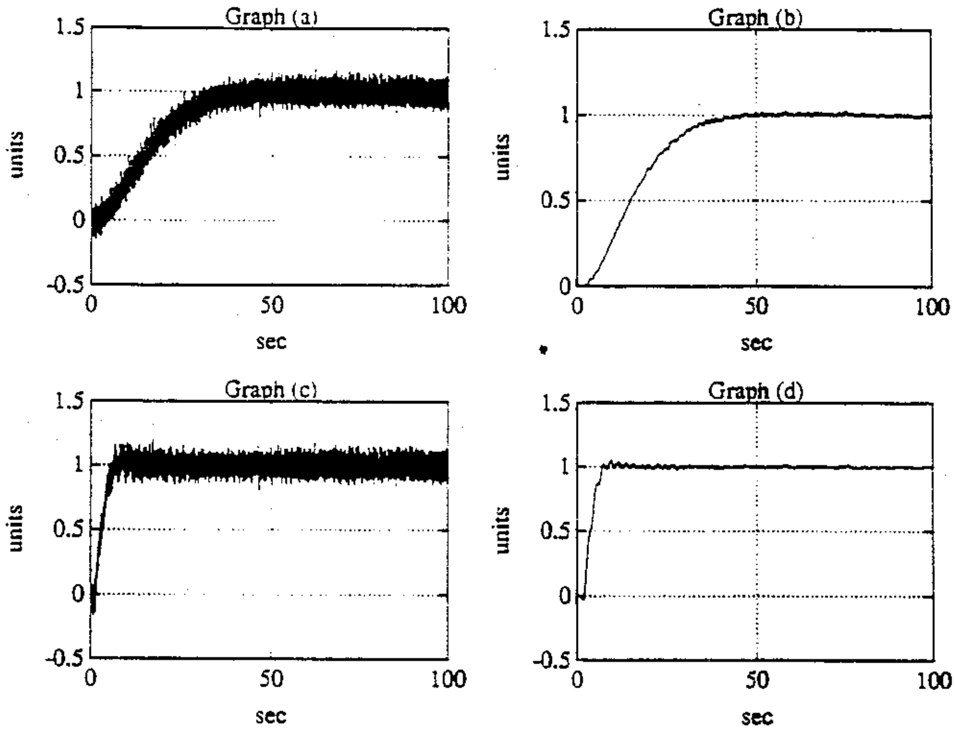

Figure 5: Simulation results 1

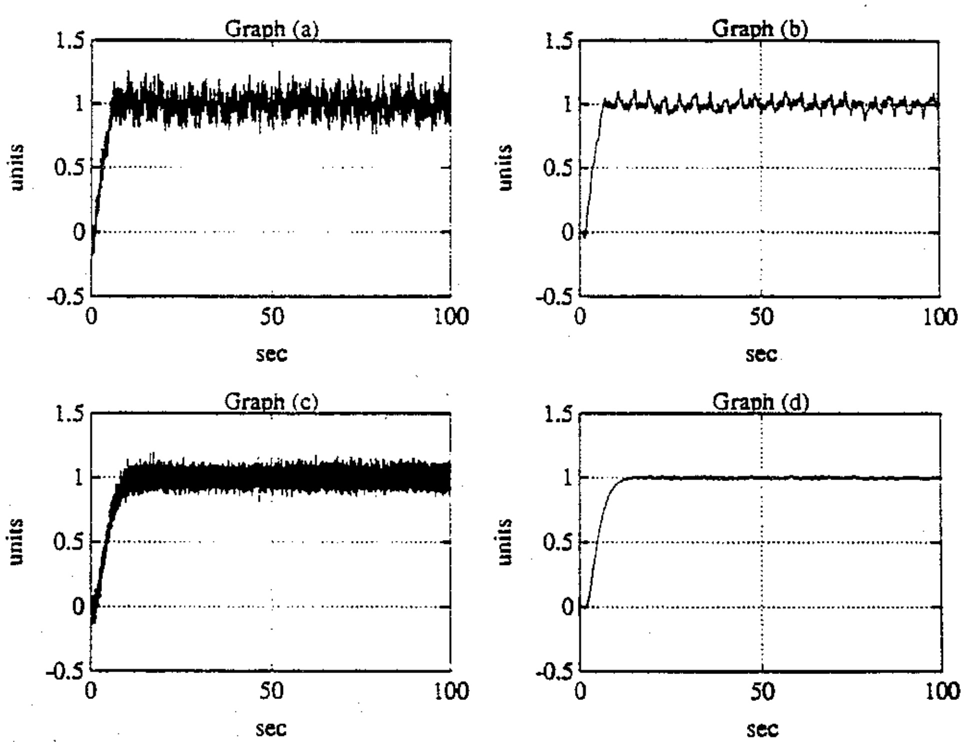

Figure 6: Simulation results 2

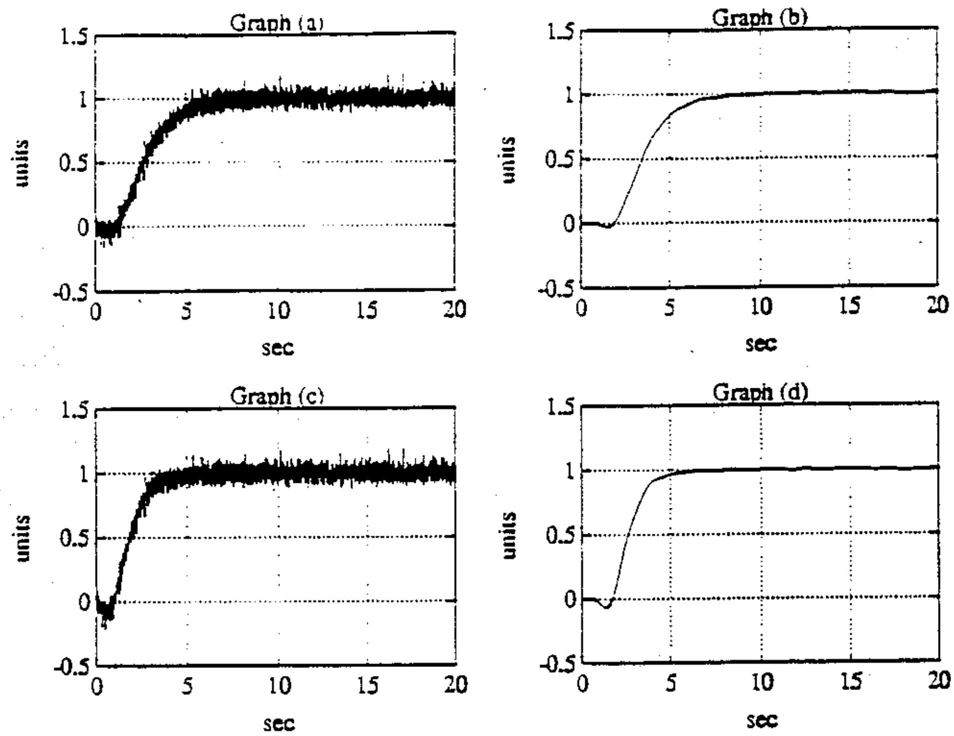

Figure 7: Simulation results 3

for the response in graph (b). Graphs (c) and (d) of figure 5 are for a nominal closed-loop bandwidth of 0.5 rad/s. Note that the response in graph (d) is oscillatory and any attempt to increase the nominal closed-loop bandwidth further is likely to lead to instability. At this stage, it is necessary to improve the accuracy of the model if we wish to increase the nominal closed-loop bandwidth further. To ensure that the signals are sufficiently exciting, low amplitude sinusoids in the relevant frequency range are superimposed on the unit step input just prior to system identification. The responses are shown in graphs (a) and (b) of figure 6. The updated model has a transfer function

$$G_1 = \frac{-0.011756s^2 - 10.668s + 15.868}{s(s^2 + 0.038756s + 8.8634)}.$$

The updated model G_1 is used to re-design a nominal closed-loop system with a bandwidth of 0.5 rad/s, and the responses are shown in graphs (c) and (d) of figure 6. By comparing graph (d) of figure 6 to that of figure 5, we observe that the response no longer has oscillations. If we continue to increase the nominal closed-loop bandwidth of the system, we obtain the responses shown in figure 7 where graphs (a) and (b) are for a bandwidth of 1 rad/s, and graphs (c) and (d) are for a bandwidth of 2 rad/s.

7 Conclusions

We have reviewed the weakness of traditional adaptive control systems. One of the possible ways to overcome the weakness is by the windsurfer approach [3]. We have shown that, for a type 1 stable plant, by starting with a (crude) initial model and a (small bandwidth) robustly stabilizing controller, the bandwidth of the closed-loop system can be increased progressively through an iterative control-relevant system identification and control design procedure.

References

[1] G.C.Goodwin, and K.S.Sin, *Adaptive Filtering, Prediction and Control*, Prentice Hall, Englewood Cliffs, NJ, 1984.

[2] C.E.Rohrs, L.Valavani, M.Athans, and G.Stein, Robustness of Continuous-Time Adaptive Control Algorithms in the Presence of Unmodeled Dynamics, *IEEE Trans. Automat. Contr.*, **30**, 881-889, (1985).

[3] B.D.O.Anderson, and R.L.Kosut, Adaptive Robust Control: On-Line Learning, *Proc CDC'91*, 297-298, Brighton, England, Dec 1991.

[4] W.S.Lee, B.D.O.Anderson, R.L.Kosut, and I.M.Y.Mareels, On Adaptive Robust Control and Control-Relevant System Identification, *Proc. ACC'92*, Chicago, Illinois, Jun 1992.

[5] W.S.Lee, B.D.O.Anderson, R.L.Kosut, and I.M.Y.Mareels, A New Approach to Adaptive Robust Control, submitted to *Int. J. Adaptive Control and Signal Processing*, Jun 1992.

[6] J.S.Freudenberg, and D.P.Looze, Right Half Plane Poles and Zeros and Design Tradeoffs in Feedback Systems, *IEEE Trans. Automat. Contr.*, **30**, 555-565, (1985).

[7] L.Ljung, *System Identification: Theory for the user*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

[8] F.R.Hansen, A Fractional Representation Approach to Closed-loop System Identification and Experiment Design, *PhD dissertation*, Stanford University, March 1989.

[9] M.Morari, and E.Zafiriou, *Robust Process Control*, Prentice Hall, Englewood Cliffs, NJ, 1989.

[10] B.D.O.Anderson, and Y.Liu, Controller Reduction: Concepts and Approaches, *IEEE Trans. Automat. Contr.*, **34**, 802-812, (1989).