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# LOSSLESS N-PORT SYNTHESIS VIA STATE-SPACE TECHNIQUES

by

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## Abstract

A new synthesis of lossless impedance matrices is presented which results in a circuit realization using a minimal number of reactive elements and often a minimal number of gyrators. The synthesis procedure relies heavily on the posing of the problem in systems theoretic terms and applying recent results on synthesis from state-space equations.

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## I. Introduction

Network synthesis is concerned with the general problem of passing from a function or a matrix of functions of a complex variable to a description of an interconnection of subnetworks, such as resistors, inductors, transmission lines, etc. Often however, an additional problem arises, namely, that of giving a network description involving a minimal number of a particular class or classes of elements.

The problem considered here is of this type: a network is sought which will realize a prescribed lossless impedance or scattering matrix; moreover the network is to contain a minimal number of reactive elements and gyrators if possible. Minimal reactive element syntheses of multiport networks have been known for some time [1], [2], [3], but minimal gyrator syntheses have not been exhibited except in the 2-port case [4]. It has previously been shown that a lossless n-port network requires at most (n-1) gyrators for its synthesis [1], while a lower bound on the number of gyrators for an arbitrary network has been set [5]. It is also conjectured [5] that this lower bound could be achieved in practice, but as yet this has not been achieved though for the lossless case this work gives some assistance toward minimal gyrator synthesis.

In Section II we review the properties of lossless impedance matrices in sufficient detail to meet our requirements. This section also contains material of a linear system theory nature which is used in the sequel. The synthesis procedure is outlined in Section III, and draws heavily upon systems theoretic ideas of Section II. In Section IV we give an alternate proof that no more than (n-1) gyrators are needed.

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It should be noted that the problem of giving a minimal gyrator synthesis is still open. Such a synthesis is known to sometimes require a nonminimal number of resistors or reactive elements [1] in the nonlossless case. Consequently it seems invalid to seek a solution via the state-space lossy synthesis techniques [6], which might seem to offer a likely approach in view of the possibilities of state-space methods in this paper. The reason is that the technique of reference [6] always gives a minimal resistor and minimal reactive element synthesis, and would thus seem to require considerable modification to establish a minimal gyrator synthesis.

## II. Preliminary Work

Without further comment, it will be assumed henceforth that we are dealing with a multiport network N composed of a finite interconnection of transformers, gyrators, and positive inductors and capacitors. We term N lossless. Such a network always possesses a scattering matrix, but may not possess an impedance matrix [7]. Nevertheless, there is no generality lost in considering the immittance synthesis problem only. From any prescribed scattering matrix, it is possible to pass to an impedance matrix which describes a network closely related to that which the scattering matrix describes. A synthesis of the network described by the impedance matrix then yields immediately a synthesis of the original network described by the scattering matrix, and vice versa [8].

We shall define a lossless impedance matrix to be the impedance of a multiport such as N above; such matrices have special properties:

Theorem 1. [8, p. 102]. Let $Z(s)$ be a rational lossless impedance matrix. Then $Z(s)$ satisfies the standard positive real constraints [8, p. 96] and also

$$Z(s) = -Z'(-s) \tag{1}$$

where the prime denotes matrix transposition.

A lossless matrix $Z(s)$ has elements whose poles are restricted to being simple and lying on the $j\omega$ axis [8, p. 122]. The point infinity is included as a possible pole, and if $Z(s)$ has a pole at infinity it is always possible to write

$$Z(s) = sL + Z_1(s) \tag{2}$$

where $L = L'$ is a nonnegative definite matrix, and $Z_1(s)$ is lossless, with $Z_1(\infty)$ finite. The matrix $sL$ can be synthesized with transformer coupled inductors [8, p. 204], reducing the problem of synthesizing $Z(s)$ to that of synthesizing $Z_1(s)$. Similar remarks of course apply to admittance matrices, mutatis mutandis.

Consider now the linear, finite dimensional, dynamical system described by the equations

$$\dot{x} = Fx + Gu \tag{3a}$$

$$y = H'x + Ju \tag{3b}$$

Here $u$ is the input vector, $x$ is the state vector, $y$ is the output vector, and $F$, $G$, $H$, and $J$ are constant matrices of appropriate dimension. The transfer function matrix relating $U(s)$, the Laplace transform of $u$, to $Y(s)$, the Laplace transform of $Y(s)$, through

$$Y(s) = W(s)\,U(s) \tag{4}$$

is

$$W(s) = J + H'(sI - F)^{-1} G \tag{5}$$

Here $I$ is the identity matrix of appropriate order.

A quadruple $\{F, G, H, J\}$ is termed a realization of $W$, and is minimal if $F$ has the least possible dimension. If $\{F, G, H, J\}$ is one minimal realization, all others are given by $\{T^{-1}FT, T^{-1}G, (T)'H, J\}$ where $T$ ranges through the set of nonsingular matrices [9]; note that $J$ is always $W(\infty)$ irrespective of $T$. All transfer function matrices

$W(s)$ with $W(\infty)$ finite possess realizations and thus minimal realizations, and since an impedance matrix is a special kind of transfer function, we can define minimal realizations (in the dynamical system sense) for such matrices. It is the interpretation of such minimal realizations that is interesting:

<u>Theorem 2</u>. Let $Z(s)$ be an impedance matrix with $Z(\infty)$ finite and let $\{F, G, H, J\}$ be a minimal realization for $Z(s)$. Consider the equation

$$\begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} J & -H_1' & -H_2' \\ G_1 & -F_{11} & -F_{12} \\ G_2 & -F_{21} & -F_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix} \qquad (6)$$

where $F$ has been partitioned arbitrarily as

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

and $G$ and $H$ have been correspondingly partitioned; $v_1, i_1, \ldots i_3$ are all vectors.

Then the matrix in Eq. (6) is the hybrid matrix of a multiport network such that if the ports associated with $v_2$ and $i_2$ are terminated in unit inductors, and the ports associated with $v_3$ and $i_3$ are terminated in unit capacitors, the impedance offered at the ports associated with $v_1$ and $i_1$ is $Z(s)$. Moreover, if the hybrid matrix can be synthesized by a passive network, the resulting passive network realization of $Z(s)$ uses a minimal number of reactive elements.

<u>Proof</u>. Figure 1 illustrates the connection described. We identify the vectors $v_j$ with Laplace-transformed voltages and $i_j$ with Laplace-transformed currents. The inductive and capacitive terminations require, in addition to Eq. (6), the relations

$$v_2 = -si_2 \tag{7a}$$

$$i_3 = -sv_3 \tag{7b}$$

When these are substituted in Eq. (6), the relation

$$v_1 = [J + H'(sI - F)^{-1}G]i_1 \tag{8}$$

follows. This proves the first part of the theorem. The second part follows from noting that the minimal number of reactive elements required to synthesize $Z(s)$ is a well-defined number called the degree of $Z(s)$, written $\delta[Z(s)]$ [8, p. 322], and $\delta[Z(s)]$ is the dimension of the $F$ matrix of a minimal (systems) realization of $Z(s)$ [10]. Further, the number of inductors and capacitors used in the realization is precisely the dimension of $F$. Thus this number is minimal. This completes the proof.

The minimal (systems) realizations of a lossless matrix $Z(s)$ possess special properties resulting from the lossless character of $Z$. Such properties are summed up in the following theorem (see reference [11] for proof):

> <u>Theorem 3</u>. Let $Z(s)$ be a lossless impedance matrix with $Z(\infty)$ finite and let $\{F, G, H, J\}$ be a minimal realization for $Z$. Then there exists a symmetric positive definite matrix $P$ such that

$$PF + F'P = 0 \tag{9a}$$

$$PG = H \tag{9b}$$

The matrix $P$ can be found in terms of the controllability matrix $W$ and an observability matrix $V$ as follows. From (9),

$$PG = H$$

$$PFG = -F'PG = -F'H$$

$$PF^2G = -F'PFG = (F')^2H$$

$$\vdots \qquad \vdots$$

$$PF^{n-1}G = (-1)^{n-1}(F')^{n-1}H$$

Thus with

$$W = [G, FG, \ldots, F^{n-1}G] = \text{controllability matrix} \tag{9c}$$

$$V = [H, -F'H, \ldots, (-1)^{n-1}(F')^{n-1}H] = \text{modified observability matrix} \tag{9d}$$

the above equations give

$$PW = V \tag{9e}$$

Now $W$ has a right inverse by the minimality of the realization (complete controllability) in which case

$$P = VW'(WW')^{-1} \tag{9f}$$

Theorem 3's systems theory characterization of the lossless property of $Z(s)$ forms the key to the synthesis procedure of the next section.

## III. Lossless Synthesis Procedure

The main result of this section is stated as:

**Theorem 4.** Let $Z(s)$ be a lossless impedance matrix.

Secondly, if

$$Z(s) = T_2'[Z_2(s) \dotplus 0]T_2 = T'Z_2(s)T \qquad (\text{with } \det T_2 \neq 0) \tag{11}$$

for some constant matrix $T$ (corresponding to the situation of Figure 2, where $T$ is the turns-ratio matrix of the transformer), then

$$\delta[Z(s)] = \delta[Z_2(s)] \tag{12a}$$

and

$$\text{rank } [Z(s) - Z'(s)] = \text{rank } [Z_2(s) - Z_2'(s)] \tag{12b}$$

Finally, if $Z(s)$ is nonsingular, so that we may define

$$Y(s) = [Z(s)]^{-1} \tag{13}$$

then

$$\delta[Y(s)] = \delta[Z(s)] \tag{14a}$$

and

$$\text{rank } [Z(s) - Z'(s)] = \text{rank } [Z(Y' - Y)Z'] = \text{rank } [Y(s) - Y'(s)] \tag{14b}$$

The degree relations are standard properties of $\delta[\ ]$, [8, p. 176], and the rank relations are easy to verify. Corresponding results naturally hold for admittance matrices. The significance of these relations lies in the following statement, which should be clear from the above analysis:

> Theorem 5. Suppose in attempting to synthesize an immittance $Z(s)$, transformers are used in the synthesis, and poles at infinity are extracted from any impedance or admittance appearing in the synthesis until at some stage it remains to synthesize an immittance $Z_r(s)$. Then the problem of giving a minimal

gyrator and simultaneously minimal reactive element synthesis for $Z(s)$ is reduced to giving such a synthesis for $Z_r(s)$.

This theorem can now be used to establish:

Theorem 6. The problem of giving a minimal gyrator and reactive element synthesis for

$$Z(s) = J + H'(sI - F)^{-1}G \tag{15}$$

with $J$ arbitrary is equivalent either to the problem of giving a synthesis of a constant immittance, or to the problem of giving a synthesis of an immittance $Z_r(s)$ such that $J_r = Z_r(\infty)$ is nonsingular.

Proof. If $J$ is nonsingular there is nothing to prove. Thus, suppose $J$ is singular. The matrix $Z(s)$ may be either nonsingular or singular. In the former case $Y(s)$ exists, but must have a pole at infinity in order that $Y(\infty)Z(\infty) = I$ should hold with a singular $Z$. Then we may write

$$Y_1(s) = Y(s) - sC \tag{16}$$

where $Y_1(\infty) = J_1$ is finite, $Y_1(s)$ is a lossless immittance, $C$ is nonnegative definite, and $Y(s)$ may be represented (assuming it is an admittance) by a network consisting of the parallel combination of a network of admittance $Y_1$ and transformer-coupled capacitors.

If on the other hand $Z(s)$ is singular, there exists a constant matrix $T$ such that

$$Z(s) = T'Z_2(s)T \tag{17}$$

with $Z_2(s)$ nonsingular [8, p. 128]. Then either $J_2 = Z_2(\infty)$ is nonsingular (and the theorem is thus proved) or we can apply the technique of the first part of the theorem, extracting the pole at infinity from $Y_2(s) = [Z_2(s)]^{-1}$ to obtain $Y_3(s)$ with $J_3 = Y_3(\infty)$ finite.

Such a process can be repeated with successive degree reductions until either we obtain an immittance $Z_r(s)$ of degree zero, (in other words $Z_r(s)$ is constant), or an immittance $Z_r(s)$ such that $J_r = Z_r(\infty)$ is nonsingular.

It only remains to be verified that a minimal reactive element and gyrator synthesis of $Z_r(s)$ yields a minimal synthesis of $Z(s)$. This follows however immediately from Theorem 5. This completes the proof.

Theorem 6 leads us to consider the problem of providing a minimal gyrator and reactive element synthesis for

$$Z(s) = J + H'(sI - F)^{-1}G \tag{18}$$

with $J$ nonsingular. (Note that the subscript $r$ has been dropped.) To present the synthesis we require a special minimal (systems) realization for $Z$. This is given by Theorem 7 where $\dot{+}$ denotes the direct sum and $0_p$ is the $p \times p$ zero matrix.

<u>Theorem 7</u>. Let $Z(s)$ be a lossless impedance matrix. Then $Z(s)$ has a minimal realization $\{F, G, H, J\}$ with

$$G = H \tag{19a}$$

and

$$F = \begin{bmatrix} 0 & \Gamma \\ -\Gamma & 0 \end{bmatrix} \dot{+} 0_p \tag{19b}$$

where $\Gamma$ is a diagonal matrix of real nonzero elements, and $p = 0$ or $1$, with $p = 0$ corresponding to the absence of the final term of the direct sum.

<u>Proof</u>. Let $\{F_1, G_1, H_1, J\}$ be an arbitrary minimal realization for $Z$. Let $P$ be the matrix defined in Theorem 3 corresponding to this realization; using $P$ we define a change of basis so that a new minimal realization of $Z(s)$ is $\{F_2, G_2, H_2, J\}$ with

$$F_2 = P^{1/2} F_1 P^{-1/2} \tag{20a}$$

$$G_2 = P^{1/2} G_1 \tag{20b}$$

$$H_2 = (P^{-1/2})' H_1 = P^{-1/2} H_1 \tag{20c}$$

Here $P^{1/2}$ is the symmetric positive definite square root of $P$ [12, p. 76]. Then it follows from Eqs. (9) that

$$F_2' + F_2 = 0 \tag{21a}$$

$$G_2 = H_2 \tag{21b}$$

Since $F_2$ is skew, there exists an orthogonal $L_2$ such that

$$L_2' F_2 L_2 = \dot{+}_i \begin{bmatrix} 0 & -\gamma_i \\ \gamma_i & 0 \end{bmatrix} \dot{+} 0_p \tag{22}$$

The poles of $Z(s)$ are given by the eigenvalues of $F_2$, and are known to be simple [8, p. 117]. Accordingly, the $\gamma_i$ may be taken as all nonzero, while $p$ is one or zero, depending on whether $Z(s)$ has a pole at the origin or not.

We use $L_2$ to define a new minimal realization

$$F_3 = L_2' F_2 L_2 \tag{23a}$$

$$G_3 = L_2' G_2 \tag{23b}$$

$$H_3 = L_2' H_2 \tag{23c}$$

In this new realization we have $G_3 = H_3$, using Eqs. (21b), (23b) and (23c).

Now define a permutation matrix $L_3$ such that Eq. (20) is taken into

$$L_3'F_3L_3 = \begin{bmatrix} 0 & -\Gamma \\ & \\ \Gamma & 0 \end{bmatrix} \dotplus 0_p \tag{24a}$$

with

$$\Gamma = \text{diag}\,\{\gamma_1, \gamma_2, \ldots\} \tag{24b}$$

$\Gamma$ is nonsingular, and $p = 0$ or 1. The matrix $L_3$ is orthogonal and allows us to define a new minimal realization

$$F = L_3'F_3L_3 = \begin{bmatrix} 0 & -\Gamma \\ & \\ \Gamma & 0 \end{bmatrix} \dotplus 0_p \qquad p = 0 \text{ or } 1 \tag{25a}$$

$$G = L_3'G_3 \tag{25b}$$

$$H = L_3'H_3 \tag{25c}$$

Finally, since $G_3 = H_3$, we have $G = H$; this proves the theorem.

It is convenient to eliminate the uncertainty in Eq. (25a) by defining

$$\hat{\Gamma} = \Gamma \quad \text{if} \quad p = 0 \tag{26a}$$

and

$$\hat{\Gamma} = \begin{bmatrix} \Gamma \\ \hline 0 \end{bmatrix} \quad \text{if} \quad p = 1 \tag{26b}$$

where the zero matrix in Eq. (26b) has one row, and as many columns as $\Gamma$ has. Then

$$F = \begin{bmatrix} 0 & -\hat{\Gamma} \\ & \\ \hat{\Gamma} & 0 \end{bmatrix} \tag{27}$$

At this point we can then consider the hybrid matrix $M$ of Eq. (6) to take the form

$$M = \begin{bmatrix} J & -H_1' & -H_2' \\ H_1 & 0 & \hat{\Gamma}' \\ H_2 & -\hat{\Gamma} & 0 \end{bmatrix} \tag{28}$$

where $H_1$ and $H_2$ are now partitioned components of $H$, and Eqs. (25) and (27) have been used. By Theorem 6 we may suppose $J$ is nonsingular, and since $Z$ is lossless, $J = Z(\infty)$ is skew.

Consider now Fig. 3, depicting the interconnection of a network described via an impedance matrix, and two transformers whose port variables satisfy the relations presented in Fig. 2a. It is straightforward to verify that the hybrid matrix of this network is $M$, as given in Eq. (28). The number of gyrators needed for Fig. 3 is equal to half of the rank of the impedance matrix

$$Z_M = \begin{bmatrix} J & -H_1 \\ -H_1 & 0 \end{bmatrix} \tag{29}$$

which is directly synthesized by transformers and gyrators. The number of inductors and capacitors (which all have unit value) needed to terminate is the minimal number, $\delta[Z(s)]$ = dimension of $F$.

## IV. Realization Using (n-1) Gyrators

For simplicity, attention will be restricted to the situation where $\hat{\Gamma}$ in (28) is a diagonal matrix. An important, but not obvious fact is that the matrix $H_1$ in (28) can be assumed to have a first column with every entry zero.

To demonstrate this, the realization $\{F_3, G_3, H_3\}$ of (23) can be used. Each block of $F_3$ of the form

$$\begin{bmatrix} 0 & -\gamma_i \\ \gamma_i & 0 \end{bmatrix}$$

is invariant under similarity transformations which are orthogonal, that is, as may readily be checked,

$$T_i^{-1}\begin{bmatrix} 0 & -\gamma_i \\ \gamma_i & 0 \end{bmatrix}T_i = \begin{bmatrix} 0 & -\gamma_i \\ \gamma_i & 0 \end{bmatrix} \qquad (30)$$

If $T_i$ is orthogonal, matrices $T_i$ may be chosen so that $T_i^{-1}$ acting on $G$ rotates the 2-vector defined by the $(2i-1)$-th and $2i$-th elements of $G$ to a position such that the $(2i-1)$-th element, after rotation, is zero.

If the basic transformation defined by $T = \dot{+}_i T_i$ is applied to the triple $\{F_3, G_3, H_3\}$ and then the permutation matrix $L_3$, see (24), is subsequently applied, so that

$$F = L_3' T^{-1} F_3 T L_3 = \begin{bmatrix} 0 & -\Gamma \\ \Gamma & 0 \end{bmatrix}, \qquad (31)$$

then the first column of the matrix $H_1$ resulting in (28) will be identically zero (recall $G_3 = H_3$).

At this stage a synthesis of $Z_M$ in (29) can be demonstrated using (n-1) gyrators, (n being the dimension of J). The synthesis is based on the following readily verifiable equality:

$$Z_M = \begin{bmatrix} J & -H_1' \\ H_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & 0 \\ H_1 J^{-1} & 0 \end{bmatrix}\begin{bmatrix} J & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & (H_1 J^{-1})' \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \\ 0 & H_1 J^{-1} H_1' \end{bmatrix} \qquad (32)$$

The impedance matrix $Z_M$ is synthesized as the series connection of two networks, the impedance matrices of which correspond to each of the terms on the right side of (32).

It is clear that the first terms may be synthesized by a network involving transformers and $\frac{n}{2}$ gyrators. That the second terms can be synthesized using transformers and at most $\frac{n-2}{2}$ gyrators (so that the total number of gyrators used will be n-1) will follow if $H_1 J^{-1} H_1'$ has rank bounded above by (n-2).

Noting that the first column of $H_1$ and row of $H_1'$ are zero, it follows that

$$H_1 J^{-1} H_1' = H_1 [0_1 \dotplus I_{n-1}] J^{-1} [0_1 \dotplus I_{n-1}] H_1' \tag{33}$$

which may be written as

$$H_1 J^{-1} H_1' = H_1 K H_1' \tag{34}$$

where

$$K = [0_1 \dotplus I_{n-1}] J^{-1} [0_1 \dotplus I_{n-1}] \tag{35}$$

The matrix K is skew, must have even rank, see [12], and by (35), must have rank less than $J^{-1}$, that is, less than n. Consequently, the rank of K is bounded above by (n-2), and from (34), the same is true of $H_1 J^{-1} H_1'$.

Evidently, this result agrees with that obtained by Oono and Yasuura [1, p. 163].

## V. Conclusions

There is little to conclude concerning the method itself, save to point to its existence. An obvious direction for future research is to extend the material presented in some way to provide a minimal gyrator synthesis, or, less ambitiously, to indicate a stronger bound on the minimal number of gyrators. For nonlossless synthesis the Bayard synthesis [13] also seems to hold some promise for using the minimum number of gyrators.

Little comment in the text has been made concerning obtaining an original realization $\{F, G, H, J\}$ on which to perform the transformations yielding the network structure. An original minimal realization is, however, readily found from the algebraic procedure of Ho [14], which in fact should allow for computer implementation of the procedure.

## VI. Acknowledgment

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## Figure Captions

Figure 1. Basic Network Representation

Figure 2. Multiport Transformer

Figure 3. Hybrid Matrix Realization

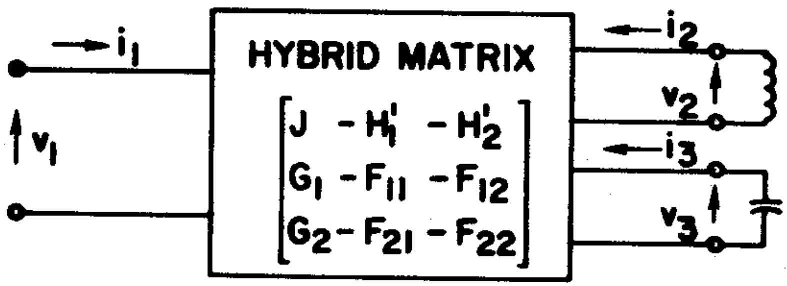


Figure 1.    Basic Network Representation

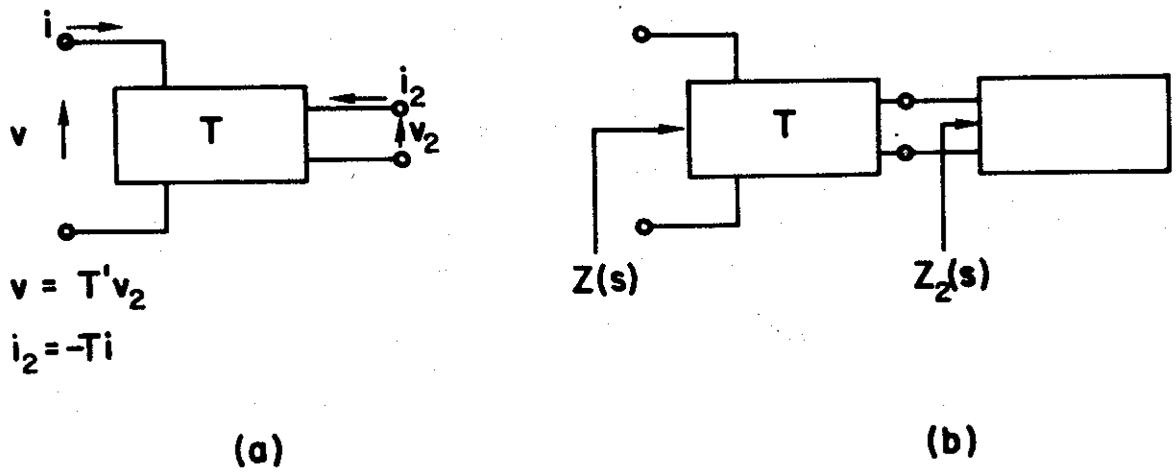


Figure 2. Multiport Transformer

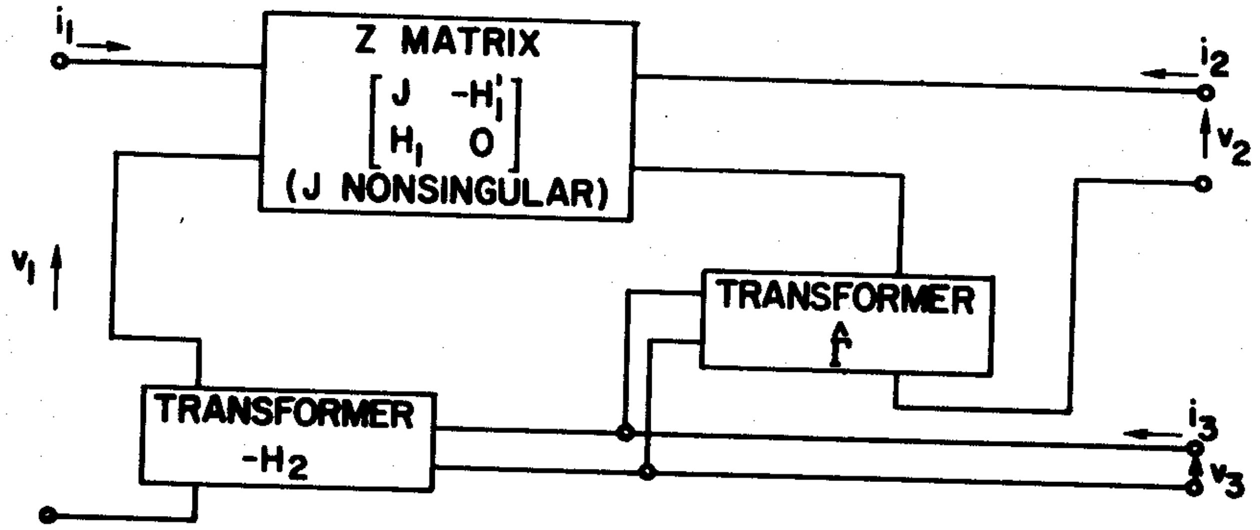


Figure 3. Hybrid Matrix Realization