ANCHORED BLIND EQUALIZATION

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Abstract

Blind equalization up to a constant gain of linear time-invariant channels is studied. Dropping the requirement of gain identification results in the elimination of a degree of freedom that causes ill-convergence of conventional blind equalizers, and affords the possibility of using simple update rules based on the stochastic approximation of output energy. Nonrecursive infinite-dimensional equalizers based on that approach exhibit global convergence for minimum-phase channels. Unlike conventional blind equalizers, truncations of those equalizers inherit the convergence properties of their infinitely parametrized counterparts. Exact finite-dimensional blind equalization with global convergence is shown for the first time. The equalizer converges to the minimum-phase part of the channel, and its coefficients can be used to identify the non-minimum-phase part of the channel.

1. Introduction

Finite-dimensional discrete-time linear time-invariant systems are popular models for digital communication channels that introduce intersymbol interference. Often intersymbol interference is removed prior to data demodulation by means of an equalizer --a linear time-invariant system whose transfer function is equal to the inverse of the channel transfer function. If the receiver does not know the actual transfer function of the channel, the need arises for an adaptive equalizer which is updated using the channel outputs. In addition, classical adaptive equalization methods [12] rely on the input being a training sequence of data which is known by the receiver.

The objective of *blind equalization* is to drop the requirement of a training sequence which in many applications (such as multiuser channels) is too cumbersome to be realistic. Thus a blind equalizer has access to the output, but not the input, of the channel.

Blind equalization imposes a specific structure on the adaptive scheme so that it can be easily implemented: the equalizer coefficients are updated according to a stochastic approximation scheme governed by a *cost function* that satisfies the following properties:

C1. it depends on the input data and the unknown channel only through the channel output.

C2. the local minima of its expected value (as a function of the equalizer coefficients) occur at systems which differ from the channel inverse transfer function by at most an arbitrary delay and a change of sign.

Because of the practical importance and conceptual interest of blind equalization, the search for an admissible cost function has attracted the attention of a number of researchers during the last fifteen years. Admissible cost functions have been found for doubly-infinite transversal equalizers [2, 5, 8, 15]. A shortcoming of those solutions is that finite-dimensional (realizable) approximations of those blind equalizers exhibiting global convergence have not been reported. Even within the domain of relatively simple classes of minimum-phase channels (e.g. first-order autoregressive), implementable blind equalization remains an open problem (see [10] for a lucid up-to-date account of the main efforts and the fundamental issues in this open problem.) In this work we study finite-dimensional implementable blind equalizers which achieve the inverse of the channel transfer function exactly. Our starting point is the observation (made previously in [18]) that condition C2 for the admissibility of the cost function is unnecessarily restrictive. Indeed, because of the symmetry of the alphabet $\{+1, -1\}$, it is irrelevant whether the data is recovered exactly or the equalizer introduces an arbitrary constant gain, even if that gain is a priori unknown. Therefore, condition 2 is replaced by

C2′. The local minima of the expected value of the cost function occur at systems which differ from the channel inverse transfer function by at most an arbitrary delay and an arbitrary gain factor. Those systems will be referred to in the sequel as *valid equalizers*.

This less restrictive condition enables us to avoid the over-parametrization of the blind equalizer which is at the root of

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the inability of previous efforts to find universal globally convergent solutions. In turn, our lower dimensional parametrization of the equalizer results in the admissibility of a very simple cost function.

2. Blind Equalization of Autoregressive Channels

Assume that the channel is described by the difference equation:

$$r_k = \sum_{i=1}^{N} a_i\, r_{k-i} + G\, x_k \tag{1}$$

where $\{x_k\}$ is the data sequence, $\{r_k\}$ is the channel output sequence, and the receiver knows that the channel is autoregressive and the value of N. If the receiver knew the channel coefficients, $(a_1, \cdots a_N)$ (a scaled version of) the input data could be simply recovered with an FIR equalizer:

$$y_k = r_k - \sum_{i=1}^{N} a_i\, r_{k-i} \tag{2}$$

In the absence of such knowledge, the output of an infinitely long nonrecursive equalizer

$$y_k = \sum_{i=0}^{\infty} \lambda_i\, r_{k-i} \tag{3}$$

will asymptotically coincide with $\pm\{x_k\}$ if the equalizer is updated according to the Godard algorithm [8] or the Shalvi-Weinstein algorithm [15]. Even though those algorithms assume a doubly infinite equalizer, in this case a causal equalizer is sufficient because the invertibility condition of [4] is satisfied for an autoregressive channel. No such convergence property has been shown for any realizable (i.e., finitely parametrized) blind equalizer. In particular, it has been shown in [4] that the FIR equalizer

$$y_k = \sum_{i=0}^{N} \lambda_i\, r_{k-i} \tag{4}$$

may end up converging to local minima instead of the desired solution $\pm\frac{1}{G}[1, -a_1, \ldots -a_N]$. For example if $N = 1$, $a_1 = -\alpha$ and $G = 1$, the Godard equalizer has four local minima located at [3]

$$\pm[1 \;\; -\alpha]$$

$$\pm[0 \;\; \sqrt{1-\alpha^4}/\sqrt{1+5\alpha^2}]$$

This behavior is a consequence of the overparametrization of the equalizer brought about by the requirement of condition C2. However, as we argued in Section 1, condition C2' is all we really need and the natural choice suggested by (2) is to fix the first equalizer coefficient to 1 and set the equalizer structure

$$y_k = r_k - \sum_{i=1}^{N} \lambda_i\, r_{k-i} \tag{5}$$

We now propose simple algorithm for updating $(\lambda_1 \ldots \lambda_N)$ based on the observation of $\{r_k\}$ which converges to $(a_1 \ldots a_N)$ as long as the channel is stable. In order to introduce our proposed cost function note that the combined transfer function of the cascade of channel and equalizer is equal to

$$G\,\frac{1 - \lambda_1 z^{-1} - \ldots - \lambda_N z^{-N}}{1 - a_1 z^{-1} - \ldots - a_N z^{-N}} = G\,(1 + \gamma_1 z^{-1} + \gamma_1 z^{-2} + \ldots)$$

and therefore the energy of the output is (since the input is iid) equal to

$$G^2 + G^2 \sum_{i=1}^{\infty} \gamma_i^2 \geq G^2$$

with equality if and only if $(\lambda_1, \ldots \lambda_N) = (a_1, \ldots a_N)$. Therefore, the output energy has a global minimum when the equalizer is equal to the inverse of the channel.

This motivates the choice of the cost function as proportional to the instantaneous value of the output energy: $\frac{1}{2} y_k^2$. Notice that this cost function cannot be used with the classical approach to blind equalization where all FIR coefficients are degrees of freedom, as the global minimum would correspond to an equalizer with zero transfer function. The gradient of the cost function with respect to the equalizer coefficients: $\boldsymbol{\lambda} = [\lambda_1, \ldots \lambda_N]^T$ is

$$\nabla \tfrac{1}{2} y_k^2 = -y_k \mathbf{r}_{k-1} \tag{6}$$

where $\mathbf{r}_{k-1} = [r_{k-1}\; r_{k-2} \cdots r_{k-N}]^T$, and the updating of the equalizer coefficients proceeds according to

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \mu y_k \mathbf{r}_{k-1} \tag{7}$$

where μ is the algorithm step size. It remains to check whether the proposed cost function satisfies condition C2'. To that end, note that the cost function is a quadratic form of the vector of equalizer coefficients with the Toeplitz matrix $\mathbf{R}$ whose (i,j) entry is $\sum_{l=0}^{\infty} h_l\, h_{l+|i-j|}$ where $\{h_0, h_1, \cdots\}$ is the channel impulse response; therefore for any N-vector $\mathbf{x}$ the quadratic form $\mathbf{x}^T \mathbf{R}\, \mathbf{x}$ is equal to the energy of the output of the channel when driven by a finite duration sequence $\cdots 0\; x_1, \cdots x_N, 0 \cdots$. If the finite length input is not identically zero, i.e., $\mathbf{x} \neq (0, \cdots 0)$ then the output sequence cannot be identically zero because the channel is causal. The conclusion is that the cost function is strictly convex in the equalizer coeffients and, thus, the equalizer in (5) updated according to (7) will converge to the inverse of the autoregressive channel regardless of the initial setting of coefficients.

3. Blind Equalization of ARMA Channels

We consider in this section the general case of a finite-dimensional linear time-invariant channel where

$$r_k = \sum_{i=1}^{N} a_i\, r_{k-i} + G\,[x_k + \sum_{i=1}^{L} b_i\, x_{k-i}] \tag{8}$$

If $L \geq 1$, the channel cannot be equalized exactly by a finite-dimensional tranversal filter. We will explore two approaches for the blind equalization of channels with zeros: (1) infinite-dimensional nonrecursive equalization (as in previous works) and (2) finite-dimensional recursive equalization.

3.1. Nonrecursive blind equalization of ARMA channels

Let us assess the feasibility of extending the approach taken in Section 2 to a nonrecursive equalizer with an infinite number of taps. For that purpose, we will consider the case where the channel inverse is causal and stable, i.e., the case where the channel is minimum-phase. Now, the equalizer in (5) is substituted by

$$y_k = r_k - \sum_{i=1}^{\infty} \lambda_i\, r_{k-i} \tag{11}$$

Let us analyze the equalizer in (11) with the updating rule in (7) (the vectors therein are now semiinfinite). The expected output energy is equal to the energy of the impulse response of the cascade of channel and equalizer which can be written as

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \lambda_i \lambda_j\, R(j-i) \tag{10}$$

where $R(l)$ is the autocorrelation of the channel impulse response, and by convention, $\lambda_0 = -1$. Notice that the cost function is a *convex* function of the equalizer coefficients $(\lambda_1\, \lambda_2, \cdots)$ and therefore any local minimum has to be a global minimum. Moreover, since the leading coefficient in the equalizer impulse response is fixed to 1, the leading coefficient in the combined response is equal to that of the channel for every $(\lambda_1, \lambda_2, \cdots)$. The minimum-energy combined response is achieved if and only if all the other coefficients are equal to zero, i.e., only if the equalizer is equal to the channel inverse (modulo gain). But since the channel is minimum-phase there is one and only one causal stable realization of the channel inverse. We note that this approach falls short of ensuring convergence to a valid equalizer if the channel does not have a causal stable inverse [19].

Let us consider the finite-length truncation of the algorithm presented here. Whereas this has been the caveat emptor of previous blind equalizers (cf. [10]), in our setting it is easy to prove global convergence to an equalizer which approximates the channel inverse up to any prespecified degree of accuracy in those cases where the infinite dimensional equalizer algorithm works (i.e., minimum-phase channels and maximum-phase FIR channels). If the nonrecursive equalizer has $K < \infty$ taps, then the cost function still admits the expression in (10) except that the summations range up to K and, thus, the function is a quadratic form of a finite-dimensional positive-definite matrix. When the leading equalizer coefficient is fixed to 1, then the cost function is strictly convex in the remaining equalizer coefficients and thus it has a unique global minimum. That minimum value will approach the minimum achieved by the infinite-dimensional valid stable equalizer as $K \to \infty$. The reason is that since the valid equalizer is stable, its distance from a K-dimensional truncation can be made as small as desired provided K is large enough. Then, the continuity of the cost function in the neighborhood of the infinite-dimensional global minimum guarantees that the cost function achieved by the K-dimensional truncation can be made as close as desired to the global minimum.

3.2. Recursive (IIR) blind equalization of ARMA channels

Let us consider the recursive equalization of minimum-phase autoregressive moving average channels. Motivated by the foregoing results we propose to equalize the channel in (8) with

$$y_k = -\sum_{i=1}^{L} \theta_i\, y_{k-i} + r_k - \sum_{i=1}^{N} \lambda_i\, r_{k-i} \tag{9}$$

Note that if $\theta^T = [\theta_1 \dots \theta_L] = [b_1 \dots b_L]$ and $\lambda^T = [\lambda_1 \dots \lambda_N] = [a_1 \dots a_N]$ then $y_k = G\, x_k$ for all k. Furthermore, since the leading coefficient in the equalizer impulse response is equal to 1, that choice of equalizer coefficients achieves the unique global minimum of the output energy. As before, the updating of θ and λ will proceed according to a stochastic approximation of the output energy. In order to obtain the gradient of the output energy with respect to λ and θ it is convenient to write (11) as the cascade of its regressive and moving average components:

$$y_k = w_k - \sum_{i=1}^{N} \lambda_i w_{k-i} \tag{12a}$$

$$w_k = -\sum_{i=1}^{L} \theta_i w_{k-i} + r_k \tag{12b}$$

It readily follows from (12a) that the gradient with respect to the numerator coefficients is

$$\frac{1}{2} \nabla_\lambda\, y_k^2 = -\,[w_{k-1} \dots w_{k-N}]^T\, y_k \tag{13}$$

In order to find the gradient with respect to the denominator coefficients let us introduce (cf. (29))

$$d_k = -\sum_{i=1}^{L} \theta_i\, d_{k-i} + y_k$$

from (12) we get

$$\frac{\partial y_k}{\partial \theta_j} = \frac{\partial w_k}{\partial \theta_j} - \sum_{i=1}^{N} \lambda_i \frac{\partial w_{k-i}}{\partial \theta_j} \tag{14a}$$

$$\frac{\partial w_k}{\partial \theta_j} = - \sum_{i=1}^{L} \theta_i \frac{\partial w_{k-i}}{\partial \theta_j} - w_{k-i} \tag{14b}$$

which implies that $\frac{\partial w_k}{\partial \theta_j}$ is the response of the equalizer to $-w_{k-j}$, or equivalently, $\frac{\partial y_k}{\partial \theta_j}$ is equal to $-d_{k-j}$. Thus, the equalizer coefficients are updated according to

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} + \mu \; y_k \; w_{k-j} \qquad j=1,...N \tag{15a}$$

$$\theta_j^{(k+1)} = \theta_j^{(k)} + \mu \; y_k \; d_{k-j} \qquad j=1,...L \tag{15b}$$

The simplicity of the cost function we are using allows us to verify that this equalizer will converge to the channel parameters regardless of its initialization (as long as it is stable) by invoking a result from maximum-likelihood identification of Gaussian linear systems due to Astrom and Soderstrom [1], and Stoica and Soderstrom [16].

Theorem. Consider the cascade of two systems:

$$r_k = \sum_{i=1}^{N} a_i \; r_{k-i} + e_k + \sum_{i=1}^{L} b_i \; e_{k-i} \tag{16}$$

$$y_k = - \sum_{i=1}^{L} \theta_i \; y_{k-i} + r_k - \sum_{i=1}^{N} \lambda_i \; r_{k-i} \tag{17}$$

Assume that $\{e_k\}$ is a second-order iid sequence. If (16) is a minimum-phase stable system without pole-zero cancellations, then the only stationary points of $E[y_k^2]$ as a function of θ and λ occur at the global minima:

$$1 + \theta_1 z^{-1} + \cdots + \theta_L z^{-L} = (1+b_1 z^{-1} + ... + b_L z^{-L})$$

$$1 + \lambda_1 z^{-1} + \cdots + \lambda_N z^{-N} = (1+a_1 z^{-1} + ... + a_N z^{-N})$$

In the case we are considering (16) and (17) become (8) and (11) respectively with $e_k = G \; x_k$, and the result of the Theorem ensures that the recursion in (15) converges to the channel coefficients **a** and **b**, provided that the algorithm is initialized at a stable system and the step size μ is suitably scheduled. Analogous results can be obtained if the equalizer (11) overparametrizes the channel [19]. In any realization of the stochastic approximation in (15), the trajectory followed by $\lambda^{(k)}$ and $\theta^{(k)}$ deviates randomly from its expected trajectory (a line of steepest descent) and in particular may wander outside the set of stable systems, in which case convergence is no longer guaranteed. This is a well-known issue in adaptive IIR filtering [9], and it is usually remedied by discarding updates that would lead to unstable systems or reflecting poles lying outside the unit circle to their reciprocals. To this end, it is customary to parametrize the recursive part of the filter as a cascade of second-order sections, the stability of which is easy to monitor. This alternative realization will turn out to be useful in dealing with nonminimum-phase channels. Let us now investigate how the adaptive updating of the equalizer coefficients works with this new parametrization. Let $M = \lceil \frac{L}{2} \rceil$. The recursive part of the equalizer will be substituted by a cascade of second-order sections

$$\frac{1}{1 + \theta_1 z^{-1} + \cdots + \theta_L z^{-L}} = \prod_{m=1}^{M} \frac{1}{\Theta^m(z^{-1})} \tag{18}$$

where $\Theta^m(z^{-1}) = 1 + \theta_1^m z^{-1} + \theta_2^m z^{-2}$. We will denote the output of the m-th stage by w_k^m (cf. Fig. 1), in particular, $w_k = w_k^M$. The gradient with respect to the numerator coefficients remains intact (cf. (13)). In order to find $\nabla_m y_k$ (the gradient of the output with respect to the coefficients of the m^{th} section) we will denote $\Lambda(z^{-1}) = 1 + \lambda_1 z^{-1} + \cdots + \lambda_L z^{-L}$, and we will use the informal notation

$$w_k^{n-1} = \Theta^n(z^{-1}) \, w_k^n \tag{19}$$

$$y_k = \frac{\Lambda(z^{-1})}{\Theta^{n+1}(z^{-1}) \cdots \Theta^M(z^{-1})} \, w_k^n \tag{20}$$

From (19), it follows that

$$0 = \nabla_n w_k^{n-1} = \begin{bmatrix} z^{-1} \\ z^{-2} \end{bmatrix} w_k^n + \Theta^n(z^{-1}) \, \nabla_n w_k^n$$

or, equivalently,

$$\nabla_n w_k^n = \frac{1}{\Theta^n(z^{-1})} \begin{bmatrix} -w_{k-1}^n \\ -w_{k-2}^n \end{bmatrix} \tag{21}$$

Putting (20) and (21) together we obtain

$$\begin{aligned} \nabla_n y_k &= \frac{\Lambda(z^{-1})}{\Theta^{n+1}(z^{-1}) \cdots \Theta^M(z^{-1})} \nabla_n w_k^n \\ &= \frac{\Lambda(z^{-1})}{\Theta^n(z^{-1}) \cdots \Theta^M(z^{-1})} \begin{bmatrix} -w_{k-1}^n \\ -w_{k-2}^n \end{bmatrix} \\ &= -\frac{1}{\Theta^n(z^{-1})} \begin{bmatrix} y_{k-1} \\ y_{k-2} \end{bmatrix} = - \begin{bmatrix} d_{k-1}^n \\ d_{k-2}^n \end{bmatrix} \end{aligned}$$

where d_k^n is the response of the n^{th} recursive stage to the equalizer output (cf. Fig. 1).

With the new parametrization, there is no longer a unique global minimum, as all permutations of second-order sections are equivalent. However, it does not give rise to spurious local minima. This is because if a given unfactored system is not a local minimum, there is a neighboring system which achieves lower cost. The factored versions of those systems exist (although they are not unique in general), attain the same respective costs and are also neighbors due to the continuity of the roots of a polynomial as a function of its coefficients [13 p. 3]. This implies that the corresponding factored system cannot be a local minimum –a conclusion that is in accordance with the general results on mean-square surfaces for cascaded IIR filters in [14].

Finally, we consider recursive blind equalization of ARMA channels that are not necessarily minimum-phase. Recall that the nonrecursive approach of Section 2 succeded in handling, in addition to minimum-phase channels, only a narrow class of non minimum-phase channels. The first obvious point to make is that (exact) equalization of a nonminimum-phase channel is impossible as it would entail an unstable filter. Even if the channel were known and its inverse could be implemented exactly, any level of receiver input noise would hopelessly garble the equalizer output. A sensible strategy is to equalize the minimum-phase part of the channel, i.e., all its poles plus its zeros inside the unit-circle, and to identify its maximum-phase part, i.e., all the zeros which lie outside the unit-circle. Once those zeros have been identified they can be handled in various ways: e.g. with a maximum likelihood sequence detector, decision-feedback equalizer or with a truncated causal approximation to the noncausal impulse response (cf. Section 2).

Consider a stable ARMA channel without all-pass factors. If we were to use the same recursive equalizer we used for a minimum-phase channel, the equalizer would converge to the inverse of the minimum-phase version of the channel, i.e., it would cancel all the poles plus all the zeros inside the unit circle and it would place a pole $p_i = \frac{1}{z_i}$ for every zero z_i outside the unit circle. The reason for this is simple: the cost function is transparent to replacing the channel with its minimum-phase version. Therefore, in the Astrom-Soderstrom Theorem we can take (16) to be the minimum-phase version of the channel rather than the channel itself. Absence of local minima is guaranteed and the global minimum occurs at the inverse of the system in (16). Note that the derivation of the gradients leading to (15) remains intact as it does not depend on the channel coefficients. So, we can view the equalizer as incorrectly assuming that the channel is minimum-phase. This will not invalidate its global convergence properties; it will simply mean that the equalizer will converge to the inverse of the minimum-phase version of the channel. Therefore, the transfer function of the cascade of channel and equalizer will converge to an all-pass filter whose zeros are those channel zeros lying outside the unit-circle. In this situation, some of the equalizer poles are indeed cancelling channel zeros whereas others are located at the reciprocal of channel zeros. If we could somehow decide for each equalizer pole whether it is indeed cancelling a channel zero or not, then we would have identified the resulting all-pass system. One possible approach to deciding which equalizer poles are cancelling channel zeros is to append at the output of the equalizer a cascade of M second-order all-pass sections such that each section can be bypassed by a switch. The transfer function of each section is

$$\frac{\Theta^m(z^{-1})}{\Theta^m(z)}$$

which admits an anticausal stable representation that can be approximated by a two-parameter FIR. The output of the switched cascade is (approximately) binary valued for one and only one switched configuation (the one that bypasses the sections corresponding to minimum phase zeros). For that configuration, equalization of the channel is achieved. In the presence of channel noise, this is still feasible as shown in [17]. In any case, the main point is that once the finite-dimensional equalizer has converged to the inverse of the minimum-phase part of the channel, then the knowledge of the location of the poles of the equalizer makes the identification of the resulting all-pass system a far easier problem than the conventional identification of an unknown linear system.

4. Summary

Comparing the anchored blind equalizers with energy cost functions and the Godard-type blind equalizers, we conclude that

(a) for doubly-infinite nonrecursive equalization Godard-type cost functions are superior as they achieve global convergence, whereas the anchored equalizers may have inadmissible global minima.

(b) for semi-infinite nonrecursive equalization, both equalizers achieve convergence if the channel is minimum-phase, otherwise, both equalizers have spurious local minima. The reason why the Godard equalizer suffers from this problem is the noninvertibility of the semi-infinite channel convolution matrix [3] in the nonminimum-phase case.

(c) for finitely parametrized equalizers, the truncation of the Godard-type equalizers leads to ill-convergence even for the simplest channels [4], whereas the anchored equalizers based on convex cost functions inherit the convergence properties of their infinite-dimensional counterparts which can be approximated as accurately as desired. Furthermore, in contrast to the Godard-type equalizers, exact finite-dimensional implementable blind equalization is achievable by anchored blind equalizers. We have shown global convergence to a factored form of the inverse of the minimum phase equivalent of the channel from which the channel zeros outside the unit circle can be identified and equalized.

The next step motivated by this paper is the study of anchored equalizers using cost functions other than energy. An equalizer has been identified in [11] which achieves essentially the same convergence properties as the doubly-infinite Godard equalizer, with the advantage that those properties are not destroyed by finite truncations.

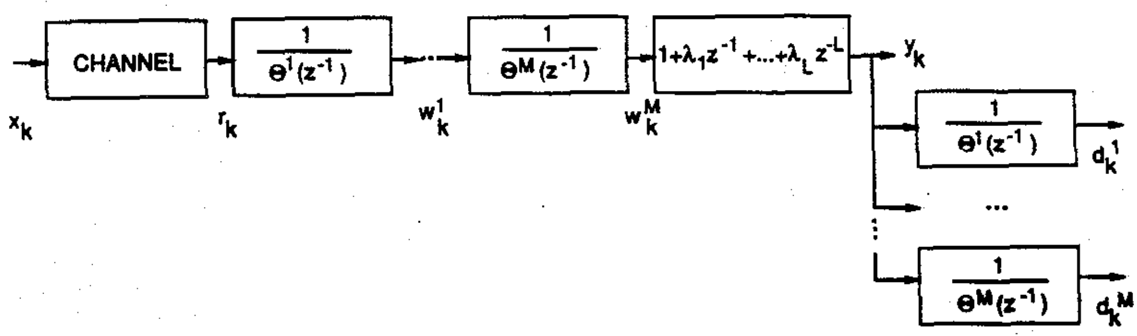

Fig. 1. Recursive blind equalizer with cascaded second-order sections

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