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Adaptive Robust Control: On-Line Learning

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Abstract A method of on-line adaptation and learning is proposed which makes use of a probing signal whose frequency content is concentrated at the bandwidth of the current controller. As the plant is learned the procedure naturally increases the learning bandwidth.

1 Introduction

It is very easy to *construct* an adaptive system: just connect a controller design rule and a model parameter estimator together. This kind of adaptive control system operates along roughly the following lines. A model for the unknown plant is assumed in which everything is known but the values of a finite number of parameters. These parameters have the property that when they are known, the controller can be defined. It too has a finite number of adjustable parameters, the values of which depend on the plant parameters. By observing the plant input and output, the plant parameters are learned and/or tracked, and the controller parameters are then set according to some design rule. Sometimes it is the controller parameters which are learned directly. Certain choices of controller parametrization lends itself to this approach, others do not.

What is absent in this approach is the recognition that the estimated plant parametric model during the learning phase can be a poor representation of the true plant. This mismatch between the plant and the estimated model can cause poor performance via such phenomena as parameter drifting and bursting. All of this has been reported in the literature and under certain conditions has been analyzed and explained, [1], [2].

In this paper we invoke a different design philosophy than that expressed by the previous reasoning. The new reasoning would have to recognize at the outset that the true plant can differ greatly from the estimated model at any one time, particularly during the initial learning stage.

Nature provides examples of this kind of adaptive control, and it seems that many such examples do not exhibit the traditional operating strategy. In particular, consider how humans learn windsurfing, where the human is the adaptive controller. Several observations can be made: (1) The problem has multiple inputs. (2) The human first learns to control over a limited bandwidth, and learning pushes out the bandwidth. (3) The human first implements a low gain controller; and learning causes the loops to be tightened (this is linked with 2). These observations suggest that one could contemplate an adaptive controller based on learning a frequency domain description of the plant, with the learning process pushing out the bandwidth over which the plant was accurately known. For such a concept to be valid and consistent with point 3 above, it would be necessary to demonstrate, at least for a broad class of plants, that a low gain controller can be contemplated for plants with significant uncertainty at high frequencies, and that reduction in the structured uncertainty progressively allow increase of the controller gain and control over an increasing frequency band; this is essentially a linear systems, as opposed to adaptive systems, exercise.

It would also be desirable to show that when the behaviour of the plant over a certain bandwidth had been learned and certain controller gains implemented, it would be natural to apply a probing signal at the upper limit of this bandwidth (perhaps in handling transients) so that the bandwidth of knowledge of the plant was expanded.

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2 Closed-Loop Identification

For the sake of expository simplicity, we shall restrict attention to scalar plants. The following result can be found in one form or another in [9] and the references therein.

Theorem 1 *Suppose that X, Y, N, D are stable transfer functions satisfying*

$$XN + YD = 1 \tag{1}$$

Then:

(i) All controllers C which stabilize the plant $P = N/D$ are in the set of transfer functions ,

$$\left\{\frac{X+QD}{Y-QN} \ : \ Q \ \text{stable}\right\} \tag{2}$$

(ii) All plants P stabilized by the controller $C = X/Y$ are in the set of transfer functions ,

$$\left\{\frac{N+RY}{D-RX} \ : \ R \ \text{stable}\right\} \tag{3}$$

Since all rational transfer functions can be expressed as a ratio of stable transfer functions , it follows that part (i) gives a parametrization of all stabilizing rational controllers of rational plants.

Statement (ii), which follows directly from (i) by interchanging the plant and controller, was developed in [3, 4] for use in closed-loop identification for the problem of experiment design. Similar results are also in [8]. In this paper we also utilize this result, but for a slightly different purpose.

Consider the feedback system,

$$\begin{aligned} y &= Gu + He \qquad (4)\\ u &= K_0(r_1 - y) + r_2 \qquad (5)\end{aligned}$$

where (y, u) are the measured output and control input, respectively, e is an unpredictable disturbance, and (r_1, r_2) are user applied inputs. It is assumed that K_0 is a stabilizing feedback compensator. This implies some knowlwdge of G, but otherwise G and H are assumed unknown. The *plant* is the pair (G, H) where G is possibly unstable and, as is standard, H and H^{-1} are stable [6]. The identification problem is to obtain estimates of (G, H) from a finite set of measured and known data $\{y, u, r_1, r_2 \ : \ 0 \leq t \leq T\}$. Following identification, the controller is to be re-designed to improve performance of the closed-loop system.

Stable Plant Let us consider the special case when the plant G is stable. Suppose also that G_0 is stable and that K_0 stabilizes G_0. Then, by Theorem 1, it can be shown that K_0 stabilizes G iff there exists a stable R and stable mini-phase S, such that

$$G = G_0 + \frac{R}{1 - RQ_0}, \quad H = \frac{S}{1 - RQ_0} \tag{6}$$

where

$$Q_0 = \frac{K_0}{1 + G_0 K_0} \tag{7}$$

Again, an interpretation is that K_0 stabilizes all plants in the set

$$\left\{ G_0 + \frac{R}{1 - RQ_0} \ : \ R \text{ stable} \right\} \tag{8}$$

As result, identification of (G, H) in closed-loop is equivalent to identification of the stable open-loop (R, S)-system,

$$\beta = R\alpha + Se \tag{9}$$

where β, α are given by

$$\beta = y - G_0 u \tag{10}$$

$$\alpha = Q_0 r_1 + (1 - Q_0 G_0) r_2 \tag{11}$$

Observe that (α, β) depend on measured and applied signals (y, u, r_1, r_2) operated on by known stable systems (G_0, Q_0).

Example To further motivate identifying the (R, S)-system, consider the following example:

$$G = \frac{9}{(s+1)(s^2 + .06s + 9)}$$

$$G_0 = \frac{1}{s+1}$$

$$Q_0 = \frac{4(s+1)}{(s+2)^2}$$

Figure 1 shows the magnitude of R and $G - G_0$ vs. frequency. These are very close showing that identification of R is close to identification of the model error $G - G_0$.

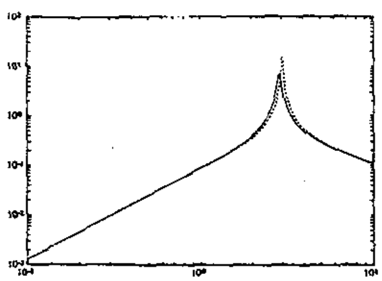

Figure 1: Magnitude plots of R and $G - G_0$ vs. frequency.

Thus, we are led to the following iterative identification algorithm for stable plants in closed-loop. A similar formulation is available for the general case where the plant is possibly unstable.

Initialize: $\hat{G} = G_{00}, \ \hat{Q} = Q_{00} = \dfrac{K_{00}}{1 + G_{00} K_{00}}$

Update $G_0 = \hat{G}, \ Q_0 = \hat{Q}, \ K_0 = \dfrac{Q_0}{1 - Q_0 G_0}$

Identification input: $u = K_0(r_1 - y) + r_2$

R – Update $\hat{R} = \arg\min_R \| y - G_0 u - R(Q_0 r_1 + (1 - Q_0 G_0) r_2) \|$

G – Update $\hat{G} = G_0 + \dfrac{\hat{R}}{1 - \hat{R} Q_0}$

ControllerDesign $\hat{Q} = \arg\min_Q \| H_{\text{desired}} - \hat{G} Q \|$

Repeat

Although we can not offer any proof at this time, we believe that this iterative procedure provides a natural approach to learning by gradually increasing the bandwidth of the controller. The essential features fall out of the fractional representation theory, in particular via the transformation from the (G, H) system in closed-loop to the (R, S)-system in open-loop, and subsequent identification of the (R, S) system to obtain estimates of (G, H).

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