Proceedings of the 29th Conference
on Decision and Control
Honolulu, Hawaii • December 1990

TA-13-1 - 9:20

H_∞ optimal controller discretization

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Abstract

A redesign method for discretizing a continuous-time controller is proposed. The resulting hybrid control system, e.g. with continuous-time plant and discrete-time controller, is stable, and performance including the system's intersampling behaviour can be optimized by approximating some chosen reference transfer function of the continuous-time control system. In order to obtain a tractable problem, the continuous-time part of the hybrid system and the reference transfer function are approximated by a discrete-time system with arbitrary fast sampling. After lifting the resulting periodic system, the approximation problem can be formulated as a standard H_∞-problem which is solved using standard software for H_∞ controller design.

1 Introduction

Since most controller design procedures allow direct discrete-time controller design, the question arises: is there any need for a controller discretization method? Several considerations suggest that it might be reasonable to design first a continuous-time controller, which is afterwards converted into a discrete-time one. Direct design of discrete-time controllers requires a predetermination of the sampling time in order to model plant, noise, process disturbances etc. Upper bounds for the sampling time can only be obtained after considering closed-loop bandwith (Franklin & Powell, [2], ch.10,) and, depending on specifications, this may only be available *after* controller design. Because this obvious dilemma does not occur in continuous-time controller design, it may be reasonable to design first a continuous-time controller, which is in a second step approximated by a discrete-time one. A second reason is that a direct discrete-time controller design only considers the system's behaviour at the sampling instants. Problems of intersampling behaviour can be solved when a continuous-time controller is first formed and then discretized, because the closed-loop properies of the desired controller design are known. A third reason for first executing a continuous-time design is that physical insight is more easily retained.

Before discussing existing discretisation methods a list of requirements for controller discretization is proposed. First of all the very basic property of closed-loop stability should be guaranteed. In most applications we are also interested in the performance of the resulting control system including its intersampling behaviour. In addition the discretization method should provide the possibility of including a possible time delay in the control law required for controller realisation. Furthermore the determination of the discrete-time controller should be a numerically tractable problem to be able to discretize also large scale systems.

An overview of existing discretization methods is given in Hanselmann [3]. The isolated or open-loop methods, for instance bilinear transformation, hold input approximation and signal invariant transformations, do not take the closed loop use of the controller into account and can therefore guarantee neither stability nor performance. Discretizations are easy to calculate but need an excessively small sampling time to perform well, (in comparison to the sampling time when other discretization methods are used). Problems of these discretization methods when used for controller controller design with relatively large sampling time were reported by Katz [4]. Methods for discrete-time implementation of linear state-feedback laws are proposed by Kuo & Peterson [9], Yackel et al. [13] and Kleinman & Rao [8]. A closed-loop redesign method is proposed by Rattan & Yeh [10] and Rattan [11], but none of these approximation methods can guarantee the elementary property of closed-loop stability after controller discretization. Kennedy & Evans [5] propose a controller redesign based on pole-zero matching of the complementary sensitivity function. Closed-loop stability and performance at the sampling instants can be achieved but intersampling behaviour is unaddressed. An extension to multivariable systems, which might not be straightforward, is not given. In Keller & Anderson [6] the discretisation problem is posed as a controller approximation problem, using ideas from Anderson & Yi [1]. The resulting controller leads to closed-loop stability but performance optimisation was not included.

The drawback of all these methods is that they do not use the knowledge of the class of all controllers stabilizing the discrete-time plant i.e. the hold-input discretization of the plant. It is clear that stability of the discrete-time system is necessary for stability of the hybrid system, i.e. the continuous-time plant and the discrete-time controller. Given the class of stabilizing controllers a particular discrete-time controller can be determined by approximating (possibly optimally) some selected mapping associated with the continuous-time system, which is chosen to represent the system's performance, e.g. controller redesign. As an error measure, the L_2-induced operator norm of the difference between the two mappings is chosen. Now because of the mixture of continuous and discrete-time entities, some of the mappings in the hybrid system are operators which have no transfer-function representation. In order to get a tractable problem, the continuous-time part of the hybrid system is approximated by a discrete-time system with arbitray fast sampling. Also the selected reference transfer function is discretised with respect to the same sampling period. This can be done in a chosen (sensible) frequency range as accurately as desired by hold-input discretisation. In this approximation the former hybrid system is replaced by a N-periodic system, if the small (fast) sampling time is chosen to be submultiple N of the controller sampling time. By lifting the N-periodic control system a time-invariant transfer-function representation is obtained. The approximation problem can then be rewritten as a standard H_∞-problem with exogenous and endogenous inputs and outputs.

In Section 2 of this study the discretization problem is formulated. In Section 3 the discrete-time approximation with respect to a relatively small sampling time is proposed. The lifting operation is exhibited and the standard H_∞-problem is derived. It is shown that the class of stabilizing controllers of the fast sampled system is equivalent to the class of stabilising controllers of the low sampling rate discrete-time plant. How additional controller delay may be included in the controller determination is shown in Section 4. The proposed approximation of hybrid control systems offers also a method for direct discrete-time controller design. This is discussed in Section 5. In Section 6 an example is presented and Section 7 contains concluding remarks.

2 Problem formulation

Let $P(s)$ be the plant transfer function, $C(s)$ the known continuous-time controller, $C_l(z)$ the unknown discrete-time controller operating at sampling time τ_l and let $H(s)$ represent a zero-order hold. As a measure of performance of the control system, the complementary sensitivity function $T(s) = PC(I+PC)^{-1}$ is chosen. Let T_h be the operator of the hybrid system corresponding to $T(s)$ and let $W(s)$ be a stable, strict proper transfer function shaping the reference signal $r(s)$. The following criterion for performance deterioration is proposed:

$$J_h \hat{=} (T_h - T)W \tag{2.1}$$

A corresponding block diagram is drawn in Figure 2.1

Let S_h be the set of controllers stabilizing T_h and S_l the set, stabilising the discrete-time plant, i.e. the hold-input discretization of $P(s)$ with respect to the controller sampling time τ_l as already defined in the introduction. A discrete-time controller C_l can be determined through the following redesign problem:

$$C_l = arg \min_{C_l \in S_h} \|J_h\| \tag{2.2}$$

where $\| \;\; \|$ is the L_2-norm induced operator norm.

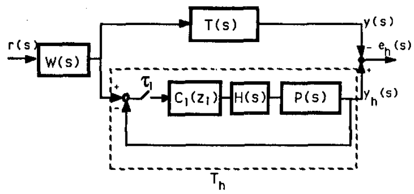

Figure 2.1: Representation of J_h

3 Approximation of J_h and transformation into an H_∞ problem

The optimization problem (2.2) contains the operator T_h which has no transfer function representation; therefore it can not be easily solved. In order to obtain a more tractable problem the criterion J_h is modified. The continuous-time transfer function $T(s)$ and the operator T_h are replaced by their hold-input discretization with respect to an arbitrarily small sampling time τ_s. τ_s is chosen to be a submultiple of τ_l, the sampling time of the final control system.

$$N\tau_s = \tau_l \tag{3.1}$$

Also the weighting $W(s)$ is replaced by a discrete-time one.

Because the resulting system has two different time scales, the notation must be clarified. Discrete-time operators mapping sequences with timescale $k\tau_s$ are denoted with supscript s, the subscript l is used for the time scale $k\tau_l$ and the subscript h denotes hybrid systems and their outputs.

Following the lines of the proof of Prop 3.1 in Keller & Anderson [6] it can be shown, that the modified error measure $\|J_{h,s}\|$ with

$$J_{h,s} \hat{=} (T_{h,s} - T_s(z_s))W_s(z_s) \quad z_s = e^{s\tau_s} \tag{3.2}$$

converges to $\|J_h\|$ as τ_s goes to zero. This convergence is more of theoretical interest, because in applications intersampling behaviour might be sufficiently characterised even for N, e.g. the number of points between the sampling instants, being relatively small.

$T_s(z_s)$ and $W_s(z_s)$ are easy to calculate, whereas the determination of $T_{h,s}$ is not obvious. The operator $T_h : y_h = T_h r$ can be written as

$$y_h(s) = PH(s)C_l(z_l)(I + P_l(z_l)C_l(z_l))^{-1}r_l^*(s) \tag{3.3}$$

with
$z_l = e^{s\tau_l}$
$r_l^* = \sum_{k=-\infty}^{\infty} r(s - jk\omega_l), (\omega_l = \frac{2pi}{\tau_l})$: sampled input at sampling time τ_l
$P_l(z_l)$: hold-input discretization of $P(s)$ with respect to τ_l

Because of the sampling device at the input of T_h, the hold-input discretisation of T_h is simply the sampled output $y_s^*(s)$. With $u_l(z_l) \hat{=} C_l(I + P_lC_l)^{-1}r_l^*(s)$ there is:

$$y_h(s) = PH(s)u_l(z_l) \tag{3.4}$$

Since $u_l(z_l)$ is already ω_s-periodic, i.e. even ω_l-periodic, there is (see for example Franklin & Powell [2]):

$$\begin{aligned} y_{h,s}(z_s) &= (PH)_s^*(s)u_s^*(z_l = e^{s\tau_l}) = (PH)_s^*(s)u_l(z_l) \\ &= PH_s(k, z_s)u_l(z_l) \quad k = 0, \cdots, N-1 \end{aligned} \tag{3.5}$$

The resulting hold-input discretization of T_h is an N-periodic system. N-periodic systems can by means of lifting be represented by time-invariant transfer function matrices in the variable z_l. These transfer function matrices operate on blocks of N successive inputs and are producing a block of N successive outputs. A more detailed description of the lifting operation and a well known state space representation of the resulting transfer functions can be found in connection with a similar application in Keller & Anderson [6]. The lifting operation is an isomorphism from $l_2[0,\infty)$ to $l_2[0,\infty)$ and does therefore not alter $\|J_{h,s}\|$. In drawing the approximation of the system in Figure 2.1 in terms of the lifted transfer functions, we denote with superscript $\tilde{}$ lifted operators. Subscripts are used, (only for lifted operators), if the notation becomes ambiguous. In the lifted representation a transfer matrix of a sampler $\tilde{S}$ can be chosen to be a row vector of length N having 1 as first element and zero on other entries elsewhere and the corresponding hold element $\tilde{H}$ (controller without time-delay) is a column vector of ones. (In the multivariable case, the ones are replaced by identity matrices, the zeros by matrices with zero entries, both of appropriate dimensions).

$$\tilde{H} \hat{=} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad \tilde{S} \hat{=} [1\, 0 \cdots 0] \tag{3.6}$$

With this (simplest) selection of $\tilde{S}$ and $\tilde{H}$, the sampling instant is being chosen to coincide with the time of the first element of the block representation. Other selections are possible but, as it can be shown, lead to the same solution for C_l.

The lifted transfer function of $PH_s(k, z_s)$, i.e. the hold element followed by $P_s(z_s)$, is:

$$\tilde{PH}(k, z_s) = \tilde{P}(z_l)\tilde{H} \tag{3.7}$$

There is a connection between the hold-input discretization $P_l(z_l)$ and an N-block representation $\tilde{P}(z_l)$ of the hold-input discretization $P_s(z_s)$. Since $P_l(z_l)$ is a hold-input discretization it can be represented with respect to z_s as a sequence of a hold element, $P_s(z_s)$ and a sampler, sampling every N^{th} signal.

$$P_l(z_l) = \tilde{S}\tilde{P}(z_l)\tilde{H} \tag{3.8}$$

The lifted operator $\tilde{J}$, shown in Figure 3.1, is now representable as a transfer function matrix:

$$\tilde{J} = \left(\tilde{P}\tilde{H}C_l(I + \tilde{S}\tilde{P}\tilde{H}C_l)^{-1}\tilde{S} - \tilde{T}\right)\tilde{W} \tag{3.9}$$

In this expression all transfer functions are functions in the variable z_l. Furthermore $\tilde{J}$ can be written as a linear fraction:

$$\tilde{J} = \mathcal{F}l\{\tilde{G}, C_l\}, \quad \tilde{G} = \begin{bmatrix} -\tilde{T}\tilde{W} & \tilde{P}\tilde{H} \\ \tilde{S}\tilde{W} & -\tilde{S}\tilde{P}\tilde{H} \end{bmatrix} \tag{3.10}$$

with

$$\mathcal{F}l\{\begin{bmatrix} \tilde{G}_{11} & \tilde{G}_{12} \\ \tilde{G}_{21} & \tilde{G}_{22} \end{bmatrix}, K\} \hat{=} \tilde{G}_{11} + \tilde{G}_{12}K(I - \tilde{G}_{22}K)^{-1}\tilde{G}_{21} \tag{3.11}$$

Since $\|J_{h,s}\| = \|\tilde{J}\| = \|\tilde{J}\|_\infty$ the controller C_l is found by solving the following standard H_∞-minimization problem:

$$C_l = arg \min_{C_l \in S_h} \|\tilde{J}\| \tag{3.12}$$

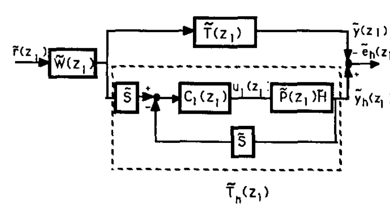

Figure 3.1: System, represented with lifted transfer functions

The H_∞-problem is of the 3-rd kind ($N > 1$). For solvability, we require no zeros of $\tilde{G}_{12}$ and $\tilde{G}_{21}$ on the unit circle, together with a stabilizability property. Zeros of $\tilde{G}_{21}$ in (3.10) on the unit circle can be avoided through a sensible selection of $W(s)$. Because of the discretization of $P(s)$ the zeros $\tilde{G}_{12}$ in (3.10) are difficult to predict. Since the transfer matrix $\tilde{P}$ is multiplied by $\tilde{H}$ from the right, possible zeros of $\tilde{P}$ at $z = 1$ ($P(s)$ strict proper) generically disappear for N sufficiently large, i.e. larger then the order of P.

Several interesting conclusions can be drawn from (3.10). The system in Figure 3.1 is stabilizable if $\tilde{G}$ in (3.10) is stabilizable with the inputs and outputs of $\tilde{G}_{22}$ (see for example Francis [2]). Since $\tilde{T}$ and $\tilde{W}$ are assumed to be stable, this condition boils down to $\tilde{S}\tilde{P}\tilde{H}$ beeing stabilizable. Because $\tilde{S}\tilde{P}\tilde{H} = P_l(z_l)$, this condition is equivalent to the well known stabilizability condition for a direct discrete-time controller design. Furthermore, the set of controllers stabilising the fast sampled system represented in Figure 3.1 is the set stabilising $\tilde{S}\tilde{P}\tilde{H}$. Since $\tilde{S}\tilde{P}\tilde{H} = P_l(z_l)$ this class is independent of N. Letting $N \to \infty$ (or $\tau_s \to 0$) the discrete-time system approaches the continuous-time one. Since the class of stabilizing controllers is independent of N, one can conclude that $\mathcal{S}_h = \mathcal{S}_l$.

4 Controllers with time delay

The realisation of controllers usually does not allow an immediate change of actuator values after the input is sampled. The computation of the new values or a measurement in a process control loop may cause a time delay, which should be incorporated in the controller design. The time delay may often be much smaller then a sampling period. Including a whole sampling period of delay may unnecessarily reduce the system's perfomance. Within the proposed controller redesign procedure, time delays, which are a multiple of the small sampling time τ_s, can easly be taken into account. Consider a time delay L of length $i\tau_s$ with transfer function z_s^{-i}. In order to insert a time-delay in the controller design, we have to calculate its lifted representation. Using the transfer matrix representation of a lifted transfer function presented in Khargonekar et al [7] there results for a scalar system:

$$\bar{L}(z_l) = \begin{bmatrix} 0_{i\times(N-i)} & z_l^{-1}I_{i\times i} \\ I_{(N-i)\times(N-i)} & 0_{(N-i)\times i} \end{bmatrix} \quad (4.1)$$

This block representation shows obviously, how the delay z_s^{-i} affects a block of N successive input elements. The first i elements of the output of the lifted delay are the i last elements of the last input block, e.g. delayed by z_l^{-1}. The remaining $N - i$ elements of the output are the first $N - i$ elements of the current lifted input vector shifted i elements down because of the delay. A similar representation can be obtained for the multivariable case with the additional possibility of different delays in the channels. This may be useful, if there are different delays in measurements. The lifted transfer function can now be inserted in the control loop either before the sampler $\tilde{S}$ or after the hold element $\tilde{H}$. In the block representation these are the only positions where it is possible to introduce a delay of less than the sampling time τ_l. $\bar{L}$ can be incorporated either in $\tilde{S}$ or $\tilde{H}$ to obtain the H_∞-problem (3.10).

5 Direct discrete-time controller design with continuous-time controller spezifications

Using the approximation idea, presented in this study, it is also possible to perform a direct discrete-time controller design for a continuous-time plant using H_∞ ideas. Figure 5.1 represents a very general problem formulation. If this hybrid system is, as before, approximated by a fast sampled, N-periodic system, followed by lifting, the following H_∞ design problem follows:

$$C_l = arg \min_{C_l \in \mathcal{S}_l} \|\tilde{J}\|_\infty \quad (5.1)$$

$$\tilde{J} = \mathcal{F}l\{\tilde{G}, C_l\}, \quad \tilde{G} = \begin{bmatrix} \tilde{W}_3\tilde{W}_4 & 0 & \vdots\; \tilde{W}_3\tilde{P}\tilde{H} \\ \tilde{W}_1\tilde{W}_4 & \tilde{W}_1\tilde{W}_2 & \vdots\; \tilde{W}_1\tilde{P}\tilde{H} \\ \cdots & \cdots & \cdots \\ -\tilde{S}\tilde{W}_4 & \tilde{S}\tilde{W}_2 & \vdots\; -\tilde{S}\tilde{P}\tilde{H} \end{bmatrix} \quad (5.2)$$

in which W_2 and W_4 have to be strictly proper

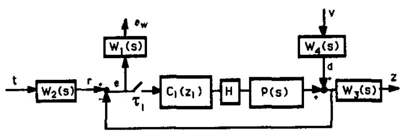

Figure 5.1: General H_∞ problem formulation

For the formal H_∞ setting, it can be shown that:

$$\mathcal{F}l\{\tilde{G}, \tilde{H}C_l\tilde{S}\} = \mathcal{F}l\{\hat{G}, C_l\} \quad \hat{G} = \begin{bmatrix} \tilde{G}_{11} & \tilde{G}_{21}\tilde{H} \\ \tilde{S}\tilde{G}_{21} & \tilde{S}\tilde{G}_{22}\tilde{H} \end{bmatrix} \quad (5.3)$$

Although intersampling behaviour of the control system is reflected through (5.2) into the direct discrete-time controller design, the problem of sampling-time selection still remains. A possible determination of the sampling time could result from a further investigation of the properties of lifted transfer functions. The idea is to choose the small sampling time τ_s reasonably small and increase N as far as the performance deterioration remains accetable. Based on bounds of the H_∞-problem (5.2) it might be possible to estimate the performance reduction without solving the H_∞ problem. These bounds could provide a measure for the determination of N and therefore also the controller sampling time τ_l.

6 Example

A discrete-time controller was determined for the H_∞-optimal controller of a large space structure designed by Safonov et al [12]. The continuous-time controller design is based on a reduced order partially controlled plant model of order 4 with 2 inputs and 2 outputs. Safonov et al propose a conformal mapping for controller discretization, and performance of the resulting control system is verified with the original 116-state plant model. In order to separate the effects of controller discretization and model reduction the discrete-time controllers of Safonov and this paper are compared using the 4-th order controller design model. The later discrete-time controller was determined by approximating the complementary sensitivity function, as per (2.1). The approximation error weighting was chosen to be a transfer function times the identity matrix. The entries are of 4-th order transfer functions consisting of a second order Butterworth filter and a second order shaping function. The frequency plot of the singular values of the complementary sensitivity function and of the weighting are drawn in Figure 6.1. The plot shows that below $100 rad/s$ a small error is forced through a relative large value of the weighting function. This was chosen to meet the design specification on the magnitude of the sensitivity function S to be smaller then 0.01, as set out in [12]. The inclusion of this design specification on S in the approximation of T is possible, since the sum of S and T is one. The Butterworth filter was used to cut off higher frequencies.

An investigation of the sampling time using bandwith considerations of T shows that the choice of sampling frequency of 3000 Hz in [12] is by at least a factor of two too small. This became also apparent in the determination of the weighting function. There was a strong trade-off beetween three goals: 1. the specification for S; 2. robustness requirements (the closed-loop singular value Bode plot of T must be bounded above by 3 db, see Safonov et al [12]); 3. minimization of inter-sampling ripple. Controller discretisation with different weightings showed that it was possible to achieve two of the goals but at the cost of the third. The proposed weighting in Figure 6.1 is a compromise between the goals. In Figure 6.2 the frequency plot of the largest singular of T_h is shown for different controllers. The continuous-time controller is compared with Safonov's controller and the controller obtained by our proposed method. The Figure shows the aspects 2 and 3. It can be seen, that Safonov's controller leads to much higher maximal values of the singular values. At half the sampling frequency a remarkable resonance peak can be seen for Safanov's discrete-time controller. Therefore, a somewhat undesirable intersampling behaviour of the control system must be expected. In Figure 6.3 and 6.4 the impulse-responses of two different outputs of the hybrid systems are shown. The plots verify our claim that with the proposed discretization method intersampling behaviour is reflected in controller discretization. In comparison to Safonov's controller intersampling ripple is significantly reduced. For the investigation of aspect 1 the sensitivity function S is plotted in Figure 6.5. The plot shows that Safonov's controller leads to a better disturbance attenuation. It shows further that the design specifications are not fullfilled with our discrete-time controller.

The discretization of the controller for twice the proposed sampling frequency shows that the trade-off between the above mentioned aspects is significantly reduced, i.e a controller meeting all performance requirements and with almost no intersampling ripple is obtained.

The H_∞-optimal discrete-time controller was of order 22. Using unweighted balanced truncation it was possible to reduce the controller order to 8. The model-data can be found in [12]; a state space realisation of the reduced order controller is given in the Appendix.

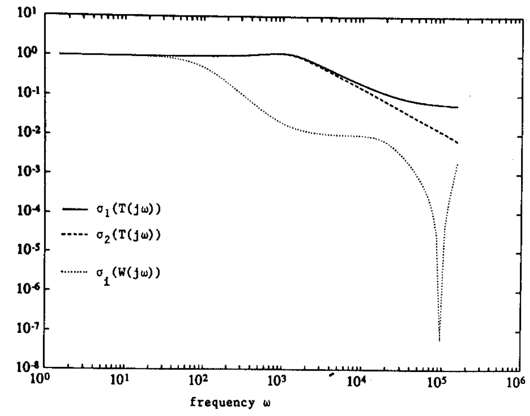

Figure 6.1: Singular value Bode plot of $T(s)$ and weighting $W(s)$

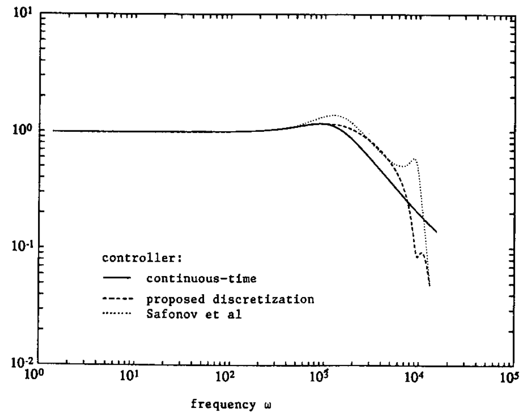

Figure 6.2: Bode plot of the largest singular value of $T(s)$ and T_h

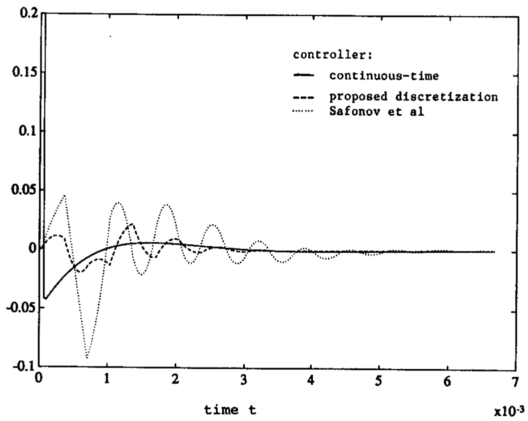

Figure 6.3: Impulse response from u_1 to y_1

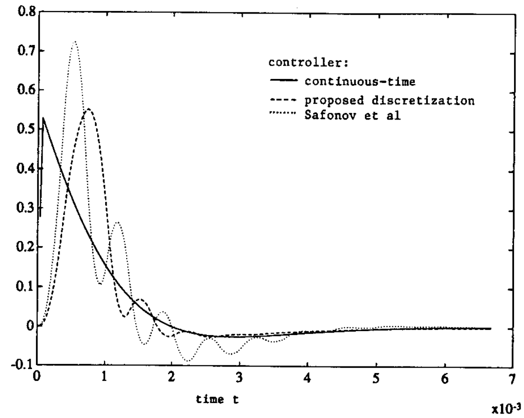

Figure 6.4: Impulse response from u_1 to y_2

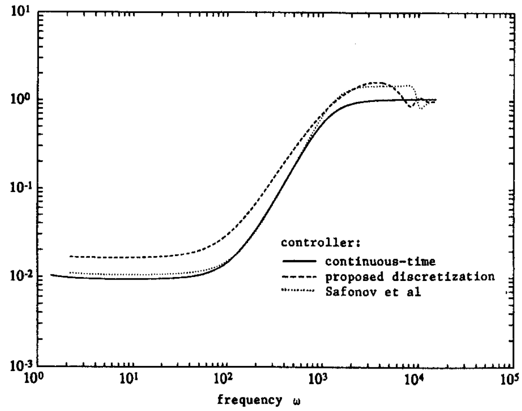

Figure 6.5: Bode plot of the largest singular value of the sensitivity function

7 Conclusion

The proposed method offers a powerful tool for controller discretization. The resulting hybrid control system is stable and its performance, including intersampling behaviour is optimized. Trade-offs resulting from the selection of sampling time are recognized, while trying to achieve a satisfactory approximation of the chosen continuous-time reference transfer function. If no satisfactory approximation can be achieved, the sampling time can be adjusted by just repeating the controller discretization and not the whole controller design procedure. Intersampling behaviour is reflected in the discretization step and does not therefore require a possibly iterative cycle of controller design (or discretization) followed by simulation. Also time-delays, necessary for controller implementation, can easily be incorporated in the controller discretization. The determination of the discrete-time controller is straightforward and can be performed with standard software used in H_∞ controller design. A drawback of the proposed redesign procedure is, that the resulting controller can be of relatively large order, e.g. the sum of plant order, order of the weighting and order of the approximated transfer function. The example shows that the order can usually be significantly reduced by applying some of the order reduction methods proposed in Anderson & Liu [1]. Also a direct discrete-time controller design, which includes intersampling behaviour, was proposed. Several aspect of this approach as for example sampling time predetermination have to be investigated. A possibly very useful additional application may be the controller design for mixed systems, e.g. systems having continuous-time and discrete-time parts.

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Appendix

State-space matrices of digital controller:

A =

-8.1619e-01	-1.6604e-01	-3.9153e-01	9.1375e-02	-5.4415e-03	-1.0489e-03	-5.5671e-04	-5.4554e-03
1.5536e-01	-8.0026e-01	-7.5841e-02	-4.0256e-01	-1.4862e-03	5.8965e-03	7.2824e-03	-1.1813e-03
3.1599e-01	2.8702e-01	-2.8929e-01	7.3621e-02	-2.4860e-02	1.9235e-02	3.2715e-02	-2.2951e-02
-2.3421e-01	3.0849e-01	-1.3529e-01	-2.1327e-01	2.6197e-02	2.4397e-02	2.5029e-02	3.5343e-02
4.8612e-03	-5.4897e-03	-2.5021e-02	2.5146e-02	9.9065e-01	3.2269e-05	5.9403e-03	-2.7139e-02
-2.8730e-03	-3.2966e-03	1.9087e-02	2.5246e-02	-8.4437e-04	9.9228e-01	-2.7496e-02	-6.2811e-03
5.1639e-03	1.2424e-03	-3.3623e-02	-2.4647e-02	-7.0229e-03	2.7220e-02	9.4166e-01	2.2162e-03
-3.2786e-03	6.6131e-03	2.1698e-02	-3.5629e-02	2.6888e-02	7.3486e-03	-2.2164e-03	9.4007e-01

B =

7.0949e+01	6.1012e+01
-5.9505e+01	7.3519e+01
3.2328e+01	1.1780e+02
-1.1638e+02	2.4194e+01
-1.0749e+00	-1.1880e+00
-9.7189e-01	1.4088e+00
1.1309e+00	-2.7974e+00
2.0609e+00	1.0408e+00

C =

3.7431e+01	-8.9202e+01	-5.4624e+00	1.1740e+02	1.0636e+00	9.9996e-01	1.3698e+00	1.9442e+00
8.8244e+01	2.0464e+01	-1.2373e+02	-6.9443e+00	1.1317e+00	-1.4449e+00	-2.7250e+00	1.2007e+00

D =

1.3571e+04	1.5373e+03
-1.6789e+01	1.3886e+04