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ERROR ANALYSIS OF THE BLOCK DFE USING FINITE STATE MARKOV PROCESSES[1]

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ABSTRACT: A method for analysing error propagation in block decision feedback equalisers (block DFEs) [2] using finite state Markov processes (FSMPs) [4] is presented. The approach is applicable whenever the binary input sequence to the channel is independent and the block DFE has a memoryless decision device. Choosing a 2-input block DFE operating on an arbitrary 2nd order channel, we show how the space of channel parameters is partitioned into regions which correspond in a one-to-one manner to an FSMP. Knowing the FSMP that applies for a given channel, we can track the effect of an initial (noise induced) decision error as it causes further errors through error propagation. We subsequently characterise the class of channels on which the block DFE is stable (has a bounded error recovery time from an initial nonzero error state) and channels where indefinite error propagation can occur (with the realisation of a particular input data sequence).

1 Introduction

The decision feedback equaliser (DFE) is a simple nonlinear equaliser used to remove the effects of intersymbol interference from noisy linear communications channels [1]. The block decision feedback equaliser (block DFE) [2] is a natural generalisation of the DFE to the vector case. It operates on blocks (of size p) of received signals to estimate blocks (of size $q \le p$) of the transmitted data. When the input blocksize $p = 1$, the block DFE is identical to a DFE. For $p > 1$, the block DFE has enhanced performance at the expense of increased complexity. As p increases, the block DFE becomes akin to a maximum likelihood sequence estimator [7] (when $p = q$) or to a maximum a posteriori symbol-by-symbol detector [8] (when $q = 1$).

The analysis of error propagation in the block DFE is difficult due to the nonlinearity of the decision device. Moon and Carley [3] give error probability calculations for a high SNR $q = 1$ block DFE assuming correct past decisions (and hence ignoring the important phenomenon of error propagation). Our approach, based on finite state Markov processes (FSMPs), follows similar lines to the error analysis of the (tuned) DFE by Kennedy et al [4]. We consider the recovery of the block DFE from an initial error condition in the absence of noise. This is reasonably complementary to [3] in that a complete performance analysis should account for both noise induced errors and their propagation. The key assumptions in our work are the *statistical independence* of the binary input sequence to the channel and the incorporation of a *memoryless* decision device. Under these assumptions, an analysis of the error dynamics of the block DFE in terms of FSMPs is possible.

2 The Block DFE

We now develop the block DFE for a FIR channel model with delay L and normalised cursor ($h_0 = 1$) with coefficients $1, h_1, h_2, \cdots, h_L$. The treatment presented here mimics that of [2]. The input to the channel is a sequence of *independent* binary random variables $\{u_k\}_{k=0}^{\infty}$, where u_k takes values in $B \triangleq \{-1,+1\}$ with equal probability (assumed for simplicity). The channel output at time k, corrupted by zero-mean white Gaussian noise n_k, gives the received signal

$$y_k = u_k + \sum_{j=1}^{L} h_j u_{k-j} + n_k.$$

If we define a state x_k as the vector of the last L channel inputs,

$$x_k \triangleq (u_{k-L}, u_{k-L+1}, \cdots, u_{k-1})'$$

where x' denotes the transpose of x, then we can express the system in state-space form:

$$\begin{aligned} x_{k+1} &= Ax_k + bu_k \\ y_k &= cx_k + u_k + n_k \end{aligned} \qquad (2.1)$$

where

$$A = \begin{bmatrix} 0 & I_{L-1} \\ 0 & 0 \end{bmatrix}; \; b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \; c' = \begin{bmatrix} h_L \\ \vdots \\ h_2 \\ h_1 \end{bmatrix}; \; d = 1$$

and where I_n is the identity matrix of order n and 0 is a matrix of zeros of the appropriate size.

By aggregating the input and output variables

$$\begin{aligned} U_k &\triangleq [u_k, u_{k+1}, \cdots, u_{k+p-1}]' \\ Y_k &\triangleq [y_k, y_{k+1}, \cdots, y_{k+p-1}]' \\ N_k &\triangleq [n_k, n_{k+1}, \cdots, n_{k+p-1}]', \end{aligned} \qquad (2.2)$$

we obtain a block processing version of blocklength p ($k = 0, p, 2p, \cdots$) for (2.1)

$$\begin{aligned} x_{k+p} &= F_p x_k + G_p U_k + N_k \\ Y_k &= Hx_k + DU_k + N_k, \end{aligned} \qquad (2.3)$$

where, taking $p = 2$ for illustration

$$F_p = \begin{bmatrix} 0 & I_{L-p} \\ 0 & 0 \end{bmatrix}; \; G_p = \begin{bmatrix} 0 \\ I_p \end{bmatrix} \qquad \text{if } p \le L-1$$

or

$$F_p = 0; \; G_p = \begin{bmatrix} 0 & I_L \end{bmatrix} \qquad \text{if } p \ge L$$

and

$$H = \begin{bmatrix} h_2 & h_1 \\ h_3 & h_2 \end{bmatrix} \qquad (2.4)$$

$$D = \begin{bmatrix} 1 & 0 \\ h_1 & 1 \end{bmatrix}, \qquad (2.5)$$

where we define $h_k = 0$ for $L + 1 \le k \le p$.

In analogy to the DFE, DU_k is the direct term of "current" channel inputs (acting as the "cursor"), Hx_k contains past inputs and acts as the "tail" of the ISI.

The block DFE (Fig. 1) assembles past decisions in a state vector estimate $\hat{x}_k = (\hat{u}_{k-L}, \cdots, \hat{u}_{k-1})'$ which is used to cancel the Hx_k term at the decision device input:

$$Z_k \triangleq Y_k - H\hat{x}_k = DU_k + HE_k + N_k \qquad (2.6)$$

where

$$E_k \triangleq x_k - \hat{x}_k \qquad (2.7)$$

is the state estimation error. The decision device is designed on the assumption that past decisions are correct ($E_k = 0$) and produces an estimate of the first q ($1 \le q \le p$) components of U_k:

$$\widetilde{U}_k = \mathcal{D}_{p,q}(Z_k) \in B^q.$$

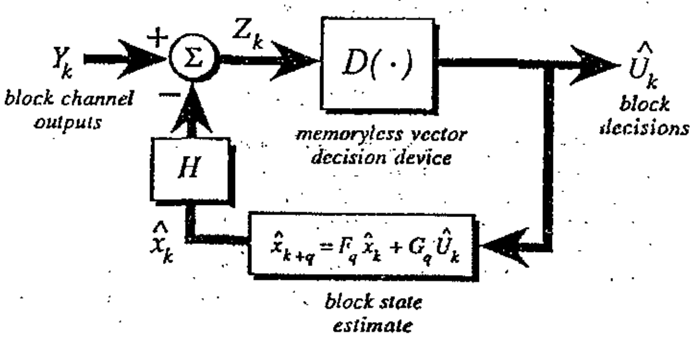

Figure 1. Block DFE

The processing advances in units of q. At each stage a current state estimate is obtained from past state and input estimates using

$$\hat{x}_k = F_q\hat{x}_{k-q} + G_q\widetilde{U}_{k-q}, \tag{2.8}$$

in which F_q and G_q are q-dimensional versions of F_p and G_p in (2.3). With a maximum *a posteriori* probability criterion assuming white Gaussian noise with variance σ^2, the decision function $\mathcal{D}_{p,q}(\cdot)$ is realised (for $k = 0, q, 2q, \cdots$) by [2]

$$\widetilde{U}_k = \arg\max_{U\in B^q} \sum_{V\in B^{p-q}} e^{-\frac{1}{2\sigma^2}\left\|Z_k - D\left[\begin{smallmatrix}U\\ V\end{smallmatrix}\right]\right\|^2}, \tag{2.9}$$

where $\|\cdot\|$ is the Euclidean vector norm, D is defined in (2.5) and Z_k in (2.6). A block DFE using (2.9) is called a *(p,q)-DFE*. For high signal-to-noise ratios (as $\sigma \to 0$) this expression reduces (when $q < p$) to [2]

$$\widetilde{U}_k \simeq [I_q|\,0] \times \arg\min_{U\in B^p} \|Z_k - DU\|^2, \tag{2.10}$$

where 0 is a $q \times (p-q)$ matrix of zeros. Note that when $p = q$, (2.9) and (2.10) are equivalent.

3 Treatment of Error Propagation

In modelling error propagation in a block DFE the quantity of interest is E_k (2.7). A nonzero entry in E_k indicates that a decision error has occurred in the last L time instants (for example due to a noise spike). We wish to track the progress of such an error as it is propagated around the feedback loop of the block DFE. For a $(p,q)-DFE$ with decision function $\mathcal{D}(\cdot)$, operating on an FIR channel, the state estimation error evolves according to

$$\begin{aligned} E_{k+1} &= \begin{bmatrix} 0 & 0 \\ I_{L-q} & 0 \end{bmatrix} E_k \\ &+ \begin{bmatrix} (I_q|\,0)U_k - \mathcal{D}(DU_k + HE_k + N_k) \\ 0_{L-q} \end{bmatrix}, \end{aligned} \tag{3.1}$$

(for $k = 0, q, 2q, \cdots$). Here it is understood that the identity matrix I_{L-q} is replaced by a zero block and the vector of zeros 0_{L-q} is omitted if $L - q < 1$.

Since we are interested in the propagation of errors and not their cause, we now set $N_k = 0$ in (3.1). If we further assume that $\mathcal{D}(\cdot)$ is a *memoryless* decision function and that $\{u_k\}$ is a sequence of *independent* binary random variables, then we can label E_k as the state in a finite state Markov process model [4] of the block DFE. Since u_k is binary, each component e_k of E_k can take only 3 values: $e_k \in \{-2, 0, 2\}$, so there are 3^L states in the FSMP, which for long impulse responses is a very large number. This treatment, however, is capable of modelling exactly the stochastic dynamics of error propagation in the block DFE.

4 Decision Function for p=2

To present a tractable analysis of error propagation, we now specialise to the $p = 2$ block DFE with high SNR decisions (2.10). Recall that

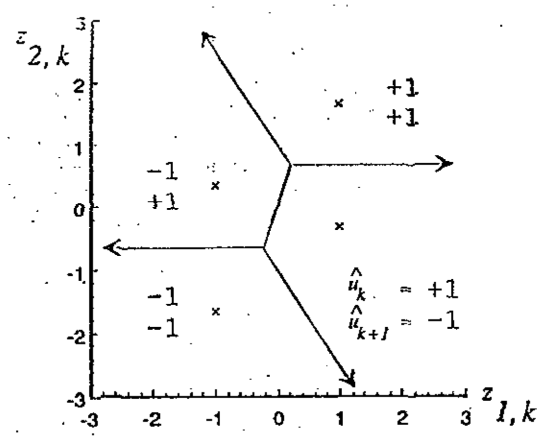

Figure 2. (2,2)-DFE decision boundaries

(2.10) is optimal in this case when $q = 2$. The inputs to the decision device, $Z_k = (z_{1,k}, z_{2,k})'$, are

$$\begin{aligned} z_{1,k} &= u_k + h_1 e_{k-1} + \cdots + h_L e_{k-L} + n_k \\ z_{2,k} &= u_{k+1} + h_1 u_k + h_2 e_{k-1} + \cdots + h_L e_{k-L+1} + n_{k+1} \end{aligned} \tag{4.1}$$

where $e_k \triangleq u_k - \hat{u}_k$. In the $p = 2$ case it is not difficult to derive explicitly the form of the decision rule in terms of sign functions [1] [2],

$$\hat{u}_k = sgn\{sgn(l_1) + sgn(l_2) + sgn(l_3)\} \tag{4.2}$$

$$\hat{u}_{k+1} = sgn\{sgn(l_4) + sgn(l_5) - sgn(h_1)sgn(l_3)\} \tag{4.3}$$

in which

$$\begin{aligned} l_1 &= h_1 z_{2,k} + z_{1,k} - h_1 \\ l_2 &= h_1 z_{2,k} + z_{1,k} + h_1 \\ l_3 &= (h_1 - sgn(h_1))z_{2,k} + z_{1,k} \\ l_4 &= z_{2,k} + h_1 \ ; \ l_5 = z_{2,k} - h_1, \end{aligned}$$

for the $(2,2)-DFE$ ($k = 0, 2, 4, \cdots$) and by (4.2) for the $(2,1)-DFE$ ($k = 0, 1, 2, \cdots$). The decision boundaries for $h_1 = \frac{2}{3}$ are shown in Figure 2.

Observing how the decision regions depend on the strips formed by the pairs of parallel lines in Figure 2 allows us to rewrite (4.2) - (4.3) in a piecewise form:

$$\hat{u}_k = \begin{cases} sgn(h_1 z_{2,k} + z_{1,k}) & , \ |h_1 z_{2,k} + z_{1,k}| > |h_1| \\ sgn(l_3) & , \ |h_1 z_{2,k} + z_{1,k}| < |h_1| \end{cases} \tag{4.4}$$

$$\hat{u}_{k+1} = \begin{cases} sgn(z_{2,k}) & , \ |z_{2,k}| > |h_1| \\ -sgn(h_1)sgn(l_3) & , \ |z_{2,k}| < |h_1| \end{cases} . \tag{4.5}$$

This simplification of the decision device will be useful in the sequel.

5 Partitioning of channel space

For the remainder of the article, we confine the discussion to an arbitrary normalised 2nd order channel $(1, h_1, h_2)$. This will allow a simple graphical interpretation of the results. With this restriction the FSMP for the $p = 2$ block DFE has only 9 states, namely

$$E_k \in \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \pm 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \pm 2 \end{pmatrix}, \begin{pmatrix} \pm 2 \\ 2 \end{pmatrix}, \begin{pmatrix} \pm 2 \\ -2 \end{pmatrix} \right\}$$

and 4 possible inputs

$$U_k \in B^2 \triangleq \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix} \right\}.$$

We treat the (more involved) $q = 2$ case first and then specialise to the $q = 1$ case.

As explained in section 3, we ignore the noise terms in (4.1). The decision device input is now

$$Z_k = \begin{bmatrix} 1 & 0 \\ h_1 & 1 \end{bmatrix} U_k + \begin{bmatrix} h_2 & h_1 \\ 0 & h_2 \end{bmatrix} e_k, \tag{5.1}$$

[1] $sgn(x) = 1$ if $x \geq 0$ and -1 if $x < 0$

with $e_k = (e_{k-2}, e_{k-1})'$ defined by (2.7) and $u_k = (u_k, u_{k+1})'$. for given values of e_k and u_k and a given condition holding in (4.2) or (4.3), we will define a *switching boundary* as the set of values (h_1, h_2) that makes the argument of the corresponding sign function zero. In the regions bounded by the intersection of the set of switching boundaries a *unique* FSMP exists. To see this, note that a change in the sign of a decision is due to a change in sign of one or more sign functions in (4.2) - (4.3), which (by definition) only occurs if a switching boundary is crossed. We will obtain these boundaries that partition (h_1, h_2) channel space, across which there is a change in the FSMP, for the various choices of E_k and U_k. Inspection of (3.1) reveals that the resulting FSMP will have the property:

$$\begin{aligned} E_k &\rightarrow E_{k+2} \text{ under } U_k \\ \Longleftrightarrow -E_k &\rightarrow -E_{k+2} \text{ under } -U_k, \end{aligned} \quad (5.2)$$

due to the odd symmetry of the decision function. ("$\rightarrow$" is to be read "transits to".) This halves the number of error-state transitions to consider and therefore facilitates the analysis. We look first at the zero-error state $E_k = (0,0)'$. Using the "strip" representation (4.4) - (4.5) of the decision function, it is easily demonstrated that under all 4 inputs $U_k \in B^2$, transitions can only be made to $E_{k+2} = (0,0)'$. For instance, let $U_k = (1,1)'$, (4.1) gives $Z_k = (1, 1 + h_1)'$ which is independent of h_2. The '>' part of (4.4) holds for all h_1 and implies $\hat{u}_k = sgn(1 + h_1 + h_1^2) = 1 = u_k$. Similarly $\hat{u}_{k+1} = u_{k+1}$ for all h_1. Thus the zero-error state is an *absorbing* state of the FSMP for any h_1 (and in fact for any FIR channel) so that once this state is reached, no further decision errors can be made due to error propagation alone. These statements also apply in the $q = 1$ case.

We next consider E_k states that have a nonzero entry. The 4 curves defined when an argument of a sign function in (4.4) - (4.5) is zero may be expressed in the form $c(h_1, h_2) = 0$, where

$$c(h_1, h_2) = a_1 h_1^2 + a_2 h_2^2 + a_3 h_1 h_2 + a_4 h_1 + a_5 h_2 + a_6$$

for real constants a_i, and are thus conic sections. As an example, to determine which part of (4.4) applies in a particular region of channel space, it is necessary to consider the values of h_1 and h_2 for which $|h_1 z_{2,k} + z_{1,k}| = |h_1|$. Applying the following identity,

$$|x| = |y| \Longleftrightarrow (x + y)(x - y) = 0$$

valid for real x and y, defines 2 conics:

$$(1 + h_1^2, h_1)U_k + (h_2, h_1 + h_2 h_1)E_k \pm h_1 = 0,$$

after substituting for Z_k in (5.1) and simplifying. The same reasoning is applicable to (4.5). Thus for each of the 32 combinations (16 if we use property (5.2)) of the 8 nonzero error states ($E_k \neq 0$) and the 4 inputs (U_k), we must plot a total of 8 curves (4 from the conditioning and 4 from the arguments of the sign functions) in order to determine the switching boundaries and hence what transitions can occur in each partition of channel space.

We choose the case $E_k = (0,2)'$, $U_k = (1,-1)'$ as a representative example. The curves in question are given by

$$c_i(h_1, h_2) \stackrel{\Delta}{=} c_i = 0 \;, i = 1, \cdots, 8,$$

where

$$\begin{aligned} c_1 &= -1 + 2h_2 \;;\; c_2 = c_1 + h_1 \;;\; c_3 = c_1 + 2h_1 \\ c_4 &= 1 + h_1^2 + 2h_1 h_2 \;;\; c_5 = c_4 + h_1 \;;\; c_6 = c_4 + 2h_1 \\ c_7 &= c_4 - c_1 \;;\; c_8 = c_4 + c_3, \end{aligned}$$

and the decisions (4.4) - (4.5) may be reexpressed as

$$\hat{u}_k = \begin{cases} sgn(c_5) & \text{if } c_4 c_6 > 0 \\ sgn(c_7) & \text{if } c_4 c_6 < 0 \text{ and } h_1 > 0 \\ sgn(c_8) & \text{if } c_4 c_6 < 0 \text{ and } h_1 < 0 \end{cases} \quad (5.3)$$

$$\hat{u}_{k+1} = \begin{cases} sgn(c_2) & \text{if } c_1 c_3 > 0 \\ -sgn(c_7) & \text{if } c_1 c_3 < 0 \text{ and } h_1 > 0 \\ sgn(c_8) & \text{if } c_1 c_3 < 0 \text{ and } h_1 < 0 \end{cases} \quad (5.4)$$

Plotting these curves and using (5.3) - (5.4) establishes the switching boundaries (Fig. 3) for this particular state/input choice. To illustrate the decision procedure, we take the point $h_1 = 0.6$, $h_2 = 0.8$

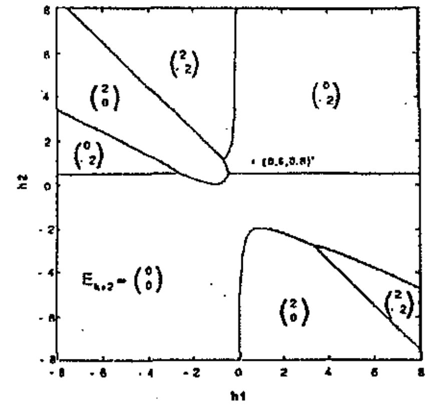

Figure 3. $E_k = (0,2)' \rightarrow E_{k+2}$ with $U_k = (1,-1)'$

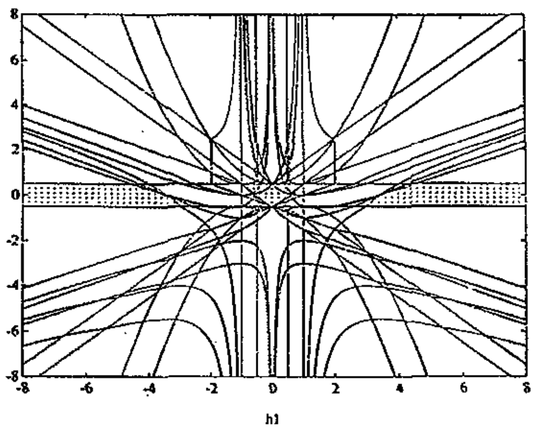

Figure 4. FSMP regions for (2,2)-DFE

(shown by the cross in Fig. 3). We compute $c_4 c_6 = 8.1664 > 0$ so the top line of (5.3) gives $c_5 = 2.92$ and $\hat{u}_k = sgn(c_5) = 1$. Similarly $c_1 c_3 = 0.28 > 0$ and (5.4) gives $c_2 = 0.8$ and $\hat{u}_{k+1} = sgn(c_2) = 1$, so that $E_{k+2} = (1,-1)' - (1,1)' = (0,-2)'$. Note that all (h_1, h_2) points in any one region of Figure 3 will result in the same E_{k+2}. These boundaries are the same for the state $E_k = (0,-2)$ with input $U_k = (-1,1)'$. Continuing this way yields the full set of switching boundaries (for all possible nonzero error states and input combinations) which, when overlayed, allow the complete determination of the FSMP for any channel indexed by (h_1, h_2). The overlay is shown in Figure 4. For instance, in the region of Figure 4 containing the point $h_1 = 0.6$, $h_2 = 0.8$ the (unique) FSMP has the state transition diagram shown in Figure 5 (with transition probabilities marked on the branches).

As the same FSMP applies to each point inside any one region, it is possible to classify classes of channels with "desirable" error recovery properties, as will be explained in the next section. By ignoring those boundaries relevant to the $\hat{u}_{k+1}$ decision in (4.5), the channel space partition for the $q = 1$ case is easily deduced from the switching boundaries of Figure 4 (see Fig. 6).

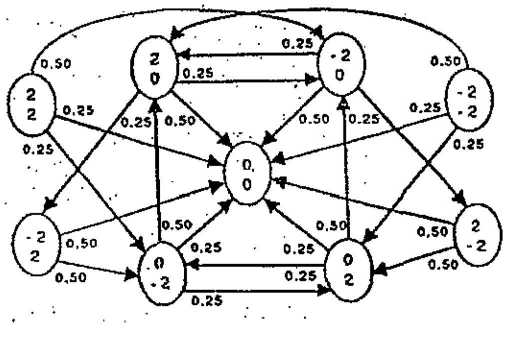

Figure 5. FSMP diagram for $h_1 = 0.6$, $h_2 = 0.8$.

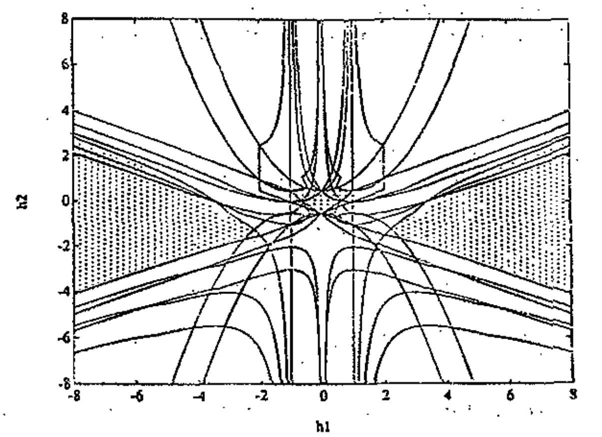

Figure 6. FSMP regions for (2,1)-DFE

6 Pathology of Error Propagation

We can gauge the performance of the block DFE on a particular channel by looking at the FSMP that governs its error propagation dynamics. For a 2nd order channel, the absorbing zero-error state is always reachable from any nonzero-error state (this can be ascertained by considering all possible FSMPs arising in Figures 4 & 6). So for any 2nd order channel, an input sequence $\{u_k\}$ can always be found which drives the block DFE from a nonzero error state to the zero error state (in a finite number of steps). This is clearly a necessary condition for the recovery of the block DFE to error-free operation.

The FSMP for certain choices of h_1 and h_2 (e.g, Fig. 5) is seen to contain groups of nonzero-error states amoung which indefinite error propagation could occur (subject to the realisation of a particularly nasty input sequence). Thus on certain channels the block DFE may suffer from long bursts of errors triggered by a single (primary) incorrect decision. In view of this, it is important to identify channels where the phenomenon of prolonged error propagation cannot occur. This motivates the notion of stability which we define after the next example.

Example

Let $h_1 = 0.6$ and $h_2 = 0.8$. If the (2,2)-DFE is initially in state $(0,2)'$ then the (period 4) input sequence

$$\{u_k\}_{k=0}^{\infty} = \{1,-1,-1,1,1,-1,-1,1,\cdots\}$$

results in the error-state sequence

$$\{E_k\}_{k=1}^{\infty} = \left\{\begin{pmatrix}0\\-2\end{pmatrix},\begin{pmatrix}0\\2\end{pmatrix},\begin{pmatrix}0\\-2\end{pmatrix},\begin{pmatrix}0\\2\end{pmatrix},\cdots\right\},$$

so that $E_k \neq (0,0)'$ for all $k > 0$, i.e., every second decision is incorrect.

In theory then, there exist channels on which the block DFE may have an *indefinite* recovery time [2] due to certain input sequences. (Of course the probability of such an input condition persisting tends to zero exponentially.) An input sequence which gives rise to the behaviour in the example is said to be *pathological*. Equivalently we define the block DFE to be *stable* on a given channel if no pathological sequences exist. (This definition of stability was introduced in [1] in connection with the DFE). The class of channels for which the block DFE is stable is readily determined from Figures 4 & 6 by examining the FSMPs that can occur. Two simple possibilities for pathological behaviour arise in FSMPs where a nonzero error state has a transition to itself or where a pair of nonzero states have transitions to each other (as in the example). In Figures 4 & 6 the block DFE is stable in the shaded regions (for a non unity cursor the axes must be scaled appropriately). In these regions, the block DFE will recover in at most 3 steps for $q = 1$ and in 2 steps for $q = 2$ (1 step for the region containing the origin).

We pause to make a comparison with the conventional DFE. The latter has a triangular stability region [4] with vertices $(0,-0.5)'$, $(-0.5,0.5)'$ and $(0.5,0.5)'$. This is a subset of the stability region for the (2,2)-DFE which is in turn contained in that of the (2,1)-DFE. We remark that for points in the outer semi-infinite regions, the block DFE is effectively using information carried by the h_1 coefficient, this is *not* a feature of the DFE.

[2] the time taken to reach the zero-error state [4]

It is curious to note that the stability region of the (2,2)-DFE lies in the strip $|h_1| < |h_0|/2$. The reason for this may be gleaned from the switching boundary diagrams (not shown) for error states $(0,2)'$ and $(0,-2)'$. These states have transitions between each other when $h_1 > \frac{1}{2}$ and transitions to themselves when $h_1 < -\frac{1}{2}$. Notice that when $|h_1| \gg |h_2| > \sqrt{2}$ in the $q = 1$ case, the dominant part of (5.1) is $Z_k \approx (h_1 e_{k-1}, h_1 u_k)'$, so that (4.4) implies

$$\hat{u}_k = sgn(h_1 e_{k-1} + h_1^2 u_k) = u_k.$$

This explains the roughly triangular shape of the outer stability regions. The above arguments run in favour of the (2,1)-DFE in terms of stability over a wider class of channels than either the (2,2)-DFE or the DFE.

7 Conclusions

We have presented a finite state Markov process analysis of error propagation in a 2-input block DFE operating on a general 2nd order channel. The question of reachability of the zero-error state has been addressed. The (stability) region corresponding to bounded error-recovery time was established for the (2,1) and (2,2)-DFE and demonstrates the improved robustness of the device over the conventional DFE in that stability is maintained over a greater region of channel space. (Conservative estimates of these regions can also be obtained calculated on minimum distance to the decision boundary.)

It remains to examine the statistics of error recovery times for the different classes of channels, although this is mechanical and proceeds analogousy to the treatment in [4]. Clearly the style of analysis presented here is not suited to the more general case where the number of inputs exceeds 2. For higher order FIR channels another approach is also warranted. The study of error propagation in block DFEs is evidently not a simple undertaking and the complexity of any analysis stems from the nonlinearity of the decision device coupled with recursion. The possibility of applying passivity theory [5] and Lyapunov techniques [6] to test the stability of the block DFE, without excessive computational burden, is also under investigation.

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