A new approach to the discretization of continuous-time controllers

J.P. Keller
B.D.O. Anderson

Department of Systems Engineering,
Research School of Physical Sciences,
Australian National University,
G.P.O. Box 4, Canberra, A.C.T., Australia.

Abstract A new method for discretizing continuous-time controllers is derived, using principles of controller approximation (Anderson & Yi [1]). It focuses on the closed-loop use of the discrete-time controller. The resulting approximation criterion is a measure for stability of the control system, provides an upper bound on the sampling time for which stability can be guaranteed and because it is based on continuous-time controller approximation it indicates the cost of discretization in terms of performance degradation. The discrete-time controller is obtained through minimization of this criterion, which can be performed with standard software used in H_∞ controller design.

1 Introduction

Most controller design procedures allow direct discrete-time controller design. Nevertheless there is some reason to design first a continuous-time controller, which is afterwards converted into a discrete-time one. Direct design of discrete-time controllers requires a predetermination of the sampling time in order to model plant, noise, process disturbances etc. Upper bounds for the sampling time can only be obtained after considering closed-loop bandwidth (Franklin & Powell, [5], ch.10,) and, depending on specifications, this may only be available *after* controller design. Because this obvious dilemma does not occur in continuous-time controller design, it may be reasonable to design first a continuous-time controller, which is in a second step approximated by a discrete-time one. A second reason for first executing a continuous-time design is that physical insight is better retained.

There are many methods available for discretizing continuous-time controllers, for instance bilinear transformation, hold input approximation and signal invariant transformations, but only a few of them take account of the closed loop use of the controller. Yet discretization of a continuous-time controller is a type of approximation, and, as argued in Anderson & Liu [1], it is logical to take closed-loop performance into account, whenever a controller is approximated. Methods for discrete-time implementation of continuous-time linear state-feedback laws are proposed by Kuo & Peterson [11], Yackel et al. [17] and Kleinman & Rao [10]. A closed loop redesign method is proposed by Rattan & Yeh [14] and Rattan [15], but none of these approximation methods can guarantee the elementary property of closed-loop stability after controller discretization. In the following a new discretization method is presented, which takes account of closed-loop issues. Using ideas from perturbation theory, stability of the closed-loop system is related to the approximation error, resulting from controller discretization, much as in some of the order reduction methods of Anderson & Liu [1]. The resulting stability criterion is derived in Section 2. The discrete-time controller is chosen to maximize the stability margin. To evaluate this margin, the value of an operator norm is required and this is hard to calculate. In order to get a tractable problem, the original criterion is approximated (arbitrary closely) by an auxiliary criterion. After some transformations an H_∞ characterisation of a stabilizing discrete-time controller, which maximizes the stability margin and approximates the continuous-time controller, is obtained. This is shown in Sections 3 and 4. In Section 5 possible applications of the discrete-time controller characterisation are proposed. In Section 6 examples are presented and Section 7 contains concluding remarks.

2 Stability of discretized control systems

Let $P(s)$ be the plant transfer function, $C(s)$ the known continuous-time controller, $C_d(z)$ the unknown discrete-time controller and let H represent a zero-order hold. Figure 2.1 shows a rearrangement of the system with the discrete-time controller. We shall assume here that $C(s)$ is open-loop stable (as in common) - removal of this assumption would presumably be possible, as discussed briefly in the conclusions. It is natural to seek to approximate $C(s)$ using a $C_d(z)$ which is also stable, and we make this assumption. In addition we assume that $C(s)$ is a stabilizing controller for $P(s)$, and, for convenience, that $P(s)$ is strictly proper. Last, we assume that $P(s), C(s)$ are rational in s.

The symbol $\Delta(s, z = e^{sT}, t)$ is used for the approximation-error operator, which is linear and periodically timevariant; it cannot be represented by a real rational transfer function. To investigate the influence of Δ on closed-loop stability an equivalent representation to Figure 2.1 is drawn in Figure 2.2.

Let:

$$J_c \doteq \Delta(I + PC)^{-1} P \qquad (2.1)$$

$$\|J_c\| = \max_{u \in L_2[0,\infty)} \frac{\|J_c u\|_2}{\|u\|_2} \qquad (2.2)$$

Notice that the stability of Δ and the closed loop formed from '' and C ensures that $\|J_c\|$ is finite.

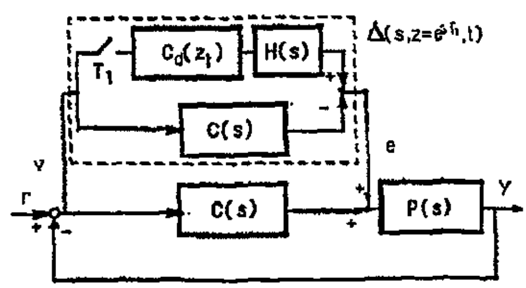

Figure 2.1: Rearrangement of feedback system with discrete-time controller

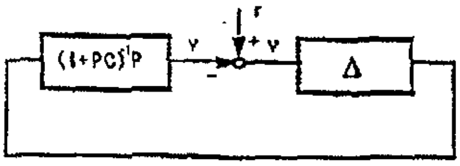

Figure 2.2: Perturbation analysis

By a variant on the small-gain theorem, see Desoer & Vidyasagar, [2] p46, the loop in Figure 2.2 is bounded-input, bounded-output stable and has a mapping from $r \in L_2[0,\infty)$ to $y \in L_2[0,\infty)$ if

$$\|J_c\| < 1 \tag{2.3}$$

Using the definition of J_c, it becomes clear how controller approximation relates to stability, and how controller approximation might be pursued. To guarantee stability, the approximation must ensure that (2.3) holds, i.e. that Δ, when weighted by $(I+PC)^{-1}P$, is suitably small in norm. It is reasonable then to seek C_d not merely to ensure that (2.3) holds, but to ensure that $\|J_c\|$ is minimized.

It is not surprising that $\|J_c\|$ is hard to calculate; however, in the next two sections, we develop a convenient approximation to $\|J_c\|$ which leads to a useful characterisation of stabilizing, discrete-time controllers. Not only can we calculate for a given $C_d(z)$ a convenient approximation to $\|J_c\|$, but we can then choose C_d to minimize this approximation error index.

Approximation of J_c

The main idea of this section is to approximate the continuous-time operators involved in forming J_c, i.e. $(I+PC)^{-1}P, H$ and C, by discrete-time operators, with arbitrary small sampling time which is a submultiple of the sampling time of C_d. It turns out that the operator J_d, obtained by replacing the continuous-time operators with their discrete time, hold-input approximation, converges to J_c as their sampling time (but not that for C_d) tends to zero. The approximation idea is illustrated in Figure 3.1, which depicts J_c, and Figure 3.2, which depicts a multirate sampled data system J_d, which is an approximation of J_c.

Two different sampling times, integrally related, appear now in J_d. Discrete-time transfer functions mapping signals sampled in intervals T_s resp. T_l will be denoted by the variable z_s resp. z_l.

In the next section, we shall show how a further step reduces the calculation of $\|J_d\|$ to a standard problem. We now turn to justifying the claim that $\|J_d\|$ approximates $\|J_c\|$. Let $W(s)$ denote $(I+PC)^{-1}P$ and suppose

$$W(s) = C_w(sI - A_w)^{-1}B_w \tag{3.1}$$
$$C(s) = D_c + C_c(sI - A_c)^{-1}B_c \tag{3.2}$$

are minimal realizations of $W(s), C(s)$. Further, let

$$C_d(z_l) = E_d + H_d(z_l I - F_d)^{-1}G_d \tag{3.3}$$

be a minimal realization of $C_d(z_l)$. The sampling rate for $C_d(z_l)$ is T_l and that for the fast sampled data approximations of $W(s)$ and $C(s)$ is assumed to be T_s, where

$$T_l = NT_s \tag{3.4}$$

for some integer N, which will be chosen large. The discrete-time replacement for $W(s)$ is its hold-input discretization with transfer function $W(z_s)$:

$$W(z_s) = H_w(z_s I - F_w)^{-1}G_w \tag{3.5}$$

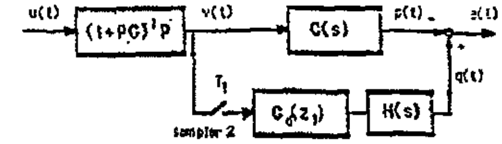

Figure 3.1: Implementation of J_c

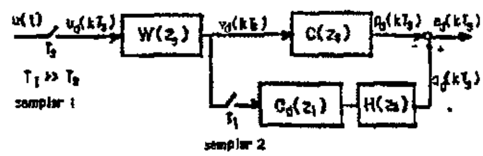

Figure 3.2: Approximation of J_c by J_d

where

$$F_w = e^{A_w T_s}, \quad G_w = \int_0^{T_s} e^{A_w \tau} B_w d\tau, \quad H_w = C_w \tag{3.6}$$

and the discrete-time hold-input replacement for $C(s)$ is

$$C(z_s) = E_c + H_c(z_s I - F_c)^{-1}G_c \tag{3.7}$$

where

$$F_c = e^{A_c T_s}, \quad G_c = \int_0^{T_s} e^{A_c \tau} B_c d\tau, \quad H_c = C_c, \quad E_c = D_c \tag{3.8}$$

The function of the block labelled $H(z_s)$ (see Figure 3.2) is easily described. H receives a discrete-time input at time intervals T_l seconds apart. Each discrete-time input pulse generates N discrete time output pulses, equal to the input pulse, starting at the time the input pulse is received and spaced apart by T_s seconds.

Proposition 3.1 :
Consider the operator J_c mapping $u(\cdot) \in L_2[0,\infty)$ to $e(\cdot) \in L_2[0,\infty)$ depicted in Figure 3.1 and the operator J_d mapping $u_d(kT_s)$, $0,1,2,.. \in l_2[0,\infty)$ to $e(kT_s), k = 0,1,2,.. \in l_2[0,\infty)$ depicted in Figure 3.2, with the relation between the blocks making up J_d and J_c as described above. Then with T_l fixed, there holds

$$\lim_{N\to\infty} \|J_d\| = \|J_c\|, \qquad NT_s = T_l \tag{3.9}$$

For the proof of this result, see Appendix A.

The intuition is clear; if we sample fast enough and use conventional discretization of the continuous time blocks, we obtain a multirate discrete-time system with the same gain (in the limit) as the hybrid system.

4 Evaluation of J_d as a standard problem

Two different sampling times appear in the operator J_d. As such, it is not immediately clear how J_d should be represented by a transfer function (matrix). However, a simple idea allows J_d to be represented by a norm-equivalent transfer function with respect to the larger sampling time. The idea has been used in the design of periodically time-varying controllers (Khargonekar et al. [9]), and is an old one in signal processing, where it is termed "blocking", see [13].

Consider for example the transfer function

$$C(z_s) = E_c + H_c(z_s I - F_c)^{-1} G_c \qquad (4.1)$$

The processing achieved by $C(z_s)$ can be viewed in an alternative way. One groups N successive inputs into an input block and similarly for the output. Input and output blocks occur at intervals of $NT_s = T_l$, and the processing is viewed as mapping input blocks into output blocks. These inputs and output blocks are vectors of dimension N in case $C(z_s)$ is a scalar transfer function, and are vectors of vectors in case $C(z_s)$ is a transfer function matrix. A state-space representation is easily obtained as:

$$x_c[(k+1)NT_s] = F_c^N x_c(kNT_s) + [F_c^{N-1}G_c \quad F_c^{N-2}G_c \ldots G_c] \begin{bmatrix} v_d(kNT_s) \\ v_d[(kN+1)T_s] \\ \vdots \\ v_d[(kN+N-1)T_s] \end{bmatrix}$$

$$\begin{bmatrix} y_d(kNT_s) \\ y_d[(kN+1)T_s] \\ \vdots \\ y_d[(kN+N-1)T_s] \end{bmatrix} = \begin{bmatrix} H_c \\ H_c F_c \\ \vdots \\ H_c F_c^{N-1} \end{bmatrix} x_c(kNT_s) + \begin{bmatrix} E_c & 0 & \ldots \\ H_c G_c & E_c & \ddots \\ H_c F_c G_c & H_c G_c & \ddots \\ \vdots & & \\ H_c F_c^{N-2} G_c & H_c F_c^{N-3} G_c & \ldots \end{bmatrix} \begin{bmatrix} v_d(kNT_s) \\ v_d[(kN+1)T_s] \\ \vdots \\ v_d[(kN+N-1)T_s] \end{bmatrix} \qquad (4.2)$$

A similar representation can be obtained for $W(z_s)$. Evidently $C(z_s)$ and $W(z_s)$ can be replaced by transfer function matrices in the variable z_l, e.g.

$$\bar{C}(z_l) = \bar{E}_c + \bar{H}_c(z_l I - \bar{F}_c)^{-1} \bar{G}_c \qquad (4.3)$$

where $\bar{F}_c, \bar{G}_c, \bar{H}_c$ and $\bar{E}_c$ are the matrices appearing in (4.2) with $\bar{W}(z_l)$ being constructed similarly. We denote the input and output vectors in 4.2 by $\bar{v}_d(kT_l)$ and $\bar{y}_d(kT_l)$.

In the block representation, the output of the sampler 2 (see Figure 3.2) is the first element of the vectorized signal $\bar{v}_d$. The transfer matrix of the sampler is therefore a row vector of length N, having 1 as first element and being zero elsewhere. The hold element $H(z_l)$ produces a sequence of N pulses equal to the input pulse which corresponds to a block vector with equal entries. It is therefore a mapping of the output of $C_d(z_l)$ by a column vector of ones.

Then the block rearrangement $\bar{J}_d$ is the time-invariant operator with transfer function matrix

$$\bar{J}_d(z_l) = \left(\bar{C}(z_l) - \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} C_d(z_l) \, [1 \, 0 \ldots 0] \right) \bar{W}(z_l) \qquad (4.4)$$

Observe also that:

$$\sum_k v_d'(kT_s) v_d(kT_s) = \sum_k \bar{v}_d'(kT_l) \bar{v}_d(kT_l) \qquad (4.5)$$

$$\sum_k e_d'(kT_s) e_d(kT_s) = \sum_k \bar{e}_d'(kT_l) \bar{e}_d(kT_l) \qquad (4.6)$$

Hence the connection between J_d, mapping $v_d(\cdot)$ to $e_d(\cdot)$ and $\bar{J}_d$, mapping $\bar{v}_d(\cdot)$ to $\bar{e}_d(\cdot)$, ensures

$$\|J_d\| = \|\bar{J}_d\| \qquad (4.7)$$

where the norms are the operator-induced norms on $l_2^p[0,\infty)$ and $l_2^{p \cdot N}[0,\infty)$. However since $\bar{J}_d$ has a transfer matrix representation, this operator norm is precisely the H_∞-norm of the transfer function matrix.

This result and that of the last section have shown how, through approximation and rearrangement, a mathematically tractable (i.e. readily computable) criterion for controller discretization is obtained, at least approximately:

Theorem 4.1 :

With quantities as defined earlier,

$$\|J_c\| = \lim_{T_s \to 0} \|J_d\| = \lim_{T_s \to 0} \|\bar{J}_d\| = \lim_{T_s \to 0} \|\bar{J}_d(e^{j\omega T_l})\|_\infty \qquad (4.8)$$

In the above analysis, we have assumed that the output of sampler 2 is the first element of a block signal. We could however have chosen to define the block signal associated with $v_d(\cdot)$ by:

$$\bar{v}_d(kT_l) = \begin{bmatrix} v_d[(kN+\alpha)T_s] \\ v_d[(kN+1+\alpha)T_s] \\ \vdots \\ v_d[(k+1)NT_s] \\ \vdots \\ v_d[((k+1)N-1+\alpha)T_s] \end{bmatrix} \qquad (4.9)$$

and similarly for $\bar{y}_d(kT_l)$. Thus the assembling of the fast signals into blocks, while still occuring every T_l seconds, occurs at times displaced from the sampling instants of the sampler 2 and the time of generation of outputs from the slow compensator. The effect is to vary $\bar{J}_d$ but, as it turns out, not to vary $\|\bar{J}_d\|$.

5 Extremizing the stability measure

Different measures for the determination of a discrete-time controller have been proposed ([4]-[8]), but most of them, except in Rattan [15], consider only performance degradation at sampling instants. Because $\bar{J}_d$ approximates the continuous-time approximation error arbitrarily closely, intersampling behaviour of the system is also reflected in the error measure used for controller discretization. The resulting advantages are that the occurence of intersampling ripple is minimized and that therefore larger sampling periods may be used, while still achieving acceptable intersampling behaviour of the continuous-time signals (e.g. output y).

By means of the representation (4.4) several discretization problems can be solved:

- Given a sampling time T_l an optimal controller $C_d(z_l)$ can be obtained through minimization of $\|\bar{J}_d\|_\infty$. The resulting controller approximates the continuous time one and is stabilizing if $\|\bar{J}_d\|_\infty < 1$ (for T_s reasonably small).

- Using the same approach, there is also the possibility of approximating a discrete-time controller by another stabilizing discrete-time controller with larger sampling time.

- A measure for the impact of the sampling time on controller discretization is the value of $\|\bar{J}_d\|_\infty$ for an optimal controller $C_d(z_l)$ and the effect of varying sampling time can be easily examined. If for an optimal controller $\|\bar{J}_d\|_\infty = 1$ an upper bound for the sampling period T_l based on a sufficient condition for stability is reached.

The minimization of $\|\tilde{J}_d\|_\infty$ is a *standard* H_∞ problem , when $C(s)$ is stable, (which it is by assumption). Controllers C_d were determined using the minimization procedure described in Safonov et al [16]. The H_∞-problem often has no unique solution. One reason becomes evident from a closer look at the underlying 4-block problem of achieving $\inf \|\tilde{J}_d\|_\infty$:

$$\inf_{C_d} \|\tilde{J}_d\|_\infty = \inf_{C_d} \| \begin{bmatrix} A - C_d M & B \\ C & D \end{bmatrix} \|_\infty \qquad (5.1)$$

A lower bound l_b for $\|\tilde{J}_d\|_\infty$ is given by:

$$l_b = max \left(\| \begin{bmatrix} B \\ D \end{bmatrix} \|_\infty, \| [C \;\; D] \|_\infty \right) \qquad (5.2)$$

The examples show that for small sampling times T_l, this lower bound is usually reached. In this case $C_d(z_l)$ is not uniquely determined even for SISO-plants. In Appendix B a set of optimal controllers C_d with equal $\|\tilde{J}_d\|_\infty$ is given. Within that set C_d can be chosen for example either to be of minimal order or to shape $\bar{\sigma}(\tilde{J}_d(j\omega))$ in a desirable way, without affecting $\|\tilde{J}_d\|_\infty$. An alternative way for the determination of an unique solution (more suitable for MIMO-systems) may be the application of super-optimal controller design proposed by Foo & Postlethwaite [4], but the advantages are not clear.

6 Examples

The new discretization method was tested with two examples, the first from Rattan [15] and the second from Katz [7]. For both examples, it was reported that common discretization methods produce either non stabilizing controllers or systems with very poor closed-loop performance. Using the proposed discretization method of this paper, controllers were calculated for various sampling times and the resulting control systems were compared with the results of Rattan [15] and Kennedy [8], who further investigated the example of Katz [7].

The following particular aspects were investigated:

- step response of the closed-loop system
- the frequency response to input signals r
- gain and phase margin of the discrete-time system
- possible increase of the sampling time above the values, proposed in [15] and [7]

For calculating a discrete-time controller, i.e. minimization of $\|\tilde{J}_d\|_\infty$, the small sampling time T_s must be chosen. As a rule of thumb, the small sampling period T_s needs to be about 20 times smaller then a sampling time determined from closed-loop bandwith considerations. (see Franklin & Powell [5]: the sampling frequency should be more than twice the closed-loop bandwith of the continuous-time system.) Smaller T_s alter $\|\tilde{J}_d\|_\infty$ only negligibly, but increase computational burden.

The transfer functions of the plants and controllers are given in Appendix C.

6.1 Rattan's example

In Figure 6.1 the approximation error $\bar{\sigma}(\tilde{J}_d(j\omega))$ is plotted for optimal controllers designed for different sampling periods T_l. The plot shows, that $T_l = 0.420s$ is close to the proposed upper bound. A further increase in sampling time leads to $\|\tilde{J}_d\|_\infty > 1$ and for a controller with $\|\tilde{J}_d\|_\infty = 1.1$ an unstable closed-loop system was obtained. For the controllers with sampling time $T_l \leq 0.157s$ relative large controller approximation errors are available to determine optimal reduced-order controllers. Their determination using balanced model reduction is given in detail in Appendix B. The order reduction leads to the characteristic that the optimal $\tilde{J}_d$ is not all-pass, i.e. $\bar{\sigma}(\tilde{J}_d(j\omega)) \neq const.$. For $T_l > 0.157s$ a negligible suboptimality was allowed to obtain a reasonable controller order and so to circumvent the problem of finding a minimal controller realisation. The sampling time proposed by Rattan is 0.157 s. In Figures 6.2 and 6.3 the step response and the frequency response are plotted. The continuous-time system and Rattan's optimization result with $T_l = 0.157s$ are compared with our results for $T_l = 0.157s$ and $T_l = 0.314s$. The plots reveal that in this example performance is maintained which is comparable to Rattan's optimization, even for a sampling time which is twice Rattan's sampling time. Gain and phase margin of the discrete-time system are analysed using a Nyquist diagram. From Figure 6.4 it can be concluded that both gain and phase margin for $T_l = 0.157s$ are similar to Rattan's result, whereas for $T_l = 0.314s$ gain margin is slightly reduced.

The continuous-time controller has degree 1, the discrete-time controller of Rattan has degree 1 and those generated by our method have degree 2.

6.2 Katz's example

According to the discussion in Katz [7], of 8 commonly used discretization methods only one, viz. prewarped bilinear transformation yields a stabilizing controller at $T_l = 0.030s$ and even then there are very poor closed-loop properties. Kennedy [8] also investigated this example and achieved good controller discretization. He proposed a procedure in which the continuous-time feedback controller is approximated by a discrete-time model-following controller, i.e. one having a feedforward and a feedback part. Our controller-discretization method will be compared with this result.

In Figure 6.5 $\bar{\sigma}(\tilde{J}_d(j\omega))$ is plotted. The controller order was determined in the same way as in the preceding example. The sampling-time $T_l = 0.030s$ is close to the proposed upper bound, which is near to 0.039 s. The relatively large value of $\min\|\tilde{J}_d\|_\infty$ for $T_l = 0.030s$ predicts therefore the discretization problems reported in Katz [7]. The step response and the frequency response are shown in Figures 6.6 and 6.7. The design of Kennedy for $T_l = 0.030s$ shows better agreement with the continuous-time step response. This is not surprising because he makes use of the additional freedom of a feedforward controller. In the frequency-response plot, it can be seen that the design of Kennedy is better at high frequencies. It is remarkable that our controller with $T_l = 0.039s$ still gives acceptable closed-loop performance. A more important difference between our design and the design of Kennedy can be seen in Figure 6.8. The Nyquist plot of the discrete-time loop-transfer function shows that the gain and phase margin of our design are considerably better than those of Kennedy even for the sampling-time $T_l = 0.039s$. The continuous-time controller, the controller of Kennedy and our controller for $T_l = 0.030s$ are second order while our controller for $T_l = 0.039s$ is first order.

7 Conclusion

Our proposed method for controller discretisation offers the advantage that it focuses on closed-loop behaviour. It is based on principles of continuous-time controller approximation and includes therefore also intersampling behaviour of the system. The examples show that the value assumed by our approximation criterion can be interpreted as a possible measure for the practical cost of discretization in terms of performance degradation. Furthermore the approximation criterion provides an upper bound on the sampling time, based on a sufficient stability criterion. The methods works well with the selected examples, which were reported to be difficult to discretize. Even for relatively large sampling time, the results are comparable to earlier proposed discretizations operating at smaller sampling time. The determination of the discrete-time controller is straightforward and can be performed with standard software used in H_∞ controller design.

Several aspects of the proposed discretization method need to be investigated. When the optimal $\tilde{J}_d$ is all-pass it may be possible to derive an exact value for the order of $C_d(z_l)$ in terms of the orders of $C(s)$ and $P(s)$. This value is an upper bound in the case when no unique optimal controller exists and the controller is determined as described in Appendix B. Another interesting point is how the resulting controllers relate to controllers obtained with conventional discretization methods, especially for extreme values of the sampling time. Two questions arise in this context: are the conventionally discretized controllers in the set of optimal controllers? and if not: how far away are they from optimality with respect to the criterion $\|J_d\|_\infty$?

In the examples presented nonuniqueness of controllers was used to minimize controller order. It is reasonable to investigate, if some other auxiliary criterion, possibly based on performance considerations, may be preferable to force an unique controller.

Using the proposed formalism, it may be possible to formulate a controller approximation problem, including other performance oriented weighting functions than stability margin as proposed for controller order reduction in Anderson & Yi [1]. Also, fractional representations of controllers may be approximated (Liu et al. [12]) allowing the discretization of unstable controllers.

The approximation of continuous-time controllers by multirate, discrete-time controllers (e.g. multivariable systems with samplers, operating with different sampling times) can be easily formalized using the block representation $\tilde{J}_d$, but for controller determination, boundary conditions ensuring causality and time-invariance must be respected. Neglecting the last mentioned constraint, time-varying multirate controllers could be obtained.

It is desirable to gain further insight into block representation optimization problems like $min\|\tilde{J}_d\|$. As a result sampling time could probably be related in a quantitative way to performance degradation resulting from system discretization and therefore provide a clear procedure for sampling time determination. Also the efficiency of more complicated control systems, for example multirate systems could be examined.

It is also possible to impose a constraint that there be some processing delay in the control $C_d(z_l)$; this inquires some development and will be explained elsewhere.

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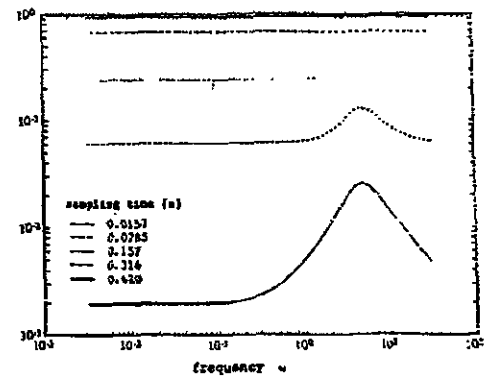

Figure 6.1: Rattan's example: $\bar{\sigma}(J_d(e^{j\omega T_1}))$

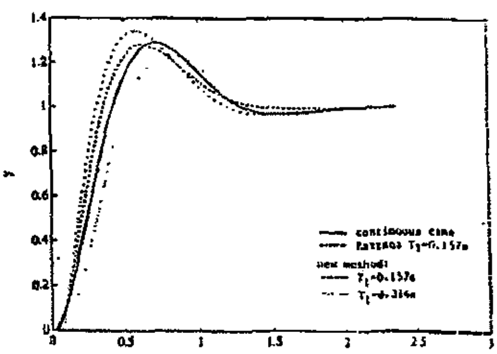

Figure 6.2: Step response.

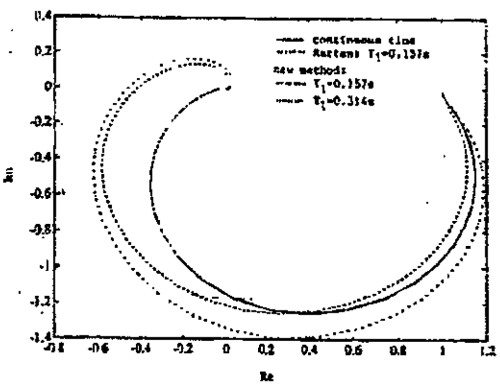

Figure 6.3: Frequency response

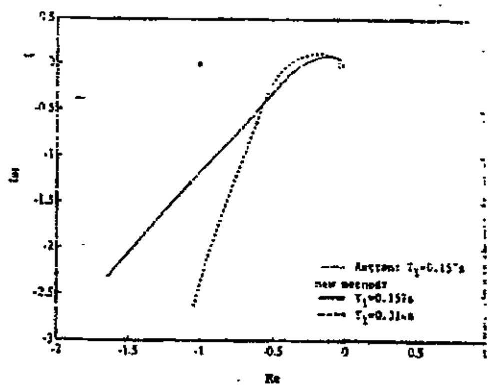

Figure 6.4: Nyquist plot of discrete-time loop-transfer function

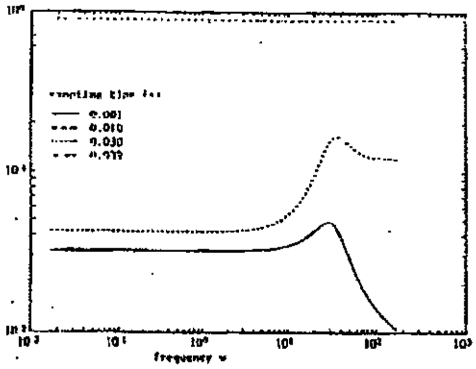

Figure 6.5: Katz's example: $\bar{\sigma}(\tilde{J}_4(e^{j\omega T_1}))$

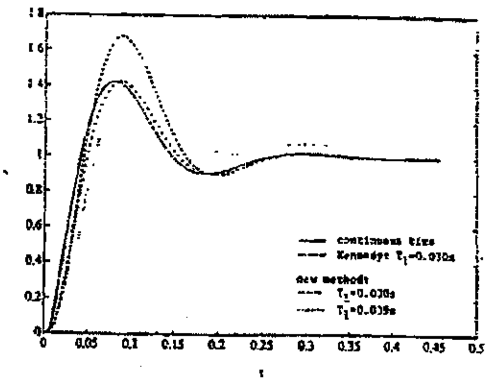

Figure 6.6: Step response

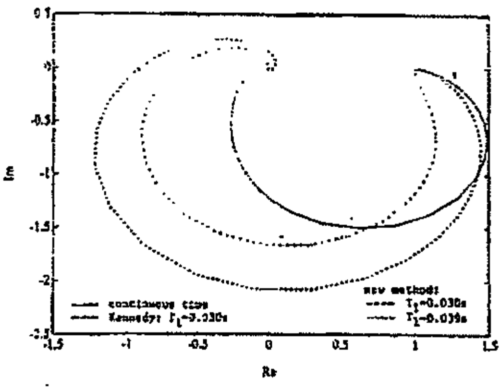

Figure 6.7: Frequency response

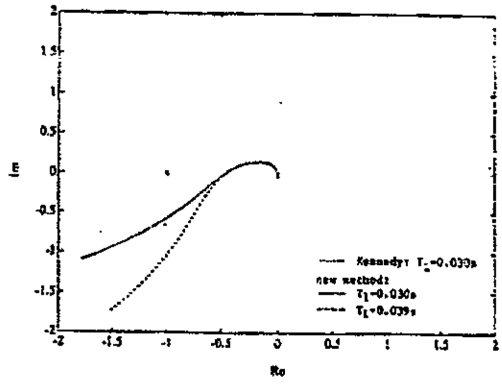

Figure 6.8: Nyquist plot of discrete-time loop-transfer function

The Appendices A, B and C can be obtained from the authors.