Proceedings of the 33rd Annual Conference of the Institute of Systems, Control and Information Engineers, ISCIE (May 17–19, 1989)

特別招待講演

On Controller Reduction

Brian D.O. Anderson

Dept. of Systems Engineering, Australian National University,
G.P.O. Box 4, Canberra, A.C.T. 2601, Australia.

Abstract

This paper considers the problem of passing from a linear time–invariant high order controller designed for a linear time–invariant plant (of presumably high order) to a low order approximation of the controller. The approximation problem is often best posed as a frequency weighted L_∞ approximation problem. Many different controller representations and approximation schemes are possible, giving different performance of the various reduction algorithms.

1 Introduction

Simple linear controllers are normally to be preferred to complex linear controllers for linear time–invariant plants: there are fewer things to go wrong in the hardware or bugs to fix in the software, they are easier to understand, and the computational requirements are less. For this reason, there is a desire to have methods available for designing low order controllers for high order plants. Such methods can broadly be divided into two classes: *direct* in which the parameters defining a low order controller are computed by some optimization or other procedure; and *indirect*, in which a high order controller is first found, and then a procedure used to simplify it.

Examples of direct methods include the work of Gangsaas *et al.* [1] (see the third case study in [1]) which draws on [2], and of Bernstein and Hyland [3-5]. Though this paper does not have such methods as its concern, they nevertheless demand comment, at least of a brief character.

The common philosophy in these methods is to seek to minimize a quadratic performance index subject to the constraint that the controller be of fixed degree (as well as stabilizing and time–invariant). This problem statement can be translated into the mathematical problem of solving a set of multivariable algebraic equations. The optimal controller is defined by one solution of these equations. Their solution is not straightforward – and the multiplicity of solutions may be very great. Nevertheless, substantial progress has been made using homotopic methods, see e.g. [6]. This more recent work gives grounds for believing that standard control design packages in the medium term could contain software suitable for a nonspecialist in numerical analysis.

There is at least one overriding theoretical issue in relation to these direct methods which deserves more attention. Often, conventional LQG design uses the performance index as a tool for achieving certain closed–loop goals relating to sensitivity, bandwidth, gain margin, etc. The LQG design process (before any controller reduction), may proceed iteratively, using results or empirical rules for adjusting weighting matrices in the light of achieved closed–loop parameters such as those noted above. The extent to which this could be possible in direct reduced order design is far from clear.

As for indirect methods, there are at least two sophisticated approaches to the design of high order controllers, LQG and H_∞, and at least for the former, a great deal of qualitative/conceptual knowledge exists which is vital in the applications of the design algorithms to practical problems. Less well developed are the procedures for reducing high order controllers to low order controllers; such procedures are the subject of this paper.

A somewhat crude approach (that nevertheless can often be successful) to controller reduction is modal reduction. In frequency domain terms, modal reduction amounts to representing a stable transfer function matrix in partial fraction form, and approximating by throwing away the summand with the smallest value of maximum $j\omega$–axis magnitude. As the poles approach the $j\omega$–axis, this method becomes equivalent asymptotically to a scheme known as approximation based on modal cost analysis (the underlying idea being to approximate so as to affect the cost to the least degree); it also becomes equivalent asymptotically to balanced realization truncation. See [7, 8, 9].

Yet another approach is to approximate the plant rather than the controller. Then a low order controller is designed using a low order of approximation of the plant, with the low order controller then used on the correct plant. There is both a general and a specific criticism of this idea. The general criticism is that in the overall design process leading to the controller, the approximation is carried out at an earlier step in the process than if the controller is approximated; each subsequent step in a design after an approximation propagates the effects of that approximation, and the ultimate effect at the end can be unclear, the more so the greater the number of design steps subsequent to the approximation. The specific criticism is that, as argued in [9], *satisfactory* approximation of the plant requires some knowledge in advance of the controller. So the designer is caught in an awkward logical loop. Iteration may be one way out. Also, the advance knowledge of the controller may be in practice not have to be precise, but equivalent to or deducible from the knowledge embedded in closed–loop specifications supplied in advance.

It is crucial to accept that the problem of controller reduction is distinct from the problem of (open–loop) model reduction, because of the presence of the plant. It is after all closed–loop performance (in the broadest sense of the word, i.e. stability, bandwidth, robustness as well as performance index value) which should be well approximated, and obviously the plant as well as the controller is a determinant. In fact one can argue that controller reduction can be regarded as a *frequency–weighted* L_∞ approximation problem, with no single procedure the best for defining the weight. Unfortunately, there are no nice algorithms available for solving such problems. There are however algorithms which appear to come close in practice, and can probably be proven to come close (closeness being measured using singular values of the error) in some cases. Such algorithms often offer less appealing performance when used on unstable controllers – this motivates a study of controller reduction using what are termed "fractional representations" of the controller.

The controller transfer function matrix is represented as a fraction, with numerator and denominator both stable; numerator and denominator are then both approximated (approximation of stable objects being easy) and a new quotient using the approximating numerator and denominator provides the reduced order controller transfer function.

Figure 1.1 displays the alternative approaches we have described above to the problem of finding a low order controller for a high order plant.

2 Controller Reduction and Frequency Weighting

We outline in this section how frequency weighted L_∞ approximation problems can arise. The choice of frequency weight is influenced by the choice of criterion thought most important in the approximation process, viz. maintenence of stability margin, performance (the term being used in a loose sense), or closed–loop transfer function.

Stability Margin Considerations for Frequency Weighting:

Let $G(s)$ be the transfer function matrix of a given linear time-invariant plant (with l inputs and m outputs), and let $K(s)$ be a stabilizing high order compensator (obtained by some standard procedure). Let $K_r(s)$ be a reduced order compensator, which we are seeking. Regard the closed loop system with $K_r(s)$ replacing $K(s)$ as being equivalent to that of Figure 2.1. It can then be concluded using this redrawing [10] (and it is now well known) that if

(i) $K(s)$ and $K_r(s)$ have the same number of poles in $\mathrm{Re}(s) > 0$ and no poles on the imaginary axis; and

(ii) either

$$\|[K(s) - K_r(s)]G(s)[I + K(s)G(s)]^{-1}\|_\infty < 1 \qquad (1)$$

or

$$\|[I + G(s)K(s)]^{-1}G(s)[K(s) - K_r(s)]\|_\infty < 1 \qquad (2)$$

then $K_r(s)$ is a stabilizing compensator. (The notation $\|A(s)\|_\infty$ means $\sup_\omega \max_i \alpha_i^{\frac{1}{2}}[A^*(j\omega)A(j\omega)]$. Here, $\lambda_i[X]$ denotes the ith eigenvalue of the matrix X).

This suggests a minimization problem: find a $K_r(s)$ satisfying (i) which at the same time minimizes the left side of (2.1) or (2.2), and has prescribed degree. The matrix $G(I + KG)^{-1} = (I + GK)^{-1}G$ acts as a *weighting matrix* in this case.

Performance Considerations for Frequency Weighting:

Consider the original closed-loop system in the presence of process and measurement noise, $w(t)$ and $v(t)$, as depicted in Figure 2.2. It is possible to compute the spectrum $\Phi_{qq}(j\omega)$ of the noise process of $q(t)$. (We assume stationarity of the excitation noises and closed-loop stability, so that the spectrum exists).

In order that a low order approximation $K_r(s)$ to $K(s)$ be a good approximation, it is important that it be most accurate in those frequency bands encountered in actual operation. Thus if $q(t)$ has little spectral energy in one band, $K(j\omega)$ need not be closely approximated there by $K_r(j\omega)$, while if the spectral energy in another band is high, approximation needs to be accurate. Let $V(j\omega)$ be a stable, minimum phase spectral factor of $\Phi_{qq}(j\omega)$. (Thus $VV^* = \Phi_{qq}$). Then the approximation problem becomes: find $K_r(j\omega)$ of nominated degree such that:

(i) $K(s)$ and $K_r(s)$ have the same number of poles in $\mathrm{Re}(s) > 0$ and no poles on the imaginary axis;

(ii) $\|[K(s) - K_r(s)]V(s)\|_\infty$ is minimized.

Note that replacement of $K(s)$ by $K_r(s)$ will change the spectrum of $q(\cdot)$ and to this extent, the choice of error is heuristic.

Closed-Loop Transfer Function Considerations for Frequency Weighting:

The closed-loop transfer function matrices with $K(s)$, $K_r(s)$ are

$$\begin{aligned} W(s) &= G(s)K(s)[I + G(s)K(s)]^{-1} = I - [I + G(s)K(s)]^{-1} \\ W_r(s) &= G(s)K_r(s)[I + G(s)K_r(s)]^{-1} = I - [I + G(s)K_r(s)]^{-1} \end{aligned}$$

Approximately, i.e. neglecting terms of second order in $K_r - K$, there holds

$$W_r(s) - W(s) = [I + G(s)K(s)]^{-1}G(s)[K_r(s) - K(s)][I + G(s)K(s)]^{-1}$$

and this suggests the following approximation problem: find $K_r(j\omega)$ of nominated degree so that

(i) $K(s)$ and $K_r(s)$ have the same number of poles in $\mathrm{Re}(s) > 0$ and no $j\omega$-axis poles;

(ii) $\|V_1(s)[K_r(s) - K(s)]V_2(s)\|_\infty$ is minimized, where $V_1 = (I + GK)^{-1}G$, $V_2 = (I + GK)^{-1}$.

Comparing (ii) with (2) shows that there is reduced weighting placed on frequencies in the high loop-gain region in this third approach as compared with the first approach.

Evidently, there are three choices possible in setting up the frequency weighted approximation problem. It turns out that further choices again exist in respect of the scheme used to evaluate the approximating K_r (including balanced realization truncation, Hankel norm approximation and covariance equivalent realization approximation), and the representation of the original controller transfer functions as the sum of a stable and unstable part, or as a factor of two-stable parts. Finally, it proves possible to move smoothly between designs which emphasize the stability and those which emphasize performance.

Examples show that no one method is universally the best.

An extended version of this paper appears in [11].

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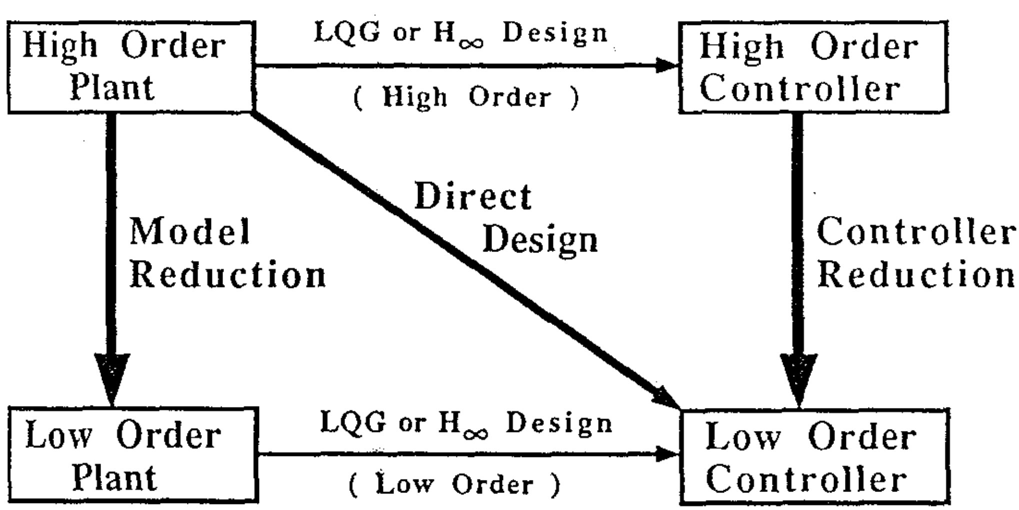

Figure 1.1

Basic principles of low order controller design

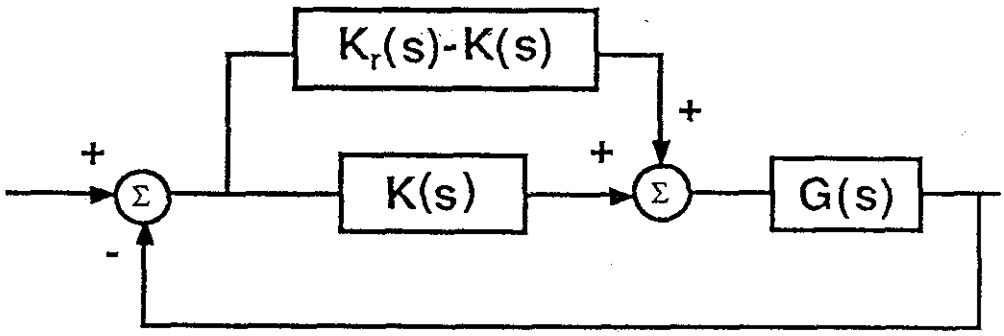

Figure 2.1

Rearrangement of feedback system with reduced order compe

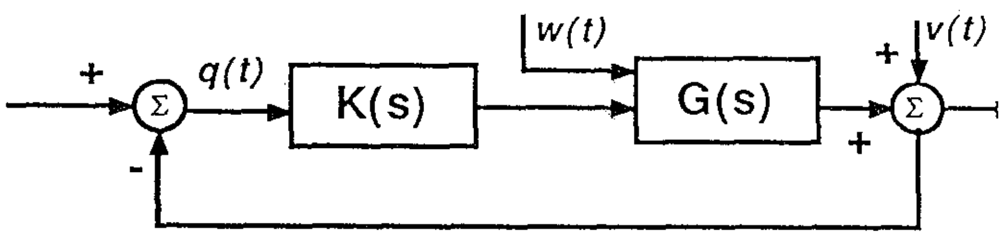

Figure 2.2

Original feedback scheme showing noise excitation