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ON THE STABILITY OF ADAPTIVE PARAMETER ESTIMATORS WITH COMPOSITE ERRORS AND SPLIT ALGORITHMS

Geoffrey A. Williamson* and C. Richard Johnson, Jr.*
School of Electrical Engineering
Cornell University
Ithaca, NY 14853, U. S. A.

Brian D. O. Anderson
Department of Systems Engineering
Australian National University
Canberra, A.C.T. 2601, Australia

ABSTRACT

For a frequently studied class of adaptive parameter estimation systems, local stability of the parameter estimator follows from satisfaction of a persistently spanning condition by the regressor together with satisfaction of a strictly positive real requirement on a key operator in the adaptive system. One may furthermore relax the operator condition by imposing a restriction on the spectral content of the regressor. However, for adaptive systems whose prediction errors have a composite form or which possess a split algorithm parameter update (termed SPACE system, for SPlit Algorithm Composite Error), these conditions are generically *insufficient* to guarantee local stability. Only when the parameter update law precisely compensates for the composite error can spectral restrictions and a persistently spanning regressor guarantee local stability. In this paper, we formally develop these results while noting that, in spite of their negative character, the results do not preclude the stability of SPACE systems. However, to achieve SPACE system stability, one requires more than a persistently spanning, spectrally restricted regressor.

1. *Introduction*

In this paper we study the local stability of a set of equations which models a wide class of adaptive parameter estimation systems. The adaptive systems in this class we term SPACE systems, for SPlit Algorithm – Composite Error system. The split algorithm and composite error are the two principal components of this model: the *composite error* is a measurable prediction error which depends on the errors in the parameter estimates, and the *split algorithm* is a recursive update of those parameter estimates, which uses in part the composite error. Integral to both components of the SPACE system are the system regressor, a vector of signals appearing in the adaptive system, and a number of linear operators which characterize the particular composite error and split algorithm at hand. We focus on the influence on parameter estimator stability that properties of the regressor and system operators have.

The equations governing a basic SPACE system are

$$e(k) = \sum_{i=1}^{n} H_i\left[x_i(k)\tilde{\theta}_i(k)\right] \qquad (1.1a)$$

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) + \mu F_i\left[x_i(k)\right]e(k), \quad i = 1,\ldots,n. \qquad (1.1b)$$

The *composite error* $e(k)$ in (1.1a) consists of a sum of filtered products of regressor elements $x_i(k)$ and parameter errors $\tilde{\theta}_i(k)$. H_i we assume to be a stable, linear, time-invariant filter. The term "composite error" reflects the fact that $e(k)$ is a composite of the (potentially) *differently* filtered regressor/parameter error products. The parameter error $\tilde{\theta}_i(k)$ is the difference $\tilde{\theta}_i(k) = \theta_i - \hat{\theta}_i(k)$ between a true or nominal value θ_i and its estimate $\hat{\theta}_i(k)$. The evolution of the parameter estimates follows the *split algorithm* (1.1b), in which the update form is "split" by permitting the different regressor filters F_i (considered to be linear, time-invariant filters which are both stable and stably invertible) in each $\hat{\theta}_i$ update law. The parameter μ is a step size which is chosen to be small to force slow, smooth adaptation.

Here we contrast the stability properties of equation (1.1) with those of a more "traditional" parameter estimation system (which we refer to as a "non-SPACE" system), governed by

$$e(k) = \sum_{i=1}^{n} H\left[x_i(k)\tilde{\theta}_i(k)\right] = H\left[X^T(k)\tilde{\Theta}(k)\right] \qquad (1.2a)$$

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) + \mu F\left[x_i(k)\right]e(k), \quad i = 1,\ldots,n. \qquad (1.2b)$$

$X(k)$ and $\tilde{\Theta}(k)$ in (1.2) are vectors of the regressor and parameter error elements, respectively. Notice that in (1.2), the filtering operations H and F are the *same*

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throughout the equations, in contrast to (1.1). This type of adaptive parameter estimation system is well-studied, and it applies to a variety of adaptive systems; see, for example, fundamental adaptive system structures in [1 – 6].

The equations in (1.1) represent a modest generalization of (1.2), and are a subset of SPACE systems as defined in [7]. In that paper, SPACE systems were introduced, and a number of applications of adaptive parameter estimation theory in which SPACE systems arose were presented. Such applications include a particular adaptive noise cancellation implementation, an adjustable model reference adaptive control problem, and the recursive identification of the parameters of an infinite impulse response tapped lattice filter. The common bond of these adaptive systems is that the model of (1.2) is inadequate to describe the evolution of their parameter errors. This fact motivated the development in [7] of a general adaptive system form encompassing the description of these situations. Here we study the stability properties of a simplified version of the general SPACE system equations in [7], namely (1.1).

Despite the seemingly modest differences between (1.1) and (1.2), conditions which have been successfully used to guarantee "good behavior" of the estimator (1.2) are insufficient to grant the same good behavior to (1.1). For our purposes, by good behavior we mean local stability at $\tilde{\Theta} = 0$ of the error system describing the evolution of the parameter errors in each equation. Averaging techniques are useful in the study of (1.2) in error system form [3], [8], [9], [10]. Essentially, for periodic excitation, local stability of (1.2) will follow from satisfaction of two conditions: (1) the regressor $X(k)$ must be persistently spanning [11], and (2) the operator composition HF^{-1} (with H and F taken from (1.2), and F assumed to be stably invertible) must be strictly positive real, or SPR. (HF^{-1} being SPR means $\mathrm{Re}[HF^{-1}(e^{j\omega})] > 0$ for all $\pi > \omega > 0$.) Furthermore, local stability of (1.2) can still result if HF^{-1} satisfies a strictly positive real condition only over a subset Ω of the frequency domain (*i.e.* for all $\omega \in \Omega$, $\mathrm{Re}[HF^{-1}(e^{j\omega})] > 0$). If a periodic, persistently spanning regressor has frequency content confined to Ω, then local stability of (1.2) follows.

However, the arguments leading to these results break down when applied to the stability analysis of (1.1). We show that when differences exist between the phase responses of the individual compositions $H_iF_i^{-1}$, no spectral restrictions will prevent the existence of periodic, persistently spanning regressors for which the error system is locally *unstable.* The implication of this fact is that for a broad class of adaptive systems decribed by (1.1), more than just spectral restrictions and persistently spanning conditions are needed to guarantee local stability of the error system for (1.1). An exception is a particularly special case of (1.1), in which the $H_iF_i^{-1}$ operators are identical. For this case, one may reapply the analysis used for (1.1). Note that the F_i and H_i operators need not be the same in this special case; only the compositions $H_iF_i^{-1}$ must agree.

Thus, in this paper we rigorously establish that only when the compositions $H_iF_i^{-1}$ are all equal (possibly up to a positive scaling factor) can one ensure local stability of the error system of (1.1) *for all* periodic, persistently spanning regressors with frequency content in a restricted set Ω. Otherwise, when differences exist between some of the operators, some persistently spanning regressors exist, for any spectral restrictions, which cause local error system instability.

2. *Averaging Analysis for non-SPACE Systems*

In this section we conduct a stability analysis of (1.2) using averaging techniques. The results are not new (see *e.g.* [8], [9], [12]), but are phrased to provide a backdrop for the analysis of SPACE systems in section 3. We confine our analysis to the case with periodic excitation.

To consider the stability analysis of (1.2), we combine (1.2a) and (1.2b) in terms of the parameter error variable $\tilde{\Theta}$. Let $Y(k) = [F[x_1(k)] \ \cdots \ F[x_n(k)]]$, and let $M = HF^{-1}$. Then (1.2) becomes

$$\tilde{\Theta}(k+1) = \Big[I - \mu R(k)\Big]\tilde{\Theta}(k) + O(\mu^2), \qquad (2.1)$$

where

$$R(k) = Y(k)\left\{\begin{bmatrix} M & & \\ & \ddots & \\ & & M \end{bmatrix} Y(k)\right\}^T. \qquad (2.2)$$

The $O(\mu^2)$ term appears since we have assumed $\tilde{\Theta}(k)$ is slowly varying (from a small step size μ) in relation to variations in X. We may then view H in (1.2a) as acting only on $x_i(k)$, with $\tilde{\theta}_i(k)$ being "roughly constant." See [8] or [10] for an explicit determination of the $O(\mu^2)$ perturbation, together with a characterization of a bound for the term.

We recall some terminology and state some definitions. We say $\{Y(k)\}$ is *persistently spanning* (PS) [11] if

$$\exists N, \alpha, \beta > 0 \text{ s.t. } \forall k_0 \quad \beta I \geq \sum_{k=k_0+1}^{k_0+N} Y(k)Y^T(k) \geq \alpha I, \qquad (2.3)$$

and we note that a linear, time invariant operator M is *strictly positive real* (SPR) if and only if

$$M(e^{j\omega}) + M^*(e^{j\omega}) = 2\mathrm{Re}\Big\{M(e^{j\omega})\Big\} > 0 \quad \forall \omega \in [0, 2\pi) \qquad (2.4)$$

and M is asymptotically stable [13]. We define E_Ω as

$$E_\Omega = \Big\{ X(k) : x_i(k) = \sum_{j=1}^{n_i} a_{ij}\cos(\omega_{ij} + \phi_{ij}), \quad \omega_{ij} \in \Omega,\ X(k) \text{ periodic} \Big\} \tag{2.5}$$

and E_{PS} as

$$E_{PS} = \Big\{ X(k) : X(k) \text{ is PS} \Big\}. \tag{2.6}$$

E_Ω is thus the set of periodic regressors whose components are finite sums of sinusoidal terms, and E_{PS} is the set of persistently spanning regressors. With these definitions in hand, we now state the following stability/instability theorem for (2.1) with $R(k)$ given by (2.2).

Theorem 2.1:

Let Ω be any open set in $(-\pi,\pi)$. If $HF^{-1} = M$ is asymptotically stable and satisfies $M(e^{j\omega}) + M^*(e^{j\omega}) > 0\ \forall \omega \in \Omega$, then for all $X \in E_\Omega \cap E_{PS}$, $\exists \overline{\mu}$ such that $\forall \mu \in (0,\overline{\mu})$, system (2.1) with (2.2) is locally stable about $\tilde{\Theta} = 0$. Furthermore, if $M(e^{j\omega_0}) + M^*(e^{j\omega_0}) < 0$ for some $\omega_0 \in \Omega$, $\exists X \in E_\Omega \cap E_{PS}$ for which $\exists \overline{\mu}$ such that $\forall \mu \in (0,\overline{\mu})$, (2.1) with (2.2) is locally unstable about $\tilde{\Theta} = 0$.

Proof:

The focus is on $\overline{R}_M[X] = \text{avg}[R(k)]$. If the real parts of the eigenvalues of $\overline{R}_M[X]$ are all positive, the homogeneous part of (2.1) is exponentially stable for small enough μ [10]. This exponential stability of the homogeneous part confers local stability to the full equation (2.1), with a bounded $O(\mu^2)$ term. If $\overline{R}_M[X]$ has an eigenvalue with a negative real part, (2.1) is *unstable* for small enough μ [10].

Assume that M is stable and satisfies the SPR condition. Given $X \in E_\Omega \cap E_{PS}$, we show that the eigenvalues of the matrix average

$$\overline{R}_M[X] = \frac{1}{\tau_X}\sum_{k=k_0+1}^{k_0+\tau_X} \begin{bmatrix} y_1(k) \\ \vdots \\ y_n(k) \end{bmatrix} \left\{ \begin{bmatrix} M & & \\ & \ddots & \\ & & M \end{bmatrix} \begin{bmatrix} y_1(k) \\ \vdots \\ y_n(k) \end{bmatrix} \right\}^T \tag{2.7}$$

all have positive real parts, where τ_X is the period of $X(k)$. Noting that τ_X is also the period of $Y(k)$ (in steady state), we consider $z^T\overline{R}_M[X]z$ for $z \in \mathbf{R}^n$:

$$z^T\overline{R}_M[X]z = \frac{1}{\tau_X}\sum_{k=k_0+1}^{k_0+\tau_X} \Big[z^TY(k)\Big]\Big(M\Big[z^TY(k)\Big]\Big). \tag{2.8}$$

Since M satisfies the SPR condition for all $\omega \in \Omega$, and $z^TY(k)$ is a sum of sinusoids with frequencies lying in Ω, (2.8) will be greater than zero as long as $z^TY(k)$ is not identically zero [13], [14]. Here, one needs to account for the effects of initial conditions, but since M is exponentially stable, such effects will die away and do not affect the conclusion of the theorem. Given that $X(k)$, and therefore $Y(k)$ (in steady state), is PS, $z^TY(k) \equiv 0$ only if $z = 0$. Therefore

$$z^T\overline{R}_M[X]z > 0 \quad \forall z \neq 0. \tag{2.9}$$

Thus we have $\overline{R}_M[X] + \overline{R}_M[X]^T > 0$, which is sufficient to guarantee that the real parts of the eigenvalues of $\overline{R}_M[X]$ are all positive. We therefore conclude that the homogeneous part of (2.1) is exponentially stable for $0 < \mu < \overline{\mu}$, for some $\overline{\mu}$. This exponential stability extends to local stability of (2.1) in its entirety when the $O(\mu^2)$ term is bounded. This proves the first part of the theorem.

Now suppose that $\text{Re}[M(e^{j\omega_0})] < 0$ for some $\omega_0 \in \Omega$. Since Ω is open and $\text{Re}[M(e^{j\omega})]$ is continuous in ω, there also exists $\omega_0' \in \Omega$ with $\text{Re}[M(e^{j\omega_0'})] < 0$ such that $\omega_0'/\pi \in \mathbf{Q}$. Hence $\cos(\omega_0' k)$ is periodic. For notational convenience, let $\omega_0' = \omega_0$. Let $X(k)$ be such that $Y(k) = [\cos(\omega_0 k)\ 0\ \cdots\ 0]^T$ in steady state. Then

$$\overline{R}_M[X] = \begin{bmatrix} \frac{1}{2}\text{Re}[M(e^{j\omega_0})] & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & & & 0 \end{bmatrix}. \tag{2.10}$$

Since $\text{Re}[M(e^{j\omega_0})] < 0$, $\overline{R}_M[X]$ has a negative eigenvalue. We may furthermore modify $Y(k)$ by replacing the last $n-1$ entries with sinusoids of frequencies in Ω different from ω_0 so that $Y(k)$ is PS, without affecting the presence of the negative eigenvalue. Then, for small μ we have the homogeneous part of (2.1) exponentially *unstable* for this $X \in E_\Omega \cap E_{PS}$, hence (2.1) is locally unstable for small μ. This proves the second part of the theorem.

$\nabla\nabla\nabla$

Theorem 2.1 shows the use of PS and SPR conditions in establishing local stability of (2.1) and (2.2) for an entire class of regressors. With $\Omega_M = \{\omega : M(e^{j\omega}) + M^*(e^{j\omega}) > 0\}$, (2.1) is locally stable *for all* $X \in E_{\Omega_M} \cap E_{PS}$. However, $X \in E_{\Omega_M} \cap E_{PS}$ is not *necessary* for local stability. For instance, having the power in X lie *predominantly* in Ω_M is sufficient to assure local stability. In $\overline{R}_M[X]$, the average of $R(k)$ from (2.2), the "positive contribution" at frequencies in Ω_M then outweighs the "negative contribution" at other frequencies. This effect is an interpretation of the average SPR condition of [12]. In this approach, one considers the positivity of eigenvalues of the symmetric part of $\overline{R}_M[X]$, which is sufficient (but not necessary) to establish positivity of the real parts of eigenvalues of $\overline{R}_M[X]$.

One may interpret Theorem 2.1 in the following way. A persistent spanning condition and a spectral restriction on the regressor X are enough to guarantee local stability. Often one may satisfy the spanning condition through the influence of external signals [15], [16]. One may meet the spectral restriction, which arises from the SPR operator condition, by similarly restricting the spectrum of external signals to lie in an appropriate frequency band. Thus, there exists a "good" frequency range (in the sense of providing local stability) for the system excitation. In the next section we find that for a general SPACE system, there is no such "good" frequency range.

3. *Spectrally-Restricted Excitation and SPACE System Stability*

We now turn our attention to the SPACE system in (1.1). To begin, we form the error system by combining the equations in (1.1). This error system again has the form

$$\tilde{\Theta}(k+1) = \left[I - \mu R(k)\right]\tilde{\Theta}(k) + O(\mu^2), \qquad (3.1)$$

though now

$$R(k) = Y(k)\left\{\begin{bmatrix} M_1 & & \\ & \ddots & \\ & & M_n \end{bmatrix} Y(k)\right\}^T. \qquad (3.2)$$

Here $Y(k) = [F_1[x_1(k)] \; \cdots \; F_n[x_n(k)]]^T$ and $M_i = H_i F_i^{-1}$. Notice that the essential difference between (3.2) and (2.2) is that the diagonal operators M_i in (3.2) now vary, while they are the same in (2.2). One may show that approximating $H_i[x_i(k)\tilde{\theta}_i(k)]$ in (1.1a) as $H_i[x_i(k)]\tilde{\theta}_i(k)$ results in the $O(\mu^2)$ perturbation of (3.1) in a fashion similar to the analogous result for (2.1) with (2.2).

Again, we apply averaging techniques to the stability analysis of (3.1) and (3.2). As in the proof of Theorem 2.1, we are concerned with the eigenstructure of averages of $R(k)$. We set $\overline{R}_{\mathcal{M}}[X] = \text{avg}[R(k)]$ when $R(k)$ is given by (3.2).

Lemma 3.1:

Let $\overline{R}_{\mathcal{M}}[X]$ be the average of $R(k)$ from (3.2), and suppose there do not exist nonzero real numbers $\{\alpha_i\}_{i=1}^n$ such that $\alpha_1 M_1 = \cdots = \alpha_n M_n$. Then given any open $\Omega \subset (-\pi, \pi)$, $\exists X \in E_\Omega \cap E_{PS}$ such that

$$\min_i \text{Re}\left[\lambda_i\left(\overline{R}_{\mathcal{M}}[X]\right)\right] < 0. \qquad (3.3)$$

Proof:

Since the operators $\{M_i\}$ are not scaled versions of each other, then for any open Ω there exists $\omega_0 \in \Omega$ for which the phase responses of $M_i(e^{j\omega_0})$ and $M_j(e^{j\omega_0})$ differ, for some i, j pair. Since Ω is open, we can find $\omega_0' \in \mathbf{Q}$ with this same property, as in the proof of Theorem 2.1. Again let $\omega_0' = \omega_0$ for convenience. Assume that $i = 1$, $j = 2$, and set $M_\ell(e^{j\omega_0}) = m_\ell e^{j\eta_\ell}$ for $\ell = 1, 2$, with $m_\ell \geq 0$. Then we have $\eta_1 \neq \eta_2$.

Now let

$$Y(k) = \begin{bmatrix} \cos(\omega_0 k) \\ \cos(\omega_0 k + \frac{\eta_1 - \eta_2}{2}) \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \qquad (3.4)$$

Now, one may calculate that

$$\begin{aligned} &\overline{R}_{\mathcal{M}}[X] \\ &= \frac{1}{\tau_X}\sum_{k=1}^{\tau_X} \begin{bmatrix} \cos(\omega_0 k) \\ \cos(\omega_0 k + \frac{\eta_1 - \eta_2}{2}) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} m_1 \cos(\omega_0 k + \eta_1) \\ m_2 \cos(\omega_0 k + \frac{\eta_1 + \eta_2}{2}) \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \\ &= \frac{1}{2}\begin{bmatrix} m_1 \cos(\eta_1) & m_2 \cos(\frac{\eta_1+\eta_2}{2}) & 0 & \cdots & 0 \\ m_1 \cos(\frac{\eta_1+\eta_2}{2}) & m_2 \cos(\eta_2) & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix}. \end{aligned} \qquad (3.5)$$

The determinant of the non-zero 2×2 block of $\overline{R}_{\mathcal{M}}[X]$ in (3.5) is

$$\begin{aligned} \det\left(\overline{R}_{\mathcal{M}}[X]\right) &= \frac{m_1 m_2}{2}\left(\cos\eta_1 \cos\eta_2 - \cos^2(\frac{\eta_1+\eta_2}{2})\right) \\ &= \frac{m_1 m_2}{4}\left(\cos(\eta_1 - \eta_2) - 1\right). \end{aligned} \qquad (3.6)$$

Since $\eta_1 \neq \eta_2$, this determinant is negative, implying $\overline{R}_{\mathcal{M}}[X]$ has a negative eigenvalue. As in the proof of Theorem 2.1, one may augment the zero entries of $Y(k)$ in (3.4) with sinusoids of frequency different from ω_0 in order to achieve $Y(k) \in E_{PS}$ without altering the presence of the negative eigenvalue in $\overline{R}_{\mathcal{M}}[X]$, thus completing the proof.

$\nabla\nabla\nabla$

Lemma 3.1 implies that without a precise match between the operator compositions $\{\alpha_i H_i F_i^{-1}\}$ in a SPACE system, one cannot guarantee local stability of the SPACE system for all (persistently spanning) regressors which have a particular spectral restriction. We formalize this statement in the following.

Theorem 3.1:

If for some i, j pair $H_i F_i^{-1} \neq \beta H_j F_j^{-1}$ $\forall \beta \in \mathbf{R}$, $\beta \neq 0$, then for every open set $\Omega \subset (-\pi, \pi)$, there exists periodic $X \in E_\Omega \cap E_{PS}$ and $\exists \overline{\mu} > 0$ such that $\forall \mu \in (0, \overline{\mu})$, (3.1) with (3.2) is locally unstable at the origin.

Proof:

Given the conditions of the theorem, one may conclude from Lemma 3.1 that for any Ω, $\exists X \in E_\Omega \cap E_{PS}$ for which $\min_i \mathrm{Re}[\lambda_i(\overline{R}_{\mathcal{M}}[X])] < 0$. Then, using averaging theory from [10], for μ small enough the homogeneous part of (3.1) with (3.2) is unstable at the origin, and hence (3.1) in full is locally unstable.

$\nabla\nabla\nabla$

The rather negative result of Theorem 3.1 tells us that we do not have a stability result for a broad class of SPACE systems similar to Theorem 2.1 for non-SPACE systems. In order to hope for stability in the sense of Theorem 2.1, we must ensure *at least* that $\beta_1 H_1 F_1^{-1} = \cdots = \beta_n H_n F_n^{-1}$. Since each $H_i F_i^{-1}$ must be SPR for such a result to hold, we would also require $\beta_i > 0$. If $H_1 F_1^{-1} = \cdots = H_n F_n^{-1}$, then the the SPACE system stability question becomes a non-SPACE one, since the operator matrix in (3.2) has in this case identical operators all along the diagonal, as in (2.2). We have the following theorem for this special case of SPACE systems.

Theorem 3.2:

If $H_i F_i^{-1} = M$ for all i, and M is asymptotically stable with $M(e^{j\omega}) + M^*(e^{j\omega}) > 0$ satisfied $\forall \omega \in \Omega$, then for all $X \in E_\Omega \cap E_{PS}$, $\exists \overline{\mu}$ such that $\forall \mu \in (0, \overline{\mu})$ system (3.1) with (3.2) is locally stable at the origin.

If $M(e^{j\omega_0}) + M^*(e^{j\omega_0}) < 0$ for some $\omega_0 \in \Omega$, then there exists $X \in E_\Omega \cap E_{PS}$ for which $\exists \overline{\mu}$ such that $\forall \mu \in (0, \overline{\mu})$, (3.1) with (3.2) is locally unstable at the origin.

Proof:

With $H_i F_i^{-1} = M$ for all i, $\overline{R}_{\mathcal{M}}[X] = \overline{R}_M[X]$, so that the proof follows from a direct application of Theorem 2.1.

$\nabla\nabla\nabla$

4. *Conclusions*

We summarize the results of sections 2 and 3 as follows. For the non-SPACE system (1.2), one can guarantee error system local stability with a persistent spanning condition and a spectral restriction on the regressor. The permitted frequency range for the spectral restriction is the set Ω for which HF^{-1} satisfies the SPR condition. However, for the SPACE system (1.1), unless each $H_i F_i^{-1}$ is a scaled version of the other compositions $H_j F_j^{-1}$, these conditions do not assure local stability of the error system. If the frequency content of the regressor is restricted to *any* (open) set Ω, there still exists a persistently spanning regressor satisfying that spectral restriction which locally destabilizes the error system.

This rather negative result leaves us with the seeming requirement of exactly matching each $H_i F_i^{-1}$ with all the other $H_j F_j^{-1}$ operators in order to achieve local stability for SPACE systems. However, in applications the H_i operators are often parametrized by the unknown parameters and are therefore themselves unknown [7]. It is thus unlikely that one will be able to specify each algorithm filter F_i to match precisely its counterpart H_i in the composite error. In the event of even a slight mismatch, Theorem 3.1 states that there is the potential for instability given a frequency content restriction to *any* set Ω. One alternative approach is based on using a gradient descent algorithm to minimize the square of the prediction error e in (1.1a). Such an algorithm results in choosing F_i in (1.1b) as $\widehat{H}_i$, a time-varying approximation of H_i based on current parameter estimates [17]. This method offers local stability once $\widehat{H}_i \to H_i$, as then each $H_i F_i^{-1}$ composition equals the identity operator.

However, even for time-invariant algorithm operators F_i, one would still expect reasonable behavior of the error system as long as the compositions $H_i F_i^{-1}$ were a *close* match, though not *exact*. Furthermore, given a collection of operators $\{H_i F_i^{-1}\}$, one would not assume the *non*-existence of regressor sequences which yield locally stable error system behavior. What our results indicate is that for SPACE systems, something more than persistent spanning conditions and spectral restrictions (for example, the exploitation of fixed functional relationships between different regressor entries) is needed to characterize the set of regressors which provide stable behavior.

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