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A SIMPLE CIRCUIT-THEORETICAL APPROACH FOR THE STABILITY TESTING OF 2-D DIGITAL FILTERS

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Abstract

A new procedure for checking the stability of 2-D digital filters is proposed. The key part of the test involves construction of related allpass sections of reduced order, stability of which is shown to be equivalent to the stability of the original function. This is a direct extension to a similar approach, developed recently in details for the 1-D case [1]. The results are shown to be equivalent to some known stability testing methods in 2-D, but have the advantage of an additional insight gained from the circuit interpretation. Explicit formulas for checking the stability in terms of the filter coefficients are derived for some special cases, and numerical examples are included to clarify the application of the proposed approach.

1. Introduction

A major concern in the design of 2-D recursive digital filters is ensuring their stability. After the original work of Shanks [2], Huang [3] proposed a simplified, often used ever since criterion for checking the stability of 2-D digital filters, which for the purpose of further reference and unified notation we state here, as modified in [4]:

Huang's theorem : A causal filter with a z - transform $H(z_1, z_2) = P(z_1, z_2)/D(z_1, z_2)$, where P and D are polynomials in z_1 and z_2, is stable in the sense that there are no values of z_1 and z_2 for which $D(z_1, z_2) = 0$; $|z_1| \geq 1$ and $|z_2| \geq 1$ iff :

i) the map of $\partial d_2 \triangleq (z_2; |z_2| = 1)$ in the z_2 plane, according to $D(z_1, z_2) = 0$, lies strictly inside $d_1 \triangleq (z_1; |z_1| \leq 1)$; and

ii) no point in $d_2 \triangleq (z_2; |z_2| \geq 1)$ maps into the point $z_1 = \infty$ by the relation $z_1^{-M} D(z_1, z_2) = 0$, where M is the maximum degree of z_1 in $D(z_1, z_2)$.

To check the conditions derived there, a number of test procedures [3], [5], [6], [7] requiring different levels of computational efforts, have been proposed with each showing equivalent results on test examples.

The object of this paper is to present a procedure for checking the stability conditions, which is believed to be conceptually simpler than the existing ones, because it provides direct physical interpretation of each step. The procedure involves successive reduction of the order of allpass sections, whose stability depends on the stability of the original filter. In addition it requires testing of functions of one variable for a strictly bounded real (SBR) property. In this paper the stability of the overall filter is assumed to depend on the denominator polynomial only and hence cases of reducible 2-D transfer functions or nonessential singularities of the second kind are not considered. The results are shown to be equivalent to those of some known methods, but have the advantage of additional insight gained from the circuit interpretation. The method can be considered as a direct extension to the 2-D case of the well developed theory for the 1-D case [1].

The organization of the paper is as follows. In Section 2 we outline the basic idea behind the method for the case of real filter coefficients. In Section 3 we explicitly derive, in terms of the filter coefficients, the conditions for stability of certain low order 2-D filters. In section 4 the equivalence of this method to the modified Jury table method in [4] is demonstrated. Numerical example of our approach is included in Section 5. Possible circuit interpretation of the method is presented in Section 6, while Section 7 contains concluding remarks.

Notations - In the following derivation the variables z_1, z_2 denote the transform domain complex variables for the discrete time systems under consideration. The explicit dependence on these variables is dropped for notational convenience in the absence of any ambiguity. The complex conjugate of given quantity is denoted by the "*" superscript. The discussion also heavily relies on the concept of "structural boundedness", which we briefly review next, see e.g.,[8]. Thus, if $H(z_1, z_2)$ is a stable digital filter transfer function, that is real valued for real z_1 and z_2, and satisfies $|H(e^{j\omega_1}, e^{j\omega_2})| \leq 1$ for $\forall\ \omega_1,\ \omega_2$, it is called a "bounded real"(BR) function. BR functions, for which strict inequality holds for $\forall\ \omega_1,\ \omega_2$ are referred to as "strictly bounded real" (SBR) functions. A BR function, for which equality holds for $\forall\ \omega_1,\ \omega_2$ is called a "lossless bounded real" (LBR) function. Thus a stable allpass function with real coefficients is LBR. The above definitions are direct extensions of the 1-D case [1]. Functions with complex coefficients, which satisfy above relations will be called "strictly bounded complex" (SBC), and "lossless bounded complex" (LBC), correspondingly.

2. The Generalized Schur-Cohn Test

Let $H(z_1, z_2) = P(z_1, z_2)/D(z_1, z_2)$ be given 2-D transfer function with real coefficients. To check the stability of this function in the sense of Huang's theorem we first form the 2-D allpass function:

$$S(z_1, z_2) = z_1^M z_2^N \frac{D(z_1^{-1}, z_2^{-1})}{D(z_1, z_2)}$$

where N is the maximum degree of z_2 in $D(z_1, z_2)$.

This function can be rewritten as a ratio of polynomials in one variable, with the coefficients being polynomials of the other variable, i.e.:

$$S(z_1, z_2) = \frac{\tilde{a}_M(z_2) + \cdots + \tilde{a}_0(z_2) z_1^M}{a_0(z_2) + \cdots + a_M(z_2) z_1^M}$$

where for $\forall\ i$, a_i and $\tilde{a}_i$ are polynomials in z_2, and the tilde sign denotes the operation of replacing z_2 by z_2^{-1} and multiplying by z_2^N. Note that in the case when the maximum degree of each $a_i(z_2)$ is equal to N, this corresponds to taking the paraconjugate [8].

This can further be rewritten as:

$$\begin{aligned} S(z_1, z_2) &= \frac{\tilde{a}_M(z_2)}{a_M(z_2)} \frac{1 + \tilde{n}_{M-1}(z_2) z_1 + \cdots + \tilde{n}_0(z_2) z_1^M}{n_0(z_2) + \cdots + n_{M-1}(z_2) z_1^{M-1} + z_1^M} \\ &= A_M(z_2) G_M(z_1, z_2) \end{aligned} \quad (1)$$

where:

$$n_i(z_2) = \frac{a_i(z_2)}{a_M(z_2)};\ \ \tilde{n}_i(z_2) = n_i(z_2^{-1});\ \ A_M(z_2) = \frac{\tilde{a}_M(z_2)}{a_M(z_2)}$$

and

$$G_M(z_1, z_2) = \frac{1 + \tilde{n}_{M-1}(z_2) z_1 + \cdots + \tilde{n}_0(z_2) z_1^M}{n_0(z_2) + \cdots + n_{M-1}(z_2) z_1^{M-1} + z_1^M}$$

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For the overall stability of $S(z_1, z_2)$ this decomposition implies the necessary condition that $A_M(z_2)$ is a stable allpass function in z_2, i.e., $|A_M(e^{j\omega_2})| = 1$ for $\forall\ \omega_2$, and has all its poles inside the unit circle. Assuming that this condition has been verified using any one of the known 1-D stability tests, we proceed to investigate the stability of the second term in (1). It is clear that $G_M(z_1, z_2)$ should be a stable allpass function.

The basic idea behind our method, in analogy with the 1-D case, is to successively reduce the order of G_M in the z_1 variable. As it is shown, the stability of the reduced order functions G_{M-i} formed in the method is directly related to the stability of G_M, so that after $M-1$ steps we can decide on the overall stability of the function from the coefficients of the remaining first order allpass, which are functions of the other variable only. Thus on each step we have to deal with functions of one variable, which is a major improvement in terms of complexity.

To this end we consider the transfer function $G_{M-1}(z_1, z_2)$, derived as follows:

$$G_{M-1}(z_1, z_2) = z_1 \frac{G_M(z_1, z_2) - k_M(z_2)}{1 - \tilde{k}_M(z_2) G_M(z_1, z_2)} \tag{2}$$

where:

$$k_M(z_2) \triangleq G_M(\infty, z_2) = \tilde{n}_0(z_2); \quad \tilde{k}_M(z_2) = n_0(z_2)$$

With this definition, we can state the following important theorem.

Theorem : $G_M(z_1, z_2)$ is a stable allpass function (and hence LBR) iff $G_{M-1}(z_1, z_2)$ is a stable allpass function (LBR), and the function $\tilde{k}_M(z_2)$ is strictly bounded real (SBR).

Proof. Assume $G_M(z_1, z_2)$ is a stable allpass. We first show that G_{M-1} is allpass. This property follows directly from (2), once we observe that for $|z_1| = 1$ and $|z_2| = 1$ the relationship becomes:

$$G_{M-1}(e^{j\omega_1}, e^{j\omega_2}) = e^{j\omega_1} e^{j\Theta} \frac{1 - k_M(e^{j\omega_2}) e^{-j\Theta}}{1 - [k_M(e^{j\omega_2}) e^{-j\Theta}]^*} \tag{3}$$

Hence, as long as $|k_M(e^{j\omega_2})| \neq 1$ for $\forall\ \omega_2$, the allpass property follows.

Also, using (1) and (2) we have:

$$G_{M-1}(z_1, z_2) = \frac{(1 - n_0\tilde{n}_0) + \cdots + (\tilde{n}_1 - n_{M-1}\tilde{n}_0) z_1^{M-1}}{(n_1 - \tilde{n}_{M-1} n_0) + \cdots + (1 - n_0\tilde{n}_0) z_1^{M-1}} \tag{4}$$

where the explicit dependence of n_i, $\tilde{n}_i$ on z_2 was dropped for notational convenience. The important result in (4) is that the function $G_{M-1}(z_1, z_2)$ is allpass of reduced order, $M-1$ in the z_1 variable.

Next we prove that if $G_M(z_1, z_2)$ is LBR, so is $G_{M-1}(z_1, z_2)$. From (2) it follows that the singularities z_{10} of G_{M-1} are solutions of

$$G_M(z_{10}, z_2) = \frac{1}{\tilde{k}_M(z_2)} \tag{5}$$

From the first condition of Huang's theorem we know that all the roots of $G_M(z_1, z_2)_{|z_2|=1} = 0$ are inside the unit circle and according to the maximum modulus theorem will attain values as [1]:

$$|G_M(z_1, z_2)|_{|z_2|=1} < 1 \text{ for } |z_1| > 1$$

unless G_M is a constant, in which case equality holds.

$$|G_M(z_1, z_2)|_{|z_2|=1} > 1 \text{ for } |z_1| < 1 \tag{6}$$

$$|G_M(z_1, z_2)|_{|z_1|=1; |z_2|=1} = 1$$

Using the definition $k_M(z_2) \triangleq G_M(\infty, z_2)$ it follows from (6) that $|G_M(z_{10}, z_2)| > 1$ for $|z_2| = 1$. This result implies that all singularities of $G_{M-1}(z_1, z_2)_{|z_2|=1}$ are inside the unit circle in the z_1 plane, which proves the first condition of the theorem to hold. The stability of G_M by the second Huang's condition implies that $n_0 = \tilde{k}_M$ is stable, and since $|\tilde{k}_M(e^{j\omega_2})| < 1$ for $\forall\ \omega_2$ it follows, that $\tilde{k}_M$ is SBR, as asserted in the theorem.

To prove the second condition in Huang's theorem for the denominator of $G_{M-1}(z_1, z_2)$, solving

$$z_1^{-(M-1)} D_{M-1}(z_1, z_2) = (1 - n_0\tilde{n}_0) = 0$$

where $D_{M-1}(z_1, z_2)$ is the denominator polynomial in (4) we observe that equality can hold only for values of z_2 with $|z_2| < 1$, which is implied by the relation $n_0(z_2) = \tilde{k}_M(z_2)$ and $\tilde{k}_M$ being SBR.

Thus, if G_M is LBR, it follows that $\tilde{k}_M$ is SBR and that G_{M-1} is also LBR.

To prove the theorem in the other direction we assume that G_{M-1} is LBR, and $\tilde{k}_M$, as defined above, is SBR. Reversing the relation in (2) we arrive at:

$$G_M(z_1, z_2) = \frac{G_{M-1}(z_1, z_2) + z_1 k_M(z_2)}{z_1 + \tilde{k}_M G_{M-1}(z_1, z_2)} \tag{7}$$

With $G_{M-1}(z_1, z_2)$ allpass it is easy to see that so is $G_M(z_1, z_2)$. Next, if z_{10} is a pole of G_M, then:

$$z_{10}^{-1} G_{M-1}(z_{10}, z_2) = -\frac{1}{\tilde{k}_M(z_2)} \tag{8}$$

When evaluated at $|z_2| = 1$ this implies

$$|z_{10}^{-1} G_{M-1}(z_{10}, e^{j\omega_2})| > 1 \tag{9}$$

Because of the LBR assumption on G_{M-1} we must then have $|z_{10}| < 1$ in order for (9) to hold, and hence the poles of G_M where $|z_2| = 1$ lie inside the unit circle in z_1, as required by Huang's first condition.

The second condition of Huang's theorem follows directly from the assumption that $\tilde{k}_M(z_2)$ is SBR.

Summarizing, a necessary and sufficient set of conditions for the allpass function $G_M(z_1, z_2)$ to be stable are therefore: *a)* the function $\tilde{k}_M(z_2)$ must be SBR, and *b)* the allpass function $G_{M-1}(z_1, z_2)$ be stable.

3. Explicit coefficient conditions for some low order transfer functions

In this section we derive the explicit conditions on the filter coefficients for some low order cases, using the outlined method.

Example 1. General case first order transfer function

Consider the case of a filter with denominator polynomial:

$$D(z_1, z_2) = c + bz_1 + az_2 + z_1 z_2$$

Following the procedure of Section 2 we first derive the associated allpass function. Denoting $c + az_2 = a_0(z_2)$; $b + z_2 = a_1(z_2)$ we get, using the previously adopted notations:

$$H(z_1, z_2) = \frac{\tilde{a}_1(z_2)}{a_1(z_2)} \frac{1 + \tilde{n}_0(z_2) z_1}{n_0(z_2) + z_1} = A_1(z_2) G_1(z_2)$$

The function $A_1(z_2) = (1 + bz_2)/(b + z_2)$ is stable allpass in z_2 for $|b| < 1$.

We next check $\tilde{k}_1(z_2) = n_0(z_2)$ for being SBR, or

$$|n_o(e^{j\omega_2})|^2 = \frac{(c + az_2)}{(b + z_2)} \frac{(c + az_2^*)}{(b + z_2^*)} < 1$$

Recognizing, that on the unit circle $z_2^* = z_2^{-1}$, and denoting $z_2 + z_2^{-1} = 2x$ as usually adopted notation, we get:

$$c^2 + a^2 + ac(z_2 + z_2^{-1}) < 1 + b^2 + b(z_2 + z_2^{-1})$$

or equivalently the condition:

$$1 + b^2 - a^2 - c^2 + 2(b - ac)x > 0 \quad for \ -1 \leq x \leq 1 \tag{10}$$

which is derived in a number of papers [3],[5].

Example 2. General case second order polynomial

Our next example is of a general case second order polynomial. Its coefficients can conveniently be expressed in the matrix notation:

$$D_2(z_1, z_2) = \begin{bmatrix} 1 & z_1 & z_1^2 \end{bmatrix} \begin{bmatrix} f & g & d \\ h & c & a \\ e & b & 1 \end{bmatrix} \begin{bmatrix} 1 \\ z_2 \\ z_2^2 \end{bmatrix}$$

The associated allpass structure can be written using the previously adopted notations as:

$$H(z_1, z_2) = \frac{\tilde{a}_2(z_2)}{a_2(z_2)} \frac{1 + \tilde{n}_1(z_2) z_1 + \tilde{n}_0(z_2) z_1^2}{n_0(z_2) + n_1(z_2) z_1 + z_1^2} = A_2(z_2) G_2(z_1, z_2)$$

We first check the stability of $A_2(z_2)$, which corresponds to checking the second Huang's condition, i.e. we check the roots of:

$$z_2^2 + bz_2 + e = 0 \tag{11}$$

If the test passes, i.e.$|z_2|_{1,2} < 1$, we proceed to check $\tilde{k}_2(z_2)$ for being SBR. Again recognizing that on the unit circle $z_2^{-1} = z_2^*$, using (11) we can express the conditions on the coefficients in terms of the real variable $x = \frac{1}{2}(z_2 + z_2^{-1})$, which are:

$$\begin{aligned} &4(e - df)x^2 + 2(b + be - dg - fg)x \\ &+ (1 + b^2 + e^2 - d^2 - g^2 - f^2 + 2df - 2e) \end{aligned} > 0 \;\; for \; |x| < 1 \tag{12}$$

If no roots of this quadratic equation are in $|x| < 1$ we can proceed to form, using (4), the reduced order polynomial $G_1(z_1, z_2)$. The condition on $\tilde{k}_1(z_2)$ to be SBR becomes, expressed in terms of the a_i's as follows:

$$(\tilde{a}_1 a_2 - a_1 \tilde{a}_0)^2 < (a_2 \tilde{a}_2 - a_0 \tilde{a}_0)^2 \tag{13}$$

After some algebra, the polynomial in the real variable x to be checked for roots in $|x| < 1$ becomes:

$$\begin{aligned} &16(F^2 - AE)x^4 + 8(2FG - AD - BE)x^3 + \\ &4(2FH + G^2 - 4F^2 - AC - BD - CE + 4AE)x^2 + \\ &2(2GH - 4FG - AB - BC - CD - DE + 3(AD + BE))x + \\ &H^2 - A^2 - B^2 - C^2 - D^2 - E^2 - \\ &2(2FH - AC - BD - CE - AE) > 0 \end{aligned} \tag{14}$$

where the quantities above are defined in terms of the filter coefficients as follows:

$$\begin{aligned} A &= ae - dh \\ B &= ab + ce - cd - gh \\ C &= a + bc + eh - fh - cg - ad \\ D &= c + bh - cf - ag \\ E &= h - af \\ F &= e - df \\ G &= b + be - dg - fg \\ H &= 1 + b^2 + e^2 - d^2 - f^2 - g^2 \end{aligned} \tag{15}$$

This fourth order polynomial can further be checked for roots inside the $|x| < 1$ by well known methods e.g.[5].

It is a common practice in filter design to impose some symmetry conditions on the coefficient matrix. In this case, assuming symmetry along the main diagonal of the coefficient matrix the computations can be simplified. The polynomial to be checked remains as in (14), but the quantities must be redefined as follows:

$$\begin{aligned} A &= ad - dg & E &= g - af \\ B &= a^2 - g^2 & F &= d - df \\ C &= a + ac + dg - fg - cg - ad & G &= a + ad - dg - fg \\ D &= c - cf & H &= 1 + a^2 - f^2 - g^2 \end{aligned} \tag{16}$$

As it can be seen the advantage of this method is in the conceptually straightforward manner in which the equations are set up and solved. It should also be noted, that derivation of the stability conditions for the general case second order polynomials using some other methods may become very complicated, whereas this circuit-theoretical approach can relatively easily be extended to higher dimensionalities as well.

4. Relation between the proposed method and the Maria - Fahmy method

The purpose of this section is to establish the equivalence between the above method, and the Maria - Fahmy method in [4]. The test procedures in [4] use rewriting the denominator of the tested function in terms of one variable only:

$$D(z_1, z_2) = a_M(z_2) z_1^M + a_{M-1}(z_2) z_1^{M-1} + \cdots + a_0(z_2)$$

where

$$a_i(z_2) = \sum_{i=0}^{n} b_{ik} z_2^i$$

The coefficients $a_i(z_2)$ are next arranged as first entries of the Jury table and are used to successively obtain the coefficient sets $b_i(z_2)$, $c_i(z_2)$,... using the relations:

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n^* & a_k^* \end{vmatrix}; \;\; c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1}^* & b_k^* \end{vmatrix}; \;\; \cdots$$

The polynomials b_0, $c_0, \cdots, t_0$ can be shown to be functions of the real variable x. The modified Jury criterion, which is a test for Huang's first condition, takes then the form:

For $-1 \le x \le 1$

$b_0(x) < 0$; $c_0(x) > 0$; $d_0(x) > 0$; $\cdots; t_0(x) > 0$ or simplified further:

i) $b_0(0) < 0$; $c_0(0) > 0$; $d_0(0) > 0 \cdots; t_0(0) > 0$

ii) the polynomials $b_0(x)$, $c_0(x)$, $\cdots, t_0(x)$ have no real roots for $-1 \le x \le 1$

The second Huang's condition is checked from solving:

$$\lim_{z_1 \to \infty} z_1^{-M} D(z_1, z_2) = 0$$

to see whether there are roots of magnitude greater than 1.

To show the equivalence of the procedures we first observe that the test for the second condition corresponds directly to checking $A_M(z_2)$ for stability. Next, the condition on $b_0(x) = a_0^2 - a_M^2 < 0$ corresponds to checking $\tilde{k}_M(z_2)$ for being SBR. Indeed from:

$$\tilde{k}_M(e^{j\omega_2}) \tilde{k}_M^*(e^{j\omega_2}) = \frac{a_0}{a_M} \frac{a_0^*}{a_M^*} < 1$$

equality holds.

The condition $c_0(x) > 0$ in [4] is as follows: $c_0 = b_0^2 - b_{M-1}^2 > 0$ or in terms of the a_i's :

$$(a_0^2 - a_M^2)^2 > (a_0 a_{M-1}^* - a_1 a_M^*)^2 \tag{17}$$

We next show that this is equivalent to $\tilde{k}_{M-1}(z_2)$ being SBR. Indeed, from (13) it follows:

$$\tilde{k}_{M-1}(e^{j\omega_2}) \tilde{k}_{M-1}^*(e^{j\omega_2}) = (1 - n_M^2)^2 > (n_{M-1}^* - n_1 n_M^*)^2$$

Expressing this in terms of the a_i's we get:

$$(a_0^2 - a_M^2)^2 > (a_0 a_{M-1}^* - a_1 a_M^*)^2$$

which is equivalent to (17). Proceeding in the same way one can show that the conditions on $d_0, \cdots, t_0$ in [4] correspond to the functions $\tilde{k}_i(z_2)$ being SBR, when using our approach.

5. Illustrative Example

We next give a numerical example, solved by Huang and Maria and Fahmy, to illustrate the use of the method. Suppose we are given the filter:

$$H(z_1, z_2) = \frac{1}{1 + \frac{1}{2}z_1^{-1} + \frac{1}{2}z_2^{-1} + \frac{1}{4}z_1^{-1}z_2^{-1} + \frac{1}{4}z_1^{-2} + \frac{1}{4}z_2^{-2}}$$

We first form:

$$D(z_1, z_2) = z_1^2 z_2^2 + \frac{1}{2} z_1 z_2^2 + \frac{1}{2} z_1^2 z_2 + \frac{1}{4} z_1 z_2 + \frac{1}{4} z_1^2 + \frac{1}{4} z_2^2$$

The coefficient matrix for this case is symmetric, hence we can directly use formulas (11),(12),(14),(16) to arrive at the conditions:

1) $z_2^2 + \frac{1}{2}z_2 + \frac{1}{4}$ must have no roots outside the unit circle, which condition is clearly satisfied.

2) The polynomial $b_0(x) = x^2 + \frac{5}{4}x + \frac{3}{4}$ should have no real roots in $|x| < 1$, which condition is also satisfied.

3) For the computation of $c_0(x)$ we can use the symmetric case formulas (14), (16) which lead to:

$$c_0(x) = x^4 + \frac{9}{4}x^3 + \frac{41}{16}x^2 + \frac{3}{2}x + \frac{27}{64}$$

The same polynomial was obtained in [4], and was shown to have no real roots in $|x| < 1$. Hence the conclusion is that the original polynomial $H(z_1, z_2)$ is stable.

This example shows, that for general, second order filter one can directly use the formulas from Section 4, to decide on the stability of the original polynomials. Extensions to higher orders are obvious and lend themselves to easy programming.

6. Circuit interpretation of the method

In this brief section we give a circuit "implementation" of the function $G_M(z_1, z_2)$ by a cascaded lattice structure, as shown in Fig. 1. In Fig. 2 is shown the first building block of the lattice structure in cascade with the function $A_M(z_2)$, to give the overall function $S_M(z_1, z_2)$. We can note, that this structure is not realizable, since by construction the multiplier function k_M is unstable. This problem can still be resolved, as shown on Fig. 3, once we realize, that the same transfer function is obtained provided:

$$A = \tilde{k}_M(z_2) = \frac{a_0(z_2)}{a_M(z_2)};\; B = \frac{\tilde{a}_0(z_2)}{a_M(z_2)};\; C = \frac{\tilde{a}_M(z_2)}{a_M(z_2)}$$

Note that the structure on Fig. 3 incorporates the function $A_M(z_2)$ as well.

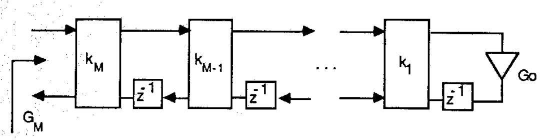

Fig. 1

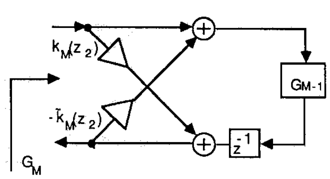

Fig. 2

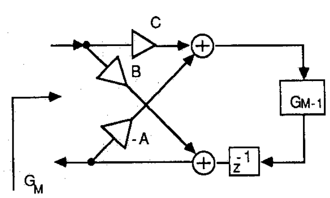

Fig. 3

7. Concluding remarks

While in this paper we concentrated on cases with polynomials with real coefficients, it is not difficult to show that only minor modifications of the procedure are necessary if they are complex. More specifically, the coefficients of the numerator polynomial have to be taken to be the complex conjugates of the corresponding coefficients of the denominator. Next, the set of functions $\tilde{k}_i(z_2)$ has to be checked for being strictly bounded complex (SBC), and the function G_0 will in general be complex, but of magnitude one.

Summarizing, a simple stability testing procedure for discrete time 2-D systems was presented in an unified manner based on lossless network synthesis. Explicit formulas in terms of the filter coefficients were derived for the stability of general case first and second order 2-D polynomials and examples were solved by the proposed method and compared to existing solutions.

Acknowledgments

The authors wish to express their gratitude to Professors M. Bellanger and E. Zeheb for their valuable comments. This work was supported in part by a National Science Grant No. MIP 85-08017.

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