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"IDENTIFICATION OF LINEAR SYSTEMS CONTAINING UNKNOWN GAIN ELEMENTS"

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Abstract. We consider the identification of stable linear systems in which all but a limited number of parameter values (usually physical in origin) are known. Such parameter values may correspond to mass, friction coefficient, reaction rate constant, mutual inductance, etc. Using a characterization of the output error between the true system and a model of it containing adjustable parameters, we describe a recursive algorithm for estimating the unknown parameter value. A persistency of excitation condition on the input is described which ensures a local convergence property for the algorithm.

Keywords. Identification; adaptive systems; sensitivity analysis; stability criteria; modelling.

1. INTRODUCTION

The great majority of recursive identification schemes are aimed at identifying the coefficients of a finite-impulse-response model, or the coefficients of the numerator and denominator of a transfer function representation of an unknown system. On the other hand, there would seem to be numerous practical situations calling for identification characterized by linear systems in which much, but not everything, is known in advance. It is this class of situation examined in this paper.

More precisely, we consider identification where (possibly only a few of) a finite number of (generally physical) parameters are unknown. Examples of such parameters include mass, friction coefficient, mutual inductance, moment of inertia, reaction constant, and so on. A crucial property is that the dependence of the transfer function on any one parameter must be of the type depicted in Fig. 1. This means that there exist transfer function matrices $G_{ij}(s)$ such that the overall system transfer function matrix from the vector input u to the vector output y is

$$W(s) = G_{11}(s) - \alpha G_{12}(s)[I + \alpha G_{22}(s)]^{-1} G_{21}(s) \tag{1.1}$$

Moreover, the $G_{ij}(s)$ are independent of α, (but may depend on some other parameter(s)).

When the feedback loop in Fig. 1 involves scalar z and w, we have a situation termed rank-1 dependence in [1] and [2], and when such dependence applies for all parameters, special procedures for identification are available, as in [1] and [2]. Rank-1 dependence can be associated with a mass or friction coefficient, but not with, for example, mutual inductance. (Actually, sometimes the inverse of the physical quantity must be used, rather than the quantity itself to secure the rank-1 property, but this is a minor point.) In a companion paper [3], we analyze structures in discrete time containing a symmetric lattice with taps; the lattice coefficients each involve a rank-2 dependence, the taps a rank-1 dependence. In this paper, we make no assumption of rank-1 dependence.

In the remainder of the paper, the following ideas are discussed. First, the form of (1.1) is generalized to cope with more than one unknown parameter. Second, we discuss large scale sensitivity calculations in section 2. Third, we present a recursive algorithm for identifying the unknown parameters in section 3. Fourth, we present a local stability result for this algorithm in section 4. (It seems too much to expect a global stability result, without further specialization of the problem.)

We now describe the system we aim to identify, with reference to Fig. 2. The matrix A^* is given by

$$A^* = diag\{\alpha_1^* I_{n_1}, \alpha_2^* I_{n_2}, \ldots, \alpha_m^* I_{n_m}\} \tag{1.2}$$

Note that, if $n_1 = n_2 = \ldots = n_m = 1$, then each α_i^* has a rank-1 dependence. The transfer function matrix from input to output is

$$W^*(s) = G_{11}(s) - G_{12}(s) A^* (I + G_{22}(s) A^*)^{-1} G_{21}(s) \tag{1.3}$$

or

$$W^*(s) = G_{11}(s) - G_{12}(s)(I + A^* G_{22}(s))^{-1} A^* G_{21}(s) \tag{1.4}$$

and is assumed stable without further explicit comment. The driving signal $u^*(\cdot)$ is assumed bounded, as are all internal signals, including $y^*(\cdot)$, $z^*(\cdot)$ and $w^*(\cdot)$. The transfer function matrices $G_{ij}(s)$ are assumed known, while the constants $\alpha_1^*, \ldots, \alpha_m^*$ appearing in A^* are the unknowns which are to be identified.

We have deliberately cast our discussion in terms of continuous time here, since we believe the problem origin will on most occasions involve continuous time. Often, it is the case that physical parameters can appear quite simply in a continuous time model, but get "smeared out" in the associated discrete-time model. One could however conceive of approximating the scheme of Fig. 2 by a discrete-time model with virtually the same structure, e.g. by using an Euler approximation for the one-step transition matrix of the discrete-time equivalent of a continuous-time system, rather than using e^{AT}.

2. LARGE SCALE SENSITIVITY CALCULATION

A great many adaptive identification procedures work by copying the structure of the plant into a model which uses an adjustable known parameter at every point where the plant has a fixed unknown parameter. This strategy is followed here. Fig. 3 depicts the adjustable model, in which

$$\hat{A}(t) = diag[\hat{\alpha}_1(t) I_{n_1}, \hat{\alpha}_2(t) I_{n_2}, \ldots, \hat{\alpha}_m(t) I_{n_m}] \tag{2.1}$$

The adjustable model is driven by the same signal $u^*(\cdot)$ as the true plant. Notice that one or more of the entries of $\hat{z}$ and z^* may coincide; in equation error identification of an entire transfer function, $\hat{z} \equiv z^*$ (the entries being delayed versions of u^* and y^*). In output error identification, some, but not all, of the entries are identical (those corresponding to delayed versions of u^*).

A vital quantity of concern in any adaptive algorithm is the output error, here $y^* - \hat{y}$. Fig. 4 sums up an important result showing how this depends on the parameter error:

Theorem. With plant as depicted in Fig. 2 and adjustable model as in Fig. 3, suppose that the adjustable model is stable with $\hat{A}(t)$ fixed at $\hat{A}$. Then each of the two arrangements depicted in Fig. 4 generates the error $y^* - \hat{y}$.

Proof. Using (1.4) in $y^* = W^* u^*$ and an expression similar to (1.3) in $\hat{y} = \hat{W} u^*$,

$$\begin{aligned} y^* - \hat{y} &= G_{12}\hat{A}(I + G_{22}\hat{A})^{-1} G_{21} u^* - G_{12} A^* (I + G_{22} A^*)^{-1} G_{21} u^* \\ &= G_{12}\{\hat{A}(I + G_{22}\hat{A})^{-1} - (I + A^* G_{22})^{-1} A^*\} G_{21} u^* \\ &= G_{12}(I + A^* G_{22})^{-1}\{(I + A^* G_{22})\hat{A} - A^*(I + G_{22}\hat{A})\} \\ &\quad \cdot (I + G_{22}\hat{A})^{-1} G_{21} u^* \end{aligned}$$

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$$= G_{12}(I + A^* G_{22})^{-1}(\hat{A} - A^*)(I + G_{22}\hat{A})^{-1}G_{21}u \quad (2.2)$$

Similarly,

$$y^* - \hat{y} = G_{12}(I + \hat{A}G_{22})^{-1}(\hat{A} - A^*)(I + G_{22}A^*)^{-1}G_{21}u^* \quad (2.3)$$

Now for Fig. 3,

$$\hat{z} = (I + G_{22}\hat{A})^{-1}G_{21}u^* \quad (2.4)$$

while for Fig. 2,

$$z^* = (I + G_{22}A^*)^{-1}G_{21}u^* \quad (2.5)$$

With inputs of zero and $(\hat{A} - A^*)\hat{z}$ used as in the first part of Fig. 4, one can establish that the upper output is given by the right side of (2.2). Similarly, with inputs of zero and $(\hat{A} - A^*)z^*$ used as in the second part of Fig. 4, one can establish that the upper output is given by the right side of (2.3). Thus the theorem claim is established. ∇∇∇

3. AN ADAPTIVE ALGORITHM

Our goal in this section is to exploit the error formula of the last section to generate an algorithm for recursively identifying the unknown coefficients α_i. We retain the notation of the last section.

We showed that

$$y^* - \hat{y} = G_{12}(I + A^* G_{22})^{-1}(\hat{A} - A^*)\hat{z}$$

$$= G_{12}(I + A^* G_{22})^{-1} diag[\hat{z}_i] \begin{bmatrix} \hat{\alpha}_1 - \alpha_1^* \\ \vdots \\ \hat{\alpha}_1 - \alpha_1^* \\ \hat{\alpha}_2 - \alpha_2^* \\ \vdots \\ \hat{\alpha}_2 - \alpha_2^* \\ \vdots \\ \hat{\alpha}_m - \alpha_m^* \end{bmatrix} \quad (3.1)$$

Note that the term $\hat{\alpha}_i - \alpha_i^*$ occurs in n_i consecutive entries for $i = 1, \ldots, m$. Suppose for simplicity that y^* is scalar. Then

$$y^* - \hat{y} = [x_1^1(t) \cdots x_1^{n_1}(t)\; x_2^1(t) \cdots x_2^{n_2}(t) \cdots x_m^{n_m}(t)] \cdot \begin{bmatrix} \hat{\alpha}_1 - \alpha_1^* \\ \vdots \\ \hat{\alpha}_2 - \alpha_2^* \\ \vdots \\ \hat{\alpha}_m - \alpha_m^* \end{bmatrix}$$

$$= [\psi_1(t) \cdots \psi_m(t)] \begin{bmatrix} \hat{\alpha}_1 - \alpha_1^* \\ \hat{\alpha}_2 - \alpha_2^* \\ \vdots \\ \hat{\alpha}_m - \alpha_m^* \end{bmatrix} \quad (3.2)$$

where

$$\psi_i(t) = \sum_{j=1}^{n_i} x_i^j(t) \quad (3.3)$$

Notice that the signals $x_i^j(t)$ and then the signals $\psi_i(t)$ can be thought of as arising through excitation of the true system (at other than a standard input point) with signals measured in the identifier, or adjustable model. More precisely, Fig. 5 and Fig. 6 illustrate the calculation of both $\psi_i(t)$ and $x_i^j(t)$.

The error $y^* - \hat{y}$ is precisely constructable using the unknown system output and the adjustable model. Because we do not know the parameters α_i^*, the individual signals $\psi_i(t)$ cannot be computed. We can however generate approximations $\hat{\psi}_i(t)$ of the $\psi_i(t)$ by using $\hat{\alpha}_i$ in place of α_i^* in the generating process. Fig. 7 shows the idea, and it should be compared with Fig. 5.

Now were the ψ_i available, the classical choice for an update algorithm would be

$$\frac{d}{dt}\begin{bmatrix} \hat{\alpha}_1 \\ \vdots \\ \hat{\alpha}_m \end{bmatrix} = \mu \begin{bmatrix} \psi_1(t) \\ \vdots \\ \psi_m(t) \end{bmatrix} [y^*(t) - \hat{y}(t)] \quad (3.4)$$

because this would imply

$$\frac{d}{dt}\begin{bmatrix} \hat{\alpha}_1 \\ \vdots \\ \hat{\alpha}_m \end{bmatrix} = -\mu \begin{bmatrix} \psi_1(t) \\ \vdots \\ \psi_m(t) \end{bmatrix} [\psi_1(t) \cdots \psi_m(t)] \begin{bmatrix} \hat{\alpha}_1 - \alpha_1^* \\ \vdots \\ \hat{\alpha}_m - \alpha_m^* \end{bmatrix} \quad (3.5)$$

which is well studied [4]-[6]. As just noted though, $\psi_i(t)$ is not available. So we use instead

$$\frac{d}{dt}\begin{bmatrix} \hat{\alpha}_1 \\ \vdots \\ \hat{\alpha}_m \end{bmatrix} = \mu \begin{bmatrix} \hat{\psi}_1(t) \\ \vdots \\ \hat{\psi}_m(t) \end{bmatrix} [y^*(t) - \hat{y}(t)] \quad (3.6)$$

Of course, μ is a positive constant, and can be replaced by $diag[\mu_1, \mu_2, \ldots, \mu_m]$ with $\mu_i > 0$. Note that the signals $\hat{\psi}_i(t)$ have been defined as if the $\hat{\alpha}_i$ were constant, and now we have allowed for time variation of the $\hat{\alpha}_i$. Obviously the definition of the $\hat{\psi}_i(t)$ is immediately extendable to this situation. But also, if μ is small enough so that the $\hat{\alpha}_i$ vary sufficiently slowly, the use of frequency-domain notation, as in, e.g., (2.4) and (3.10) below, (carrying with it the implication of constant $\hat{\alpha}$) can be justified [6].

Above we have defined signals $\psi_i(t)$ and $\hat{\psi}_i(t)$. For future reference, we define a third signal $\psi_i^*(t)$. Fig. 8 illustrates the idea:

$$\psi_1^*(t) = G_{12}(I + A^* G_{22})^{-1}[z_1^* \cdots z_{n_1}^*\; 0 \ldots 0]' \quad (3.7)$$

We note that the quantities $\hat{\psi}_i(t)$ and $\psi_i^*(t)$ have interpretations as sensitivity coefficients

$$\frac{\partial y^*(t)}{\partial \alpha_i^*} = -\psi_i^*(t) \quad (3.8)$$

$$\frac{\partial \hat{y}(t)}{\partial \hat{\alpha}_i} = -\hat{\psi}_i(t) \quad (3.9)$$

References [7] and [8] discuss the calculation of such quantities in certain special cases via procedures similar to those of this paper, and indicate their relevance for adaptive control.

For completeness, we also note an alternative arrangement for generating $\psi_i(t)$. As an alternative to (3.1), we have (from the last section)

$$y^* - \hat{y} = G_{12}(I + \hat{A}G_{22})^{-1} diag[z_i^*] \begin{bmatrix} \hat{\alpha}_1 - \alpha_1^* \\ \vdots \\ \hat{\alpha}_1 - \alpha_1^* \\ \hat{\alpha}_2 - \alpha_2^* \\ \vdots \\ \hat{\alpha}_2 - \alpha_2^* \\ \vdots \\ \hat{\alpha}_m - \alpha_m^* \end{bmatrix} \quad (3.10)$$

Now (3.2) also remains valid. It follows then that $\psi_i(t)$ can be generated by replacing A^* and $\hat{z}_i$ in Fig. 5 by A and z_i^* (see Fig. 9).

4. LOCAL CONVERGENCE OF THE ADAPTIVE ALGORITHM

We retain the notation of the previous section, but we shall look first at the convergence of an error equation which is *not* that applying for our algorithm; this equation is

$$\frac{d\underline{x}}{dt} = -\mu \hat{\underline{\psi}} \hat{\underline{\psi}}^T \underline{x} \tag{4.1}$$

where

$$\hat{\underline{\psi}} = [\hat{\psi}_1 \cdots \hat{\psi}_m]^T \tag{4.2}$$

This is of course a well-studied equation. It is well known that the key to establishing exponential stability is a persistency of excitation condition on $\hat{\psi}$ [4], [6], [9]-[12]

$$\int_s^{s+\delta} \hat{\underline{\psi}}(\sigma)\hat{\underline{\psi}}^T(\sigma)d\sigma > \lambda_0 I > 0 \quad \text{some } \delta, \forall s \tag{4.3}$$

We shall now discuss when (4.3) can be guaranteed. Adopt the notation $\hat{W}(j\omega)$ ($= W(j\omega,\hat{\alpha})$) to denote the transfer functions from u^* to $\hat{y}$; then $W(j\omega,\alpha^*)$ is equivalent to $W^*(j\omega)$, defined in (1.3). Let us assume that $u^*(t)$ is a linear combination of N sinusoids, thus

$$u^*(t) = \sum_{k=1}^{N} \mathrm{Re}[\exp(j\omega_k t)u_k] \quad u_k \neq 0,\ \omega_k \neq \omega_j \text{ for } k \neq j. \tag{4.4}$$

Then

$$\hat{y}(t) = \sum_{k=1}^{N} \mathrm{Re}[\hat{W}(j\omega_k)\exp(j\omega_k t)u_k] \tag{4.5}$$

and

$$\hat{\psi}_i(t) = \sum_{k=1}^{N} \mathrm{Re}[\frac{\partial \hat{W}(j\omega_k)}{\partial \hat{\alpha}_i} u_k \exp(j\omega_k t)]. \tag{4.6}$$

Now $\hat{\psi}_i(t)$ is almost periodic. It is possible to show [13]-[14] that (4.3) cannot be satisfied for any (λ_0, δ) pair if and only if there exists a constant m-vector γ such that

$$\sum_{i=1}^{m} \frac{\partial \hat{W}(j\omega_k)}{\partial \hat{\alpha}_i} u_k \gamma_i = 0 \quad \forall k = 1, 2, \ldots, N \tag{4.7}$$

If $u^*(t)$ and thus u_k is a scalar, this is equivalent to

$$\frac{d}{d\epsilon} W(j\omega_k ; \hat{\underline{\alpha}} + \epsilon\underline{\gamma})|_{\epsilon=0} = 0 \quad k = 1, 2, \ldots, N. \tag{4.8}$$

Equation (4.8) is a type of local unidentifiability condition. It says that at a number of frequencies, $\omega_1, \ldots, \omega_k$, $\hat{W}(j\omega) = W(j\omega; \hat{\underline{\alpha}})$ is independent of first order variations of $\hat{\underline{\alpha}}$, made in a direction $\underline{\gamma}$. Fairly evidently, if N is large enough and $W(j\omega; \hat{\underline{\alpha}})$ is rational in ω, i.e. if the $G_{ij}(s)$ are rational in ω, (4.8) will be equivalent to

$$\frac{d}{d\epsilon} W(s; \hat{\underline{\alpha}} + \epsilon\underline{\gamma})|_{\epsilon=0} = 0 \cdot \quad \forall s \tag{4.9}$$

The derivative is readily evaluated as

$$\frac{d}{d\epsilon} W(s; \hat{\underline{\alpha}} + \epsilon\underline{\gamma})|_{\epsilon=0} = -G_{12}(I + \hat{A}G_{22})^{-1}\Gamma(I + G_{22}A)^{-1}G_{21} \tag{4.10}$$

where $\Gamma = diag(\gamma_1, \ldots, \gamma_m)$. This expression may or may not be convenient to use in checking whether or not (4.9) can hold.

These arguments have established the following

Proposition. Assume that the local unidentifiability condition (4.9) is not fulfilled for any $\hat{\underline{\alpha}}$ under consideration, or any $\underline{\gamma} \in R^m$. Assume moreover that the $G_{ij}(s)$ are rational in s. Then if the input $\underline{u}^*(\cdot)$ comprises a sufficiently large number of sinusoids, the signal $\hat{\underline{\psi}}(\cdot)$ is persistently exciting, in that it satisfies (4.3).

Though the above argument was developed for scalar $u^*(\cdot)$, it can be varied to accommodate vector $u^*(\cdot)$ as well.

Now we return to the stability of the algorithm of Section 3. From (3.2) and (3.6), we have

$$\frac{d}{dt}\begin{bmatrix} \hat{\alpha}_1 \\ \cdot \\ \cdot \\ \cdot \\ \hat{\alpha}_m \end{bmatrix} = -\mu \begin{bmatrix} \hat{\psi}_1(t) \\ \cdot \\ \cdot \\ \hat{\psi}_m(t) \end{bmatrix} [\psi_1(t) \cdots \psi_m(t)] \begin{bmatrix} \hat{\alpha}_1 - \alpha_1^* \\ \cdot \\ \cdot \\ \hat{\alpha}_m - \alpha_m^* \end{bmatrix}$$

$$= -\mu \hat{\underline{\psi}}(t)\underline{\psi}^T(t)\tilde{\underline{\alpha}} \tag{4.11}$$

With obvious definition of $\tilde{\underline{\alpha}}$, the error equation is

$$\frac{d\tilde{\underline{\alpha}}}{dt} = -\mu \hat{\underline{\psi}}(t)\underline{\psi}^T(t)\tilde{\underline{\alpha}} \tag{4.12}$$

Now we have established that

$$\frac{d\underline{x}}{dt} = -\mu\hat{\underline{\psi}}(t)\hat{\underline{\psi}}^T(t)\underline{x} \tag{4.13}$$

is exponentially stable for suitable $u^*(\cdot)$, viz. those guaranteeing persistency of excitation for $\hat{\underline{\psi}}(\cdot)$. As a particular case, we can choose $\hat{\underline{\psi}}(t) = \underline{\psi}^*(t)$ (corresponding to $\hat{\underline{\alpha}}(t) \equiv \underline{\alpha}^*$). Thus,

$$\frac{d\underline{x}}{dt} = -\mu\underline{\psi}^*(t)\underline{\psi}^{*T}(t)\underline{x} \tag{4.14}$$

is also exponentially stable with suitable $u^*(\cdot)$. Now observe (see especially Figs. 5, 7, 8 and 9) that

$$||\hat{\underline{\psi}}(t) - \underline{\psi}^*(t)|| = O(\sup_{s \le t} ||\tilde{\underline{\alpha}}(s)||) \tag{4.15}$$

$$||\underline{\psi}(t) - \underline{\psi}^*(t)|| = O(\sup_{s \le t} ||\tilde{\underline{\alpha}}(s)||) \tag{4.16}$$

when all signals remain bounded (as they do with stability assumptions, and assumptions on $u^*(\cdot)$). Noting that $\psi^*(\cdot)$ is independent of $\hat{\alpha}(\cdot)$, it follows that (4.14) is a linearized version of (4.12). Accordingly, (4.12) inherits the exponential stability property, but only in a local sense [6] [15]-[17]. Now we are guaranteed a local stability property for our algorithm.

5. CONCLUSION

This paper has presented a very general framework for structured identification problems, using a recursive identifier. We have described how the regressor vector can be computed, relating this computation process, in figures, to quantities arising in the algorithm. A condition for local convergence has been established which relies on the satisfaction of an intuitively reasonable identifiability condition stemming from the parameterization, together with a sufficient richness condition on the excitation.

We have not indicated conditions for non-local (large region or global) convergence. It would seem futile to seek these without imposing more structure on the problem.

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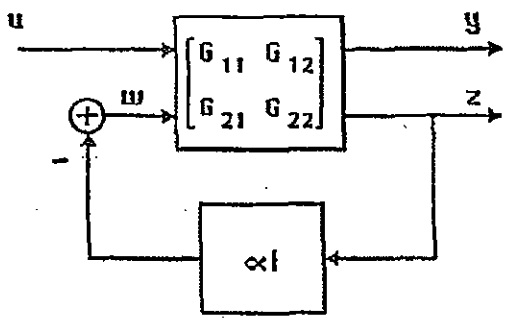

Figure 1: System input and output are u and y, and ∝ denotes an unknown parameter

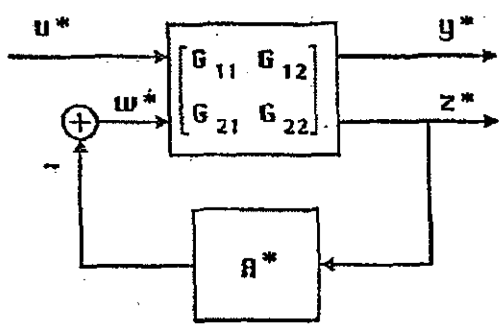

Figure 2: System input and output are u and y, and A* contains a number of unknown parameters

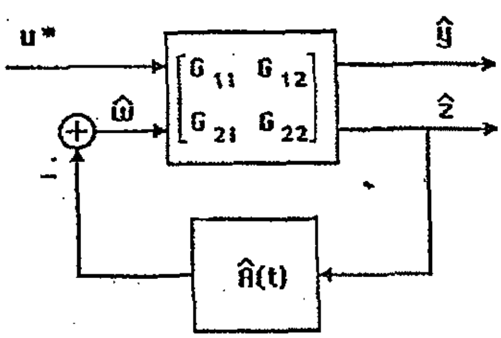

Figure 3: The adjustable model

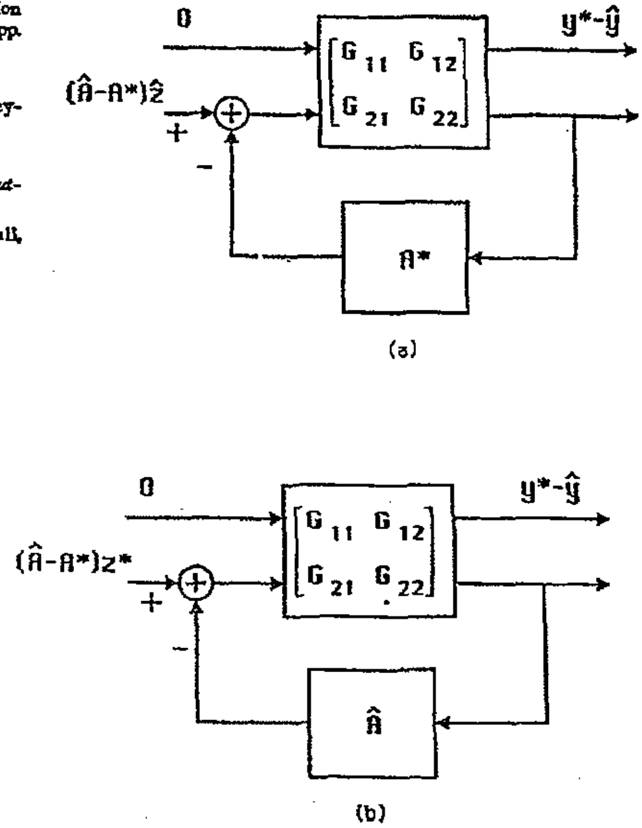

Figure 4: Two representations of the large scale sensitivity

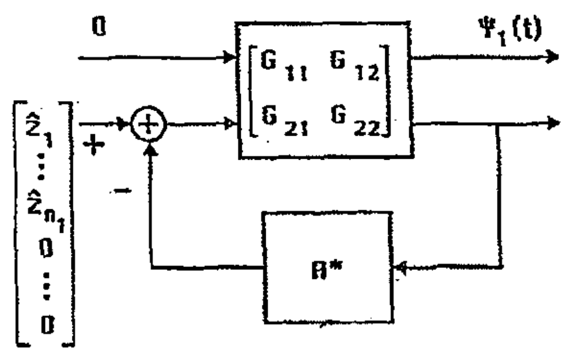

Figure 5: Generation of $\Psi_i(t)$

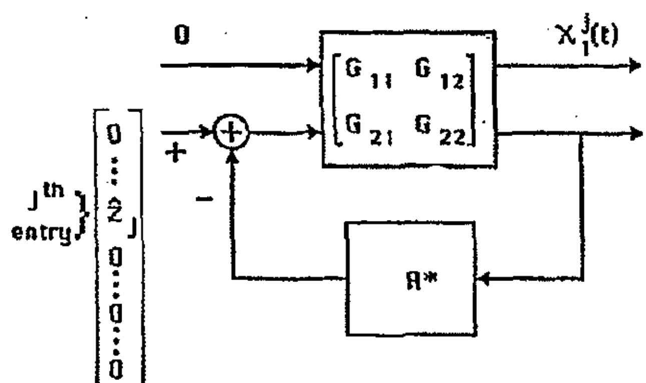

Figure 6: Generation of $X_i^j(t)$

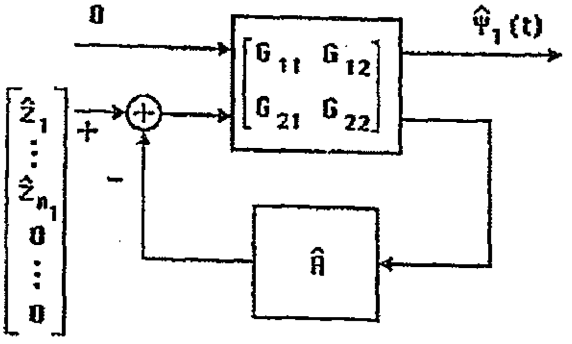

Figure 7: Generation of $\hat{\Psi}_1(t)$

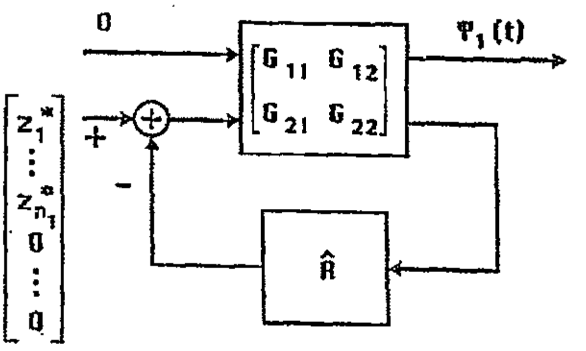

Figure 9: Alternative arrangement for generating $\Psi_1(t)$

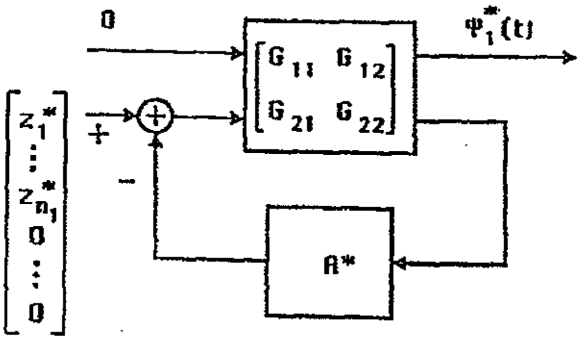

Figure 8: Generation of $\Psi_1^*(t)$