MATRIX RATIONAL INTERPOLATION AND REALIZATION

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Given are N pairs

$$(x_i, Y_{i,j-1}),\ j \in \underline{\nu_i},\ i \in \underline{\theta},$$

where the x_i's are (distinct) scalars, the sum of the θ multiplicities ν_i is equal to N, and the $Y_{i,j-1}$'s are $p \times m$ matrices. We are looking for all rational $p \times m$ matrices $Y(x)$ interpolating the above pairs, i.e. satisfying

$$D^{j-1}Y(x_i) = Y_{i,j-1},\ j \in \underline{\nu_i},\ i \in \underline{\theta},$$

where D denotes differentiation with respect to x. The solutions are to be parametrized in terms of the MacMillan degree $\delta(Y)$ of Y. There is no apriori properness requirement imposed on Y.

As in the scalar case $p = m = 1$, which is reported in the first reference below, it turns out that the main tool for studying the matrix interpolation problem with the MacMillan degree δ as parameter, is the *block Löwner* or *divided differences matrix* L. For unity ν_i's, L is defined as follows:

$$L = \left[\frac{Y_i - Y_j}{x_i - x_j}\right].$$

One of the main results states that the minimal degree of the interpolating functions $Y(x)$ is a simple function of the fine structure of L, that is of the minimal column and row indices of L. (The latter are defined in the same way as the minimal indices of block Hankel matrices in the realization problem.)

Classical realization theory, i.e. the determination of all rational matrices of minimal MacMillan degree, given their partly defined Laurent expansion at infinity, can be viewed as an *interpolation problem*; in this problem the value and a certain number of consecutive derivatives of the function at a single point (called the *Markov parameters*) are preassigned. Recall that the main tool for the study of the realization problem is the *block Hankel* matrix of the Markov parameters. In our framework a further result shows that in case of interpolation at a single point of multiplicity N, the corresponding block Löwner matrix reduces to a block Hankel matrix. This shows that interpolation theory as developed in the two references given below, generalizes indeed the well-known realization theory.

The last part of the talk will be devoted to recursive interpolation, i.e. a computation of the minimal MacMillan degree solutions which is recursive with respect to the number of points N. This, as in the scalar case, is accomplished by means of matrix linear fractional representation formulae.

REFERENCES.

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