REVISITING THE MIT RULE FOR ADAPTIVE CONTROL

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ABSTRACT
The MIT rule is a scalar parameter adjustment law which was proposed in 1961 for the model reference adaptive control of linear systems modelled as the cascade of a known stable plant and a single unknown gain. This adjustment law was derived by approximating a gradient descent procedure for integral error squared performance criterion. For the early part of the 1960s this rule was the basis of many adaptive control schemes and a considerable wealth of practical experience and engineering folklore was amassed.

The MIT rule is in general not globally convergent nor stable, has a performance determined by several factors such as algorithm gain, reference input magnitude and frequency, and the particular transfer function appearing in the cascade. These restrictions on the MIT rule slowly came to be discerned through experimentation and simulation but effectively were without theoretical support until some novel algorithm modifications and stability analysis, so-called Lyapunov redesign, due to Parkin. Our aim in this paper is to pursue a theoretical analysis of the original MIT rule to support the existing simulation evidence and to indicate mechanisms for treating questions of robustness of MIT-rule-based adaptive controllers with undermodelling effects.

The techniques that we apply to solve this problem centre on root locus methods, Nyquist methods and the application of the theory of averaging. Stability and instability results are presented and, using pertinent theories for different regimes of the gain-frequency plane, we approximate the experimentally derived stability margins, but with a generally narrower signal class than simple periodic inputs. The mechanisms of instability and stability for these adaptive systems are highlighted and allow us to emulate guidelines for the MIT rule to work. It is a pleasing by-product of this theoretical analysis that these guidelines coincide to a large degree with those advanced in earlier times on experimental and heuristic grounds.

1. INTRODUCTION

The MIT rule of adaptive control is a scalar parameter adjustment law which was formulated in the late fifties and early sixties as a model reference adaptive control law for linear systems modelled as a cascade of known stable plant and a single unknown gain. The name was generally associated with this formulation are Whitaker, Gahura and Keizer [1,2] and the initial intended application was to the control of aircraft dynamics where the single unknown system parameter was related to dynamic pressure. The basis for this adaptive control law was an explicit performance criterion minimization carried out by an on-line gradient search.

In the history of adaptive control - or at least in its folklore - the MIT rule represents a watershed. The method was simply formulated in the model-reference framework, was easily appreciated, and was applicable. Consequently, this approach to self-optimizing systems was taken up by theorists and practitioners alike as a potential route to enhanced performance. In application trials with aircraft dynamics, however, the MIT rule adaptive controller led to unpredicted instability with a considerable associated loss of face of and confidence in ad hoc adaptive control rules.

Simulation studies provided some guidelines [3] to the rule's stability properties and indicated the likely complexity of any analysis. Also, engineering guidelines or rules of thumb were developed to indicate factors affecting performance [4]. Indeed the broad principles enunciated by Donaldson and Leondes [4] bear a disturbingly close resemblance to currently emerging "modern" notions of suitable operating conditions for the applicability of adaptive control algorithms. Also, particularly with regard to averaging techniques [5-8] and time-scale separation. Theoretical tools dealing with the MIT rule have been lacking, however, and it is in this area where we are now able to reappraise and affirm the earlier results on the MIT rule by utilising essentially these latter techniques. In particular, we address the stability issues of the rule to describe the underlying stability and instability mechanisms.

The broad field of adaptive control has moved on from the criterion minimization approach primarily to a stability based rationale. This "Lyapunov redesign" of adaptive control schemes was originally proposed by Butchart and Shackcloth [9] and beautifully espoused, extended and promulgated by Parkin [10]. (The Lyapunov redesign also heralded the appearance of strictly positive real conditions in adaptive control.) Our aim here is not to develop new adaptive control schemes but rather, by revisiting the well-documented MIT rule, to demonstrate the efficacy of some recently developed tools and to assess their agreement with experimental and simulation evidence.

2. THE MIT RULE

The setup under consideration is depicted in Figure 1. The plant to be controlled is modelled by a known, time-invariant, linear system with transfer function $k_p(s)$ in cascade with an unknown scalar gain $k_u$ of known sign, here assumed (without loss of generality) to be positive. The control objective is to adjust a feedforward control gain $k_c$ so that the plant output $y_p(t)$ tracks the reference model output $y_f(t)$ determined by the parallel model system with transfer function $k_{mp}(s)$. Here, $k_m$ is a known gain, assumed positive for convenience. The bounded reference input signal $r(t)$ is the same to both the reference model and the controller plant systems. (In the original aircraft dynamics problem $k_p$ was related to the dynamic pressure which approximately alters the aircraft dynamics in this fashion and changes with altitude and Mach number.)

The MIT rule is derived by attempting to minimize the integral squared error
\[ V = \lim_{T \to \infty} \frac{1}{T} \int_0^T e^t(k_0) dt \]  
(2.1)

where \( e(t, k_0) \) is the output error
\[ e(t, k_0) = y_p(t) - y_d(t). \]  
(2.2)

Whitaker proposed to adjust \( k_0 \) using a gradient formula to attempt to minimize \( V \), viz.
\[ k_0 = -g \left[ e^t(k_0) \right] \]  
(2.3)

This may be implemented as
\[ k_0 = -g \left[ y_p(t) - y_d(t) \right] y_m(t) \]  
(2.5)

where the sign of \( g \) is the same as that of \( k_0 \) - here assumed positive - and whose magnitude scales the adaptation speed. This is the MIT rule for adaptive feedforward control.

[We have used above a somewhat transparent mixture of notations with which we should be careful. When \( Z(s) \) is a strictly stable transfer function, \( Z(s)(r(t)) \) represents the time signal generated by passing \( r(t) \) into \( Z(s) \). We ignore initial conditions, effects since they decay exponentially and our analysis is linear. These seeming abuses may be formalized easily but provide a convenient notation.]

Other variants on the rule (2.5) are equally possible by considering windowed criteria in place of (2.1) such as
\[ \frac{1}{2T} \int_{t-T}^t e^t(k_0, t) dt, \text{ rectangular window} \]  
(2.6)

or by considering different controller structures.

These former variants yield adaptive laws:
\[ k_0 = -g \left[ y_m(t) [y_p(t) - y_d(t)] dt \right. \]  
(2.8)

The latter variants generally add complexity without new insight. We shall now move on to consider instability mechanisms for these schemes.

3. Instability Mechanisms

Because the most remarkable feature of the MIT rule was its unpredicted instability, we begin our analysis by first investigating mechanisms to cause this behaviour. We study three main mechanisms: high adaptation gain, resonance effects and model mismatch.

3.1 Large Adaptation Gains

The gain of the MIT rule (2.5) or (2.8), (2.9)) scales with \( g \) and the magnitude squared of the reference signal \( r(t) \). Thus this instability can arise due to either large algorithm gain or large reference inputs. To demonstrate the possibility of high gain instability we consider a simple constant input \( r(t) = R \). This corresponds to set-point regulation.

When \( r(t) = R \) and \( k_p \) is constant the MIT rule reduces to
\[ \dot{k}_0(t) = -g \left[ Z_p(s) k_p k_0(t) R - Z_p(s) k_p R \right] \]  
(3.1)

where we have taken \( Z_p(s) = 1 \), without loss of generality. This in turn may be written as
\[ k_0(s) = \frac{g k_p R^2}{s k_p + k_p R} \]  
(3.2)

and root locus arguments may be applied directly to establish the boundedness of \( k_0(t) \). The gain parameter is \( g k_p R^2 \) and our first global result is:

Lemma 1: The MIT rule with \( r(t) \) constant has infinite gain margin (i.e. for all positive \( g \) and \( R \), the adaptive law is stable independent of \( k_p \) if and only if
\[ \Re \left[ \arg Z_p(j\omega) \right] < \Re \left[ \arg \frac{g k_p R^2}{s k_p + k_p R} \right] \]  
(3.3)

Remark 3.1: As our simple structure produces a linear differential equation for \( k_0(t) \), the converse of this lemma is that, unless (3.3) is satisfied by all \( g \) and \( R \), the adaptive control scheme is unstable. Condition (3.3) will not be satisfied by many \( Z_p(s) \) containing nonminimum-phase zeros and having relative degree greater than one. For example, with \( Z_p(s) = \frac{1}{s^2} \), (3.3) is unstable for \( g^2 R^2 > 2 k_p k_p R \). It is worthwhile to note, however, that (3.3) is satisfied by all strictly positive real \( Z_p(s) \).

Remark 3.2: Whenever (3.3) is exponentially stable for \( r(t) = R \) the adaptive feedforward gain \( k_p \) becomes asymptotically optimal as desired, i.e. \( k_0(t) \to k_p R \) as \( t \to \infty \).

Remark 3.3: Notice the nonlinear manner in which the input enters the algorithm gain and is proportional to \( R^2 \). The nonlinear dependence of the algorithm's behaviour on the input is at the core of the problem in understanding the MIT rule. This becomes more pronounced in later developments.

Remark 3.4: For the alternative algorithms (2.8), (2.9) equivalent conditions can be derived although only that from (2.9) communicates much: the exponentially weighted MIT rule has infinite gain margin for constant \( r(t) = R \) if and only if \( Z_p(s) \) satisfies (3.3). This modification (and also the modification appropriate for (2.8)) only serve to make stability more difficult, so that the algorithm may not perform adequately even with strictly positive real \( Z_p(s) \).

3.2 Resonance Effects

By considering periodic input signals \( r(t) \), as opposed to constant inputs in the previous subsection, another broader class of instabilities is displayed. We shall proceed by using an example.

Let \( k_p = k_m = 1 \) and \( Z_p(s) = 1/(s+1) \) which incidentally is SPR. This avoids the high gain instability as \( Z_p(s) \) satisfies (3.3). Now we take \( r(t) = \cos(ut) \) and investigate the effect of altering \( u \).

The MIT rule (2.5) may then be written in terms of \( x_1 \), the state of the plant \( Z_p(s) \), and \( x_2 = k_0(t) - k_0 \), the parameter error, as
\[ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos ut & 0 \\ -g y_p(0) \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \end{bmatrix} \]  
(3.4)

where
\[ y_p(t) = \Re \left[ Z_p(j\omega)e^{jut} \right] = (u^2+1)^{-\frac{1}{2}} \cos(u(t) - \tan^{-1}(w)). \]
Equation (3.4) is a linear ordinary differential equation with periodic coefficients—note the similarity with the classical Mathieu equation. Its stability properties may be studied using Floquet theory as was done for this adaptive control problem by James using numerical integration methods [3]. The results displayed in Figure 2 depict the stability domain in the frequency (ω)-gain (g) parameter plane and exhibit the extreme complexity of the stability/instability boundary characteristic of this class of equations—again recall the Mathieu equation and the extraordinary difficulty of describing analytically its stability properties. The gain margin for this $Z_p(s)$, which is infinite when ω0, is drastically reduced around the cut-off frequency (ω1) of the plant. More complicated $Z_m(s)$ demonstrate this and more complicated behaviour. Similarly, introduction of forgetting factors and integration produces more complex analysis, and produces more complex behaviour. Replacing $Z_p(s)$ by $P'(sP(s))$ is equivalent to scaling ω or g respectively and so does not alter affairs. Again, further examples of resonance phenomena or "pumping up" can be derived with different periodic r(t).

3.3 Modelling Errors

Under normal circumstances, the plant's transfer function $Z_p(s)$ is not exactly known to the designer and only an approximate representation $Z_m(s)$ is available. This approximation must be used to create the desired reference trajectory $Y_d(t)$. These modelling errors alter the previous analysis and, in particular, can drastically affect control performance. In the previous example, if $Z_p(s) = (s^3/9)(s+1)r(t)$ and $Z_m(s)$ remains as before, then the resulting stability/instability boundary is as in Figure 3. Notice in particular that the algorithm becomes unstable for ω = π/2 and that the stability margin is further reduced. These three instability mechanisms demonstrate seemingly disparate phenomena which do not auger well for the adequate performance of the MIT rule in almost any situation. However, a rudimentary analysis of these examples indicates that operation with small adaptation gain g may be necessary for good performance. Small g corresponds to slow adaptation which will be specifically pursued in greater detail to rescue the MIT rule.

4. STABILITY ANALYSIS VIA AVERAGING: RESCUING THE MIT RULE

Abiding by the warnings of the instability mechanism of the previous section, we shall seek now to consider the time-scale separation between the plant and the adaptation. This restriction allows us to use averaging and/or singular perturbation techniques to obtain some intuitively appealing sufficient requirements for good performance.

Throughout this section we assume that the parallel model transfer function $Z_m(s)$ is only an approximation of the plant transfer function $Z_p(s)$, and we shall derive both instability and instability results where possible. We consider two different types of timescale separation where alternatively (i) the adaptation is slow relative to plant and reference signals and (ii) the reference input is slow relative to the plant and adaptation.

4.1 Slow Adaptation

Writing

$$\dot{y}_p(t) = k_gZ_m(s)[r(t)]$$

(4.1)

$$\dot{y}_p(t) = Z_m(s)[k_p\phi(r(t))]$$

(4.2)

the MIT rule (2.5) is

Assuming that $g$ is small, i.e., $k_g$ is slowly time-varying, it is reasonable to approximate (4.3) by formally treating $k_g$ as a constant in the right hand side, i.e., with $k_g$ approximating $k_p$.

$$\dot{y}_p(t) = \frac{-g[Z_m(s)[k_p\phi(r(t))]]Z_m(s)[k_p\phi(r(t))]Z_m(s)[k_p\phi(r(t))]}{\text{det}(Z_m(s)[k_p\phi(r(t))])}$$

(4.3)

For sufficiently small g (h, n) and (4.3) will have similar stability properties. In particular, exponential stability or instability of (4.3) for sufficiently small g will imply the same for (4.3), see [6,8].

Notice now that (4.4) is a linear time-varying first-order differential equation whose stability properties are assessed quite readily. We have

Lemma 2: The homogeneous part of (4.4) is exponentially stable for bounded r(t) and stable $Z_m(s)$ and $Z_p(s)$ if and only if:

$$\lim \inf \frac{1}{T} \int r(t)[Z_m(s)[k_p\phi(r(t))]]Z_m(s)[k_p\phi(r(t))]] Z_m(s)[k_p\phi(r(t))] dt > 0$$

(4.5)

Remark 4.1: If the homogeneous part of (4.4) is exponentially stable, r(t) is bounded, and $Z_m(s)$ is stable, then $k_p(t)$ converges to a bounded limiting function $k_p(t)$ independent of initial conditions but not necessarily constant.

Remark 4.2: The condition (4.5) is a variant on the usual persistence of excitation conditions—both $Z_m(s)$ and $Z_p(s)$ are now involved. For stable $Z_m(s)$, we simply require r(t) to be persistently exciting but (4.5) embodies an additional requirement that the energy in r(t) be localised where $Z_m$ and $Z_p$ have similar frequency responses, or at least phase responses.

We may use averaging theory to transfer this result directly to (4.3).

Theorem 1: Under the condition that $Z_m(s)$ and $Z_p(s)$ are strictly stable, that r(t) is bounded and that (4.5) is satisfied, there exists a positive constant $g_0$ such that for all $g(0,g_0)$ the gain $k_p$ adjusted by the MIT rule (2.5) is bounded and converges exponentially fast to $k_p(t) + 0(g)$ as $t \to \infty$.

Remark 4.3: The constant $g_0$ above may be quantified in terms of $\omega$ in (4.5) and further characterization of $r(t)$. Averaging theory (or, more generally, singular perturbation theory) permits us only to look with some degree of safety to $g_0$ and gives us no further information about the properties of (4.3) in terms of (4.8). Thus this bound makes little quantitative sense.

Remark 4.4: Subject to small g we have an essentially unstable result that, under conditions of strictly stable $Z_m(s)$, $Z_p(s)$ and bounded r(t) with the integral on the left of (4.5) having strictly negative limit superior, there exists $g_0$ such that $g(0,g_0)$ yields $k_p$ unbounded.

Remark 4.5: The time invariance of $k_0$ has not been invoked up to this point and, given the usual rationale for adaptation systems of adjustment to slowly-varying parameter values, one can envisage seeking to admit $k_0$ variation, provided at least that $k_0$ does not change sign. Using the same averaging principles in allowing $k_0$ to vary more slowly than the adaptation speeds, it is possible to split the timescales into three distinct components and (4.5) becomes

$$\lim \inf \frac{1}{T} \int_{-T}^T r(t)[Z_m(s)[k_p\phi(r(t))]Z_m(s)[k_p\phi(r(t))]] Z_m(s)[k_p\phi(r(t))] dt = \infty$$

(4.6)

We now ask under what conditions (4.6) is satisfied and for clarity consider this condition for inputs of the kind
become adaptive schemes and indicates that the MIT rule will, i.e., negative. Thus for small adaptation gain and adaptive systems instability here is due to model errors since the integral squared error criterion is small, stable

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4.2 Slowly Time-varying Inputs

Starting from the MIT rule description (2.5) and now assuming that $r(t)$ and $k_p$ are slowly time varying we may appeal once again to averaging and attempt to obtain the extension of the root locus arguments used in Section 2 with constant $r(t)$. Just as (4.4) approximates (4.3) with $g$ small, we can find the following equation to approximate (4.3) with $r(t)$ and $k_p$ slowly varying.

\[
K_p = g(k_p)^r(t)[Z_p(0) + k_p(t)] + g k_p(t)
\]

(4.10)

where we have again taken $Z_p(0) = Z_p(0) = 1$. In justifying the approximation, in addition to $d/dt(k_p r)$ being small, we require $Z_p(0)$ and $Z_p(0)$ strictly stable. We do not require that $g$ be small. Then $k_p(t)$ and $k_p(t)$ will be close as $t \to \infty$ in a way to be made precise shortly.

The stability properties of (4.10) may be derived simply as an extension of the root-locus analysis of subsection 3.1. We have

Lemma 3: Provided the zeros of

\[
s + g k_p(r_p(s))
\]

(4.11)

have real parts less than a negative constant, $-\sigma$, for some positive $g$ and $k_p$, and for all $s \in s^2$, where $s$ and $r$ are fixed positive quantities. The approximation (4.12) for the homogeneous part of (4.10) is exponentially stable for all inputs $r(t)$ and gains $k_p$ satisfying

\[
\sigma^2 + k_p^2 \leq R^2
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\[
\sigma^2 + \int_0^\infty |k_p(t)|^2 dt \leq \sigma, \quad \forall \sigma \geq 0.
\]

(4.13)

Remark 4.5: Notice that (4.12) requires $r(t)$ to be non-zero and that the range of $r(t)$ depends on the values taken by $k_p$. This result represents a slight modification of [11, p. 125-127] in that (4.13) is an integral bound on the derivative magnitude which can also jump as opposed to a strict bound. This result follows directly from the proof in [11] using integration by parts before appealing to Gronwall's Lemma.

We may now apply the results of Lemma 3 to ascertain the behavior of the MIT rule with slowly-varying inputs.

Theorem 2: Under the conditions of Lemma 3, and the condition that $Z_p(0)$ and $Z_p(0)$ are strictly stable, there exists a positive $s^*$ such that for $k_p(t)$ satisfying (4.12) and (4.13) for $G(0,p^*)$, the MIT rule (2.5) is stable. Moreover,

\[
k_p(t) + k_p(t) + (p) as t \to \infty
\]

exponentially fast.

Remark 4.6: Theorem 2 explains the stability properties of the MIT rule for slowly time-varying inputs. In particular, it predicts the stability in regions $I$ of Figures 2 and 3. It does not specifically address the problem of model errors since we have good low frequency matching. (Recall that $Z_p(0) = Z_p(0) + 1$.) This result may be seen as providing sufficient conditions to avoid high gain instability for very low frequency inputs. Lemma 1 arises as a special case.

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The stability properties of (4.10) may be derived simply as an extension of the root-locus
application of sound theoretical tools to engineering principles voiced in [1], of 1963, where timescale separation is routinely invoked to allow a heuristic analysis to proceed as well as to accommodate engineering design intuitions. To a very large extent our results are a reaffirmation of these fundamental principles and concepts of adaptation in general. The most realistic scenario is that of slow adaptation where the stability objectives for the MIT rule are achieved, per Theorem 1, by taking small adaptation gain g and insisting that r(t), Zp(s), Zp(s) jointly satisfy (4.5). This latter condition, in turn, requires that r(t) have a persistence property and that its dominant energies should be located at frequency bands where \( Z_p \) is close to \( Z_p \). Indeed, these may very well be envisaged as conditions necessary for the well-posedness of an adaptive solution using the MIT rule.

The case of slowly time-varying inputs is also of interest in that it highlights how these theoretical tools may be applied to gradually validate more of the stability region generated using numerical methods and Floquet theory. It helps us to isolate the generic aspects of the problem which may then be incorporated into engineering design guidelines.

Although our analysis has primarily been to use the MIT rule as a showcase for those methods, we should reiterate that more recently developed algorithms are amenable to these analytical tools as well. A desire to avoid the instability mechanisms of the MIT rule was behind the development of these new algorithms as is discussed by Parks [10,12]. The primary modification there is to replace criterion minimization by Lyapunov function specification and the affect on the adaptive law is, typically, to replace (2.5) by

\[
k_0 = -g r(t)(y_p(t) - y_m(t))
\]

and to utilize SIR \( Z_p(s) \) to allow generation of a quadratic Lyapunov function.

We have revisited the MIT rule for adaptive feedforward control and displayed some of its instability mechanisms and produced hard stability results relying on time-scale separation and good model selection. In some ways this has demonstrated the reasons for the loss of confidence in this rule and also suggested potential remedies necessary to reemulate it.

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REFERENCES


Figure 1: The MIT rule's problem

Figure 2: Stability/Instability boundary

Figure 3: Stability/Instability boundary