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Spectral Factorization By Algebra

by

B. D. O. Anderson

September 1966

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SYSTEMS THEORY LABORATORY

STANFORD ELECTRONICS LABORATORIES

STANFORD UNIVERSITY · STANFORD, CALIFORNIA

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ABSTRACT

The problem of giving a spectral factorization of a class of matrices arising in Wiener filtering theory and network synthesis is tackled via an algebraic procedure. A quadratic matrix equation involving only constant matrices is shown to possess solutions which directly define a solution to the spectral factorization problem. A spectral factor with a stable inverse is defined by that unique solution to the quadratic equation which also satisfies a certain eigenvalue inequality. Solution of the quadratic matrix equation and incorporation of the eigenvalue inequality constraint are made possible through determination of the eigenvectors of a matrix formed from the coefficient matrices of the quadratic equation.

CONTENTS

I. INTRODUCTION

Multivariable filtering problems [1] and problems of network synthesis [2] have both generated the requirement of giving a certain type of factorization (termed spectral factorization) of a class of matrices whose elements are real rational functions of a complex variable s.

Specifically, there is given a square matrix $\Phi(s)$ satisfying

$$\Phi(s) = \Phi'(-s) \tag{1}$$

and

$$\Phi(j\omega) \geq 0 \qquad \text{for all} \quad \omega \tag{2}$$

(the notation "≥ 0" being short for "is nonnegative definite"). A matrix $W(s)$ is sought which satisfies

$$W'(-s)W(s) = \Phi(s) \tag{3}$$

Normally, it is required that $W(s)$ be analytic in the right half plane $\text{Re}\ s > 0$, and in addition it may be required that $W(s)$ have constant rank there, or equivalently, that $W(s)$ possesses a right inverse $W_r^{-1}(s)$ which is analytic in the right half plane.

Two distinct solutions have been given to the problem as presented in [3], [4], and minor modifications have been discussed, see e.g. [5]. It has been established [3] that there are many solutions $W(s)$ for (3), but that any two solutions $W_1(s)$ and $W_2(s)$ are related through

$$W_1(s) = V(s)W_2(s) \tag{4}$$

where $V(s)$ is a paraunitary matrix, that is, a matrix satisfying

$$V'(-s)V(s) = I \tag{5}$$

(here I is the unit matrix). Moreover [3], if W_1 and W_2 both possess right inverses which are analytic in the right half plane, then V must be constant.

The approach taken here to solve (3) is an algebraic one. Specifically, a quadratic matrix equation (involving only constant matrices) is presented, whose solution immediately defines a solution to (3), (section 2). A solution to (3) possessing the additional property of constant right half plane rank is determined via a solution to the matrix equation which also satisfies an additional matrix eigenvalue inequality (section 3).

Some minor difficulties associated with the situation where $\Phi(\infty)$ is singular are dealt with in section 4, where it is shown how to reduce the factorization problem of such a Φ to the problem of factorizing a matrix Φ_r with the same properties as Φ, save that $\Phi_r(\infty)$ is nonsingular.

In section 5 the determination of all solutions of the quadratic equation is discussed, together with a procedure for determining what particular solution satisfying the eigenvalue inequality. The key feature of the algorithm presented is the determination of the eigenvalues of a matrix formed from the coefficient matrices in the quadratic equation [6].

II. SPECTRAL FACTORIZATION FOR MATRICES NONSINGULAR AT INFINITY

In this section the problem is considered of factorizing $\Phi(s)$, a matrix of rational functions of s, subject to

$$\Phi'(-s) = \Phi(s) \tag{1}$$

$$\Phi(j\omega) \geq 0 \tag{2}$$

and the following condition, to be relaxed later

$$\det \Phi(\infty) \neq 0 \tag{6}$$

The following assumption will also be made: $\Phi(s)$ has no poles on the $j\omega$ axis.

While it is possible to extend the results to include this case, the extension is awkward; moreover the situation where Φ does have $j\omega$-axis poles is unusual from the physical point of view, at least if Φ is conceived as being the output covariance matrix of some dynamical system driven by white noise; such a system must then have a $j\omega$-axis pole in its transfer function.

The factorization proceeds in several steps. First, $\Phi(s)$ is written as

$$\Phi(s) = Z(s) + Z'(-s) \tag{7}$$

where $Z(s)$ is a positive real matrix of rational functions; this requires Z to satisfy the following conditions [2]:

1. $Z(s)$ is analytic in $\mathrm{Re}\ s > 0$ and has only simple poles on $s = j\omega$. Residue matrices for such poles are nonnegative definite hermitian.
2. $Z'(-j\omega) + Z(j\omega) \geq 0$ (8)

Because of (2) and (7), Eq. (8) is automatically satisfied. Using the assumption concerning the lack of $j\omega$-axis poles for Φ, it will be shown that $Z(s)$ can be chosen with no such poles. Then, if $Z(s)$ is chosen to satisfy (7) and to have elements which are analytic in the right half plane, $Z(s)$ will be positive real.

A minimal realization [7] in the control systems sense is then formed for Z. This minimal realization is a collection of four constant matrices $\{F, G, H, J\}$.

A matrix equation involving F, G, H and J, and an unknown symmetric matrix P, is then constructed by using some recently found results on positive real matrices [8]. Any solution of this equation is shown to directly define a matrix $W(s)$ such that (3) holds.

Consider then Eq. (7). Let Φ be written as

$$\Phi(s) = \frac{\Phi_o(s)}{D(s)} + \Phi(\infty) \tag{9}$$

where D is the least common denominator of the elements of Φ. The order of the polynomial D is greater than that of any element in the polynomial matrix $\Phi_o(s)$. It is evident that $D(s)$ will consist of a product of terms of the form $(s - s_1)$, $(s + s_1)$, $(s - s_2)$, $(s + s_2)$, ..., where the s_i may in general be complex and not necessarily distinct. It is also evident that $\operatorname{Re} s_i < 0$ may be assumed.

A partial fraction expansion of Φ is then formed so that

$$\Phi(s) = \Phi(\infty) + \frac{\Phi_1}{s - s_1} + \frac{\tilde{\Phi}_1}{s + s_1} + \frac{\Phi_2}{s - s_2} + \dots \tag{10}$$

where Φ_i and $\tilde{\Phi}_i$ are constant matrices for simple poles and matrix functions of s for multiple poles. For convenience, it is assumed that multiple poles are absent. Then Φ_1 is given by

$$\Phi_1 = \lim_{s \to s_1} \frac{(s - s_1)\Phi_o(s)}{D(s)} = \left. \frac{\Phi_o(s_1)}{2s_1 \dfrac{D(s)}{s^2 - s_1^2}} \right|_{s = s_1} \tag{11}$$

while

$$\tilde{\Phi}_1 = \lim_{s \to -s_1} \frac{(s + s_1)\Phi_o(s)}{D(s)} = \left. \frac{\Phi_o(-s_1)}{-2s_1 \dfrac{D(s)}{s^2 - s_1^2}} \right|_{s = s_1} \tag{12}$$

Because of the form of $D(s)$ the denominators of (11) and (12) are the same except for sign. Equation (1) applied to (9) yields

$$\Phi_o'(-s) = \Phi_o(s) \tag{13}$$

and thus

$$\Phi_1 = -\tilde{\Phi}_1' \tag{14}$$

Consequently (10) becomes

$$\Phi(s) = \Phi(\infty) + \sum_i \frac{\Phi_i}{s - s_i} + \sum_i \frac{-\Phi_i'}{s + s_i} \tag{15}$$

Defining

$$Z(s) = \frac{\Phi(\infty)}{2} + \sum_i \frac{\Phi_i}{s - s_i} \tag{16}$$

(7) is fulfilled, and $Z(s)$ be some earlier remarks is positive real since the s_i are all in the left half plane. In the pathological case of Φ possessing multiple poles, an analogous procedure can be used to define $Z(s)$.

Note that, conceptually, the procedure for obtaining $Z(s)$ from $\Phi(s)$ is simple: a partial fraction expansion of $\Phi(s)$ is made, and that part with right half plane poles is simply thrown away to leave $Z(s)$.

The next step in the factorization is to write down a minimal realization for Z, that is, a set of four matrices $\{F, G, H, J\}$ such that

$$Z(s) = J + H'(sI - F)^{-1}G \tag{17}$$

and such that F has the least possible dimension. Procedures for doing this may be found in, for example, [7]. Note that in any realization the relation $J = (1/2)\Phi(\infty)$ will hold.

At this stage we require the following lemma [8]:

Lemma 1. Let $Z(s)$ be a positive real matrix such that $Z(\infty)$ is finite, with $\{F,G,H,J\}$ constituting a minimal realization for Z. Then there exist matrices P, L, W_o with P positive definite symmetric such that

$$PF + F'P = -LL' \tag{18a}$$

$$PG = H - LW_o \tag{18b}$$

$$J + J' = W_o'W_o \tag{18c}$$

In the proof of the lemma the existence of P is established by a spectral factorization. For our purposes it is sufficient to know that P exists.

Suppose that in (18), W_o is nonsingular. Then

$$PF + F'P = -LL' = -(PG - H)\, W_o^{-1}\, W_o'^{-1}\, (PG - H)'$$

$$= -(PG - H)(J + J')^{-1}(PG - H)' \tag{19}$$

For the $Z(s)$ resulting from $\Phi(s)$, we have that $J = [\Phi(\infty)/2]$. Further, $J' = J$ since $\Phi'(\infty) = \Phi(\infty)$ by (1). Hence, $J + J' = \Phi(\infty)$ is nonsingular, and

$$PF + F'P = -(PG - H)\, \Phi^{-1}(\infty)(PG - H)' \tag{20}$$

The lemma guarantees that this equation has a positive definite symmetric solution, but actually any symmetric solution to this equation defines a matrix $W(s)$ satisfying (3), with

$$W(s) = \Phi^{\frac{1}{2}}(\infty) - \Phi^{-\frac{1}{2}}(\infty)(PG - H)'(sI - F)^{-1}G \tag{21}$$

where $\Phi^{\frac{1}{2}}(\infty)$ is the square root of $\Phi(\infty)$, uniquely defined since $\Phi(\infty)$ is positive definite. For by explicit calculation,

$$\begin{aligned} W'(-s)W(s) &= [\Phi^{\frac{1}{2}}(\infty) - G'(-sI - F')^{-1}(PG - H)\ \Phi^{-\frac{1}{2}}(\infty)] \\ &\qquad [\Phi^{\frac{1}{2}}(\infty) - \Phi^{-\frac{1}{2}}(\infty)(PG - H)'(sI - F)^{-1}G] \\ &= \Phi(\infty) - (PG - H)'(sI - F)^{-1}G - G'(-sI - F')^{-1}(PG - H) \\ &\quad + G'(-sI - F')^{-1}(PG - H)\Phi^{-1}(\infty)(PG - H)'(sI - F)^{-1}G \\ &= \Phi(\infty) - (PG - H)'(sI - F)^{-1}G - G'(-sI - F')(PG - H) \\ &\quad + G'(-sI - F')^{-1}\ [P(sI - F) + (-sI - F')P]\ (sI - F)^{-1}G \qquad \text{using (20)} \\ &= Z(s) + Z'(-s) \qquad \text{using (17)} \\ &= \Phi(s) \qquad (22) \end{aligned}$$

We have thus established the following theorem:

Theorem 1. Let $\Phi(s)$ be a matrix satisfying (1), (2), and (6), and let $Z(s)$ be the positive real matrix such that (7) holds. Moreover, let $Z(s)$ have a minimal realization $\{F, G, H, J\}$. Then any solution of the equation

$$PF + F'P = -(PG - H)\ \Phi^{-1}(\infty)(PG - H)' \qquad (20)$$

defines a matrix $W(s)$ satisfying (3) through

$$W(s) = \Phi^{\frac{1}{2}}(\infty) - \Phi^{-\frac{1}{2}}(\infty)(PG - H)'(sI - F)^{-1}G \qquad (21)$$

Note the importance of the lemma in pointing out not merely the form of (20), but also that (20) has at least one solution. Note also that though this solution is known to be positive definite, any symmetric solution of (20) will define a suitable $W(s)$. Such solutions must, however, be at least nonnegative definite: this is because the right hand side of (20) will always be nonpositive definite, and this fact, combined with the stability of F, guarantees by the well-known lemma of Lyapunov [9, chapter 3] that P is nonnegative definite.

Thus far, no indication has been given of the determination of $W(s)$ such that it has constant rank in the right half plane. This is the topic of the next section. Neither have we discussed means of solving (20); such a discussion appears in Section 5.

III. DETERMINATION OF FACTOR WITH A STABLE INVERSE

Consider $W(s)$ as in (21). Explicit calculation will show that

$$W^{-1}(s) = \{I + \Phi^{-1}(\infty)(PG - H)' \, [sI - F - G\Phi^{-1}(\infty)(PG - H)']^{-1}G\} \, \Phi^{-\frac{1}{2}}(\infty) \qquad (23)$$

and thus W^{-1} will have no poles in the right half plane if and only if

$$\mathrm{Re}\, \lambda_i \, [F + G\Phi^{-1}(\infty)(PG - H)'] < 0 \qquad (24)$$

the notation indicating that the eigenvalues of $F + G\Phi^{-1}(\infty)(PG - H)'$ must have negative real parts.

Equation (24) can be regarded as an additional constraint on P, when P is used to define $W(s)$, i.e., in addition to requiring that P satisfy the quadratic equation (20), P must also satisfy (24).

Youla's analysis [3] shows that $W(s)$ is uniquely determined to within an arbitrary left orthogonal matrix multiplier, when W and W^{-1} are both analytic in the right half plane. This suggests, and we prove it below, that any P satisfying both (20) and (24) is unique.

To establish this uniqueness, suppose P_1 and P_2 both satisfy

$$PF + F'P = -(PG - H)\Phi^{-1}(\infty)(PG - H)' \qquad (20)$$

subject to

$$\mathrm{Re}\, \lambda_i \, [F + G\Phi^{-1}(\infty)(PG - H)'] < 0 \qquad (24)$$

Define

$$F_1 = F + G\Phi^{-1}(\infty)(P_1G - H)' \qquad (25)$$

and similarly for F_2. Then explicit calculation yields

$$P_1F_1 + F_1'P_1 = -HH' + P_1G\Phi^{-1}(\infty)G'P_1 \qquad (26a)$$

$$P_2F_2 + F_2'P_2 = -HH' + P_2G\Phi^{-1}(\infty)G'P_2 \qquad (26b)$$

Now observe that

$$\begin{aligned}
&(P_1 - P_2)F_1 + F_2'(P_1 - P_2) \\
&= P_2(F_2 - F_1) + (F_2' - F_1')P_1 \\
&\quad + P_1F_1 + F_1'P_1 - P_2F_2 - F_2'P_2 \\
&= P_2G\Phi^{-1}(\infty)G'P_1 + P_2G\Phi^{-1}(\infty)G'P_2 \\
&\quad - P_2G\Phi^{-1}(\infty)G'P_1 - P_1G\Phi^{-1}(\infty)G'P_1 \\
&\quad + P_1G\Phi^{-1}(\infty)G'P_1 - P_2G\Phi^{-1}(\infty)G'P_2 \\
&= 0 \qquad (27)
\end{aligned}$$

The second line of (27) follows by using (25) and (26). It is known that the equation $AX + XB = 0$ has only the solution $X = 0$ when $\lambda_i(A) + \lambda_j(B) \neq 0$ for all i,j [10, chapter VIII]. But since F_1 and F_2 have eigenvalues with negative real part, $\lambda_i(F_1) + \lambda_j(F_2) \neq 0$ for all i and j, and from (27) $P_1 = P_2$.

Hence we have

Theorem 2. With the same hypothesis as Theorem 1, there exists a unique solution of (20) which also satisfies

$$\mathrm{Re}\ \lambda_i\ [F + G\Phi^{-1}(\infty)(PG - H)'] < 0 \qquad (24)$$

This solution defines a matrix $W(s)$ whose inverse is analytic in the right half plane.

IV. EXTENSION TO GENERAL CASE

In this section the problem is considered of factoring $\Phi(s)$ satisfying (1) and (2), and

$$\det \Phi(\infty) = 0 \tag{28}$$

A procedure is described for reducing the problem of factoring such a Φ to that of factoring a Φ_r, with

$$\det \Phi_r(\infty) \neq 0 \tag{29}$$

The procedure of section 2 can be used to define a positive real matrix $Z(s)$ from $\Phi(s)$. Then

$$\det Z(\infty) = 0 \tag{30}$$

Either $\det Z(s)$ is zero for all s, or just for isolated values of s. If $\det Z(s) = 0$, then it is possible to write [2]

$$Z(s) = T_1' \begin{bmatrix} Z_1(s) & 0 \\ 0 & 0 \end{bmatrix} T_1 \tag{31}$$

where $Z_1(s)$ is nonsingular, and T_1 is constant and nonsingular. If

$$Z_1(s) + Z_1'(-s) = W_1'(-s)W_1(s) \tag{32}$$

then

$$Z(s) + Z'(-s) = W'(-s)W(s) \tag{33}$$

with

$$W(s) = \begin{bmatrix} W_1(s) & 0 \\ 0 & 0 \end{bmatrix} T_1 \tag{34}$$

Hence, it only remains to show how to factorize in the situation where $Z(s)$ is nonsingular except for isolated values of s, including ∞. In this instance, one may form

$$Z_1(s) = Z^{-1}(s) \tag{35}$$

and observe that $Z_1(\infty)$ must be nonfinite. Accordingly [2] it is possible to write

$$Z_1(s) = sL + Z_2(s) \tag{36}$$

with $Z_2(\infty)$ finite, and L nonnegative definite, and *with the degree of a minimal realization of Z_2*, i.e., the dimension of the F matrix, less *that for $Z(s)$*. If

$$Z_2(s) + Z_2'(-s) = W_2'(-s)W_2(s) \tag{37}$$

then explicit calculation shows that

$$Z(s) + Z'(-s) = W'(-s)W(s) \tag{38}$$

where

$$W(s) = W_2(s)Z(s) \tag{39}$$

If $Z_2(\infty)$ is nonsingular, a spectral factorization for $Z(s) + Z'(-s)$ results. If not, the process originally applied to $Z(s)$ can be applied to $Z_2(s)$, reducing the spectral factorization problem to that of factorizing a matrix with a minimal realization of lower degree. This process must eventually terminate on reaching some $\Phi_r(s) = Z_r'(-s) + Z_r(s)$ either in an easily factorizable constant matrix (i.e., a matrix with minimal realizations of degree zero) or in a matrix whose value at infinity is nonsingular, and which can then be factorized by the method of section 2. Moreover, through a succession of applications of equations like (34) and (39), a factorization of the original Φ matrix can be obtained from that of Φ_r.

V. SOLUTION OF THE QUADRATIC MATRIX EQUATION

To solve (20) it is convenient to rewrite it in the form

$$PG\Phi^{-1}(\infty)G'P + P\,[F - G\Phi^{-1}(\infty)H'] + [F' - H\Phi^{-1}(\infty)G']P + H\Phi^{-1}(\infty)H' = 0 \quad (40)$$

or

$$PCP + PB + B'P + A = 0 \quad (41)$$

where $C = G\Phi^{-1}(\infty)G'$, etc. The solution of such equations is discussed in [6]:

Lemma 2 (Potter). Let $a_1, a_2, \ldots, a_n$ be n eigenvectors of the matrix, not necessarily diagonalizable,

$$M = \begin{bmatrix} B' & A \\ -C & -B \end{bmatrix} \quad (42)$$

where M is $2n \times 2n$, B' is $n \times n$, etc. Suppose that a_i is partitioned as $a_i' = [b_i', c_i']$ where b_i and c_i are n vectors. Then, a solution to (41) is given by

$$P = [b_1, b_2, \ldots, b_n][c_1, c_2, \ldots, c_n]^{-1} \quad (43)$$

assuming $[c_1, c_2, \ldots, c_n]$ is nonsingular.

Proof of Lemma 2.

Let T be a matrix which reduces M to its Jordan canonical form. Thus, with an obvious partitioning

$$\begin{bmatrix} B' & A \\ -C & -B \end{bmatrix} \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} \begin{bmatrix} J_1 & J_2 \\ 0 & J_4 \end{bmatrix} \quad (44)$$

The diagonal elements of J_1 and J_4 are the eigenvalues of M, while the columns of T are (generalized) eigenvectors of M. Then

$$B'T_1 + AT_3 = T_1J_1 \tag{45a}$$

and

$$-CT_1 - BT_3 = T_3J_1 \tag{45b}$$

from which

$$B'T_1T_3^{-1} + A = T_1J_1T_3^{-1} \tag{46a}$$

and

$$-T_1T_3^{-1}CT_1T_3^{-1} - T_1T_3^{-1}B = T_1J_1T_3^{-1} \tag{46b}$$

Whence it follows that $T_1T_3^{-1}$ satisfies (41), as required.

This completes the Proof of Lemma 2.

Accordingly, if P is of dimension n, it is possible to determine a solution to (40) by performing the procedure of finding the eigenvalues of a 2n × 2n matrix, a rather lengthy process for large n. An alternative mode of solution would be to attempt an iterative procedure, presumably on a computer. Such a procedure would be hard put to iterate on to that P which satisfies the eigenvalue inequality of section 4, while the eigenvector method above, as we now show, can lead straight to this unique P.

Theorem 3 (Anderson). With the same hypothesis as lemma 2, the maxtix P satisfying (40) and the eigenvalue inequality (24) is uniquely determined by choosing those eigenvectors of M corresponding to the eigenvalues with positive real parts.

Proof of Theorem 3.

Before proceeding with the proof we comment that [6] contains a similar theorem, valid when the matrices A and -C of Eq. (41) are nonnegative definite. Here we require A and +C to be nonnegative definite. A fact proved in [6] for the case where A and -C are nonnegative definite is still valid here, however, namely that if λ is an eigenvalue of

M, so is $-\lambda$. This means that if M has no imaginary eigenvalues (which we assume to be the case), J_1 in (44) may be taken to have all positive diagonal entries. Then from (45b),

$$-CT_1 - BT_3 = T_3J_1$$

so that

$$-CT_1T_3^{-1} - B = T_3J_1T_3^{-1}$$

or

$$-CP - B = T_3J_1T_3^{-1} \tag{47}$$

Thus, the matrix $CP + B$ has all eigenvalues with negative real parts. Substituting for C and B their equivalences in terms of F, G, etc. leads to (24), the eigenvalue inequality.

<u>This completes the Proof of Theorem 3</u>.

VI. CONCLUSIONS

A statement of the spectral factorization problem has been given in algebraic form. To solve the problem, a quadratic matrix equation must be solved, and to obtain a factor with the additional property of having a stable right half plane inverse (i.e., an inverse analytic in the right half plane), a particular solution satisfying an eigenvalue inequality must be found. One way of solving this equation is to determine the eigenvectors of a matrix whose dimension is twice that of the unknown matrix. For large dimension matrices it is thus clear that an iterative computer solution might present a reasonable alternative, though it would appear difficult to incorporate the eigenvalue inequality constraint in such an iterative procedure.

It also seems probable that the quadratic matrix equation could be viewed as the limit of a differential matrix Riccati equation. The limit of a solution to this Riccati equation, obtained by straightforward iteration, would be a solution of the quadratic equation.

REFERENCES

1. N. Wiener and L. Masani, "The Prediction Theory of Multivariate Stochastic Processes," pts. 1 and 2, Acta Mathematics, Vol. 98, June 1958.

2. R. W. Newcomb, Linear Multiport Synthesis, McGraw Hill, New York, 1966.

3. D. C. Youla, "On the Factorization of Rational Matrices," IRE Transactions on Information Theory, Vol. IT-7, No. 3, July 1961, pp. 172-189.

4. M. C. Davis, "Factoring the Spectral Matrix," IEEE Transactions on Automatic Control, Vol. AC-8, No. 4, October 1963, pp. 296-305.

5. A. C. Riddle and B. D. Anderson," Spectral Factorization-Efficient Computation," IEEE Transactions on Automatic Control, Vol. AC-10, No. 4, October 1966, to appear.

6. J. E. Potter, "Matrix Quadratic Solutions," J. SIAM Appl. Math., Vol. 14, No. 3, May 1966, pp. 496-501.

7. R. E. Kalman, "Mathematical Description of Linear Dynamical Systems," Journal of the Society for Industrial and Applied Mathematics Control, Series A, Vol. 1, No. 2, 1963, pp. 152-192.

8. B. D. O. Anderson, "A System Theory Criterion for Positive Real Matrices," J. SIAM Control, to appear.

9. W. Hahn, Theory and Application of Liapunov's Direct Method, Prentice-Hall, New Jersey, 1963.

10. F. R. Gantmacher, The Theory of Matrices, Chelsea, New York, 1959.

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