DEADBEBAT CONTROL USING PERIODIC FEEDBACK

A. S. Morse

Center for Systems Science
Yale University
P. 0. Box 2157 Yale Station
New Haven, Connecticut 06510-2157

R.D.O. Anderson

Dept. of Systems Engineering
The Australian National Univ.
P. O. Box + Canberra
ACT 2600 Australia

W. A. Wolovich

Div. of Engineering
Brown University
Providence
Rhode Island 02912

Summary

Current efforts to develop simple control algorithms suitable for microprocessor implementation have led to renewed interest in the effect of periodic output feedback laws

\[ u_i = f_i y_i, \quad i \geq 1 \]  

on the closed-loop response of single-input, single-output, \( n \)-dimensional, discrete-time processes

\[ x_{i+1} = A x_i + b u_i \]

\[ y_i = c x_i \]

Relative ease of implementation makes control laws such as (1) an attractive alternative to conventional dynamic compensation for process regulation [1].

One problem of interest is to determine the \( f_i \) so that deadbeat control results when (1) is applied to (2). More precisely, (1) is a deadbeat control law for (2) if the closed-loop state transition matrix of (2) over a period \( T \) is zero; i.e.

\[ T = (A + bf(c)) = 0 \]  

where \( f \) satisfies

\[ \dim(\ker(\pi)) = \dim(\ker(I - M_{T^*})) \]  

where \( M = \text{diag}(f_1 f_2 \ldots f_{T^*}) \). The proposition implies that (3) is true, just in case

\[ \text{rank}(I - M_{T^*}) = T - \pi \]

Using this fact, it is possible to show that for \( n = 2 \), \( T \) may be either 2 or 3 depending on the particular system, whereas, for \( n = 3 \), \( T \) may be 3, 4, 5 or 7, again depending on the particular system. In all but exceptional cases \( T \leq 3 \) when \( n = 2 \) and \( T \leq 6 \) when \( n = 3 \).

It is also possible to show that for almost every \( n \)-dimensional system, \( T \leq n(n+1)/2 \), and it is conjectured that this is actually an equality. In general, the problem of computing \( T \) is unsolved.

References


