DEADBEAT CONTROL USING PERIODIC FEEDBACK†

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Summary

Current efforts to develop simple control algorithms suitable for microprocessor implementation have led to a renewed interest in the effect of periodic output feedback laws

$$u_i = f_i y_i \ , \ i \geq 1 \tag{1}$$

on the closed-loop response of single-input, single-output, n-dimensional, discrete-time processes

$$\begin{aligned} x_{i+1} &= Ax_i + bu_i \\ y_i &= cx_i \end{aligned} \tag{2}$$

Relative ease of implementation makes control laws such as (1) an attractive alternative to conventional dynamic compensation for process regulation [1].

One problem of interest is to determine the f_i so that dead-beat control results when (1) is applied to (2). More precisely, (1) is a deadbeat control law for (2) if the closed-loop state transition matrix of (2) over a period T is zero; i.e.

$$\prod_{i=1}^{T} (A + bf_i c) = 0 \tag{3}$$

Conditions for deadbeat control to be possible are known [2-4]. Mullis [2] has proved that there exists an integer T and a sequence $f_1, \ldots, f_T$ for which (3) holds if and only if $\alpha(0) \neq 0$, where $\alpha(z)/\beta(z)$ is the transfer function of (A,b,c). In addition, it has been shown [4] that if $\alpha(0) \neq 0$, T need not be selected larger than $n(3n/g + q - \sigma)$ where $g = \gcd I$, $I = \{i : cA^{i-1}b \neq 0, i \leq n^2\}$, σ is the least value of i for which $ig \in I$, and q is the smallest integer in the additive semigroup $\mathcal{I}$ generated by I for which $\{gq, g(q+1), \ldots, g(q+\sigma-1)\}$ $\mathcal{I}$. It is the purpose of this short paper to develop better bounds for T for various types of systems.

For n = 1, the smallest period T^* for which deadbeat control is possible is obviously $T^* = 1$. To obtain additional results for n > 1, use will be made of

<u>Proposition 1:</u> *Let $A_{n\times n}, b_{n\times 1}, c_{1\times n}$ be a fixed canonical system with A nonsingular; and for T > 0 write M_T for the upper-triangular Toeplitz matrix*

$$M_T = \begin{bmatrix} cA^{-1}b, & cA^{-2}b & \cdot & \cdot & \cdot & cA^{-T}b \\ 0 & cA^{-1}b & \cdot & \cdot & \cdot & cA^{-(T-1)}b \\ \vdots & \vdots & \cdot & \cdot & \cdot & \vdots \\ 0 & \cdot & \cdot & \cdot & 0 & cA^{-1}b \end{bmatrix}_{T\times T}$$

Then, for each sequence of real numbers $f_1, \ldots, f_T$,

$$\dim(\text{kernel}(\prod_{i=1}^{T} (A + bf_i c)) = \dim(\text{kernel}(I - M_T F_T))$$

where $F_T = \text{diag}[f_1, f_2, \ldots, f_T]_{T\times T}$.

The proposition implies that (3) is true, just in case

$$\text{rank}[I - M_T F_T] = T - n$$

Using this fact, it is possible to show that for n = 2, T^* *may be either* 2 or 3 depending on the particular system, whereas, for n = 3, T^* may be 3,4,5 or 7, again depending on the particular system. In all but exceptional cases $T^* = 3$ when n = 2 and $T^* = 6$ when n = 3. It is also possible to show that for almost every n-dimensional system, $T^* \leq n(n+1)/2$, and it is conjectured that this is actually an equality. In general, the problem of computing T^* is unsolved.

References

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