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IDENTIFICATION OF ECONOMICALLY PARAMETRIZED SYSTEMS

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Abstract The problem of identifying a partially known linear, time invariant system is considered where the unknowness is that associated with a limited number of physical components comprising the system or with physical parameters affecting part of the system. This unknowness translates to structural conditions on the system transfer function or a state variable representation of the system, and the associated identification problem is multilinear in the unknown parameters. Algorithms which use measurements of the input and output and knowledge of the polynomial coefficients of the multilinear combinations of the system parameters are then described. Persistence of excitation conditions on the input for computability of these algorithms are derived, and uniform asymptotic stability under a variety of settings established.

Keywords Identification; persistence of excitation; robustness; convergence; multilinear.

1. INTRODUCTION

A large number of adaptive algorithms for identifying linear, time invariant systems of known finite order are available in the literature. The proposed algorithms mostly assume a complete lack of knowledge about the unknown system (aside sometimes from the degrees of the numerator and denominator polynomials in the system transfer function) and ignore all additional information available to the modeller. Thus many of these estimate the numerator and denominator coefficients of the transfer function, having first assumed all the coefficients to be unknown. In practice, however, a great deal of partial knowledge is often available, which if exploited should give rise to convenient parametrisations involving fewer unknowns and better identification schemes.

Consider a single input single output, linear time invariant system with a state space representation described by the quadruple $\{F,G,H,j\}$. Suppose such a system has N unknown parameter values $\{k_1,\ldots k_N\}$ and is otherwise known a priori. In many cases, for example, when the k_i are values of a moment of inertia, a capacitor or a frictional coefficient, it can be shown that the system exhibits a rank-1 dependence on the unknown parameters. The definition of rank-1 dependence is as follows.

Definition:

Let $\beta_i = k_i$ or $\beta_i = 1/(a_i k_i + b_i)$ for some a_i and b_i and i=1,2,...,N. Then the parameters k_i appear in a rank-1 fashion in the system described by the quadruple $\{F,G,H,j\}$ if

(i) the β_i occur multilinearly in the elements of F,G,H and j and

(ii) the matrices

$$\frac{\partial}{\partial \beta_i}\left[\begin{array}{c|c} sI - F & G \\ \hline -H^T & j \end{array}\right] \quad i = 1,2,\ldots N$$

have rank 1.

The rank 1 property on its part results in transfer functions which are ratios of two polynomials having polynomial coefficients multilinear in the unknown parameters. For example, with three unknown parameters, the transfer function has the form

$$T(s,k_1,k_2,k_3) = P(s,k_1,k_2,k_3)/Q(s,k_1,k_2,k_3)$$

where

$$P = p_0(s)+k_1p_1(s)+k_2p_2(s)+k_3p_3(s)+k_1k_2p_{12}(s)$$

$$+k_2k_3p_{23}(s)+k_1k_3p_{31}(s)+k_1k_2k_3p_{123}(s)$$

$$Q = q_0(s)+k_1q_1(s)+k_2q_2(s)+k_3q_3(s)+k_1k_2q_{12}(s)$$

$$+k_2k_3q_{23}(s)+k_1k_3q_{31}(s)+k_1k_2k_3q_{123}(s)$$

In general we have found that rank-1 systems occur naturally in circuits, electrical or otherwise, which do not have unknown elements with cross-coupling between different energy storage devices.

In this paper, we develop a parameter estimation procedure and persistence of excitation (p.e.) conditions necessary and sufficient for its implementation. As the p.e. conditions closely resemble the conditions for the case where only two parameters are unknown (Dasgupta, Anderson and Kaye, 1983), we do not discuss them in any great detail. In Section 3, the stability properties of the algorithms are established. Our primary goal is to secure uniform asymptotic convergence of the parameter estimation error to zero in the ideal case (i.e. no noise, no unmodelled modes or nonlinearity, etc), in order that the algorithms perform in a robust (or a more or less acceptable) manner in the face of modest departures from the idealizing assumptions. As with most identification schemes for linear systems, the uniform asymptotic stability is conditional on the satisfaction of p.e. conditions (Anderson 1977,

Kreisselmeier 1977, Morgan and Narendra 1977, Sondhi and Mitra 1976). However, we are in effect facing a nonlinear problem in which questions of both local and global stability are relevant. Our results demonstrate the following properties:

(i) algorithm is locally uniform asymptotically stable (u.a.s.);

(ii) it is globally u.a.s. whenever the unknown parameter vector falls within a fixed region of space;

(iii) a modification of the algorithm is u.a.s. in the large, irrespective of the values of $\underset{\sim}{k}$;

(iv) the unmodified algorithm is u.a.s. if the parameter estimates are initialized to the correct signs, or in the absence of sign information, to zero.

All proofs have been omitted due to space constraints.

2. PARAMETER ADJUSTMENT LAW

Consider a stable, proper single input-single output, time-invariant system with transfer function

$$T(s) = \frac{P(s,\underset{\sim}{k})}{Q(s,\underset{\sim}{k})} \tag{2.1}$$

where P and Q are polynomials in s with coefficients which depend multilinearly on the elements of the unknown parameter vector $\underset{\sim}{k} \triangleq [k_1, k_2, \ldots k_N]^T$. Thus with S defined as

$S = \{1,2,3, \ldots N\}$ we have that

$$\left.\begin{aligned} P &= p_0(s) + \sum_{r \subset S} \Big(\prod_{i \in r} k_i \Big) p_r(s) \\ Q &= q_0(s) + \sum_{r \subset S} \Big(\prod_{i \in r} k_i \Big) q_r(s) \end{aligned}\right\} \tag{2.2}$$

where the $p_r(\cdot)$ and $q_r(\cdot)$ are known polynomials. We shall assume that T is proper, P and Q are coprime and $\delta[q_0] = n > \delta[q_r]$ $\forall r \subset S$. In this section an adaptive algorithm for identifying $\underset{\sim}{k}$ from the measurements of the input signal u and the output signal y and knowledge of the coefficient polynomials $p_r(\cdot)$ and $q_r(\cdot)$ is presented. The algorithm produces at time t an estimate $\hat{\underset{\sim}{k}}(t) \triangleq [\hat{k}_1(t), \ldots, \hat{k}_N(t)]^T$ of the unknown vector $\underset{\sim}{k}$. Define the parameter error $\underset{\sim}{x}(t) = \hat{\underset{\sim}{k}}(t) - \underset{\sim}{k}$, vectors $\underset{\sim}{K}, \hat{\underset{\sim}{K}} \in R^{2^N-1}$ as having the multilinear combinations of $\underset{\sim}{k}$ and $\hat{\underset{\sim}{k}}$ as their elements, respectively and $\underset{\sim}{D}(\underset{\sim}{x},\underset{\sim}{k}) = \hat{\underset{\sim}{K}}(t) - \underset{\sim}{K}$. Thus for a 3-parameter system $\underset{\sim}{K} = [k_1,k_2,k_3,k_1k_2,k_2k_3,k_3k_1,k_1k_2k_3]^T$, the ordering of the elements of different vectors being consistent.

Define also the matrix $\underset{\sim}{\Lambda} = \mathrm{diag}\{\lambda_r \mid r \subset S\}$, with

$\lambda_1, \ldots, \lambda_N > 0$ and $\lambda_{12}, \ldots, \lambda_{123\ldots N} > 0$.

Let

$$L(\underset{\sim}{x},\underset{\sim}{k}) = \underset{\sim}{D}^T(\underset{\sim}{x},\underset{\sim}{k})\underset{\sim}{\Lambda}\underset{\sim}{D}(\underset{\sim}{x},\underset{\sim}{k}) \tag{2.3}$$

then the proposed parameter update law is

$$\dot{\hat{\underset{\sim}{k}}}(t) = -\frac{\partial L(\underset{\sim}{x},\underset{\sim}{k})}{\partial \underset{\sim}{x}^T} \tag{2.4}$$

In the remainder of this section we shall first describe the on-line implementation of (2.4) showing in particular how the right side of (2.4) can be derived from measurements and the conditions under which this implementation is possible. Then we shall describe the rationale behind the law and indicate a possible modification to it.

2.1 Implementation of the adjustment law

Define for each $r \subset S$,

$$v_r(s) = p_r(s)u(s) - q_r(s)y(s)$$

and

$$h_r(s) = \frac{v_r(s)}{(s+\gamma)^n} \tag{2.5}$$

where $\gamma > 0$. We have thus introduced state variable filters in the fashion of (Lion, 1967) to avoid explicit differentiation of the measurements. In order to simplify presentation, the notation will be abused by writing, for example, $h_i(s)$ to denote the Laplace transform of $h_i(t)$. Let for $t > 0$, $\underset{\sim}{H}(t)$ be a vector with elements $h_r(t)$ for all $r \subset S$. (The ordering of entries is the same as the ordering of entries of $\underset{\sim}{K}$.) Then the input-output relation in (2.1) can be rewritten through (2.5) as

$$h_0(t) + \underset{\sim}{K}^T\underset{\sim}{H}(t) \equiv 0 \tag{2.6}$$

Let us now consider the vector $\underset{\sim}{r}(t)$ and the matrix $\underset{\sim}{R}(t)$ given by

$$\underset{\sim}{r}(t) = \int_0^t e^{-\alpha(t-\tau)} h_0(\tau)\underset{\sim}{H}(\tau)d\tau \tag{2.7}$$

and

$$\underset{\sim}{R}(t) = \int_0^t e^{-\alpha(t-\tau)} \underset{\sim}{H}(\tau)\underset{\sim}{H}^T(\tau)d\tau \tag{2.8}$$

where $\alpha > 0$. Under certain persistence of excitation conditions and certain restrictions on the polynomials $p_r(s)$ and $q_r(s)$ $\underset{\sim}{R}(t)$ is nonsingular for all $t > t_0$ and some $t_0 > 0$. If $\underset{\sim}{R}(t)$ is nonsingular, then the following result shows how it can be used in conjunction with $\underset{\sim}{r}(t)$ to compute the right hand side of (2.4).

Theorem 2.1

Consider the system described by (2.6) with $\underset{\sim}{K} \in R^{2^N-1}$ containing the multilinear combinations of the elements of the vector $\underset{\sim}{k} \in R^N$. Then with $\hat{\underset{\sim}{k}}$ an estimate of $\underset{\sim}{k}$, $\hat{\underset{\sim}{K}}$ an appropriately constrained estimate of $\underset{\sim}{K}$, $\underset{\sim}{x} = \hat{\underset{\sim}{k}} - \underset{\sim}{k}$,

$$\underset{\sim}{D}(\underset{\sim}{x},\underset{\sim}{k}) = \hat{\underset{\sim}{K}}(t) - \underset{\sim}{K},$$

$$L(\underset{\sim}{x},\underset{\sim}{k}) = \underset{\sim}{D}^T(\underset{\sim}{x},\underset{\sim}{k})\underset{\sim}{\Lambda}\underset{\sim}{D}(\underset{\sim}{x},\underset{\sim}{k})$$

for some diagonal matrix $\underset{\sim}{\Lambda}$ constrained as noted earlier, $\underset{\sim}{R}(t)$ and $\underset{\sim}{r}(t)$ defined as in (2.7) and (2.8) and $\underset{\sim}{R}(t)$ invertible, we have that

$$\frac{\partial L}{\partial \underset{\sim}{x}^T}(\underset{\sim}{x}(t),\underset{\sim}{k}) = 2\,\frac{\partial \hat{\underset{\sim}{K}}(t)}{\partial \hat{\underset{\sim}{k}}(t)}\,\underset{\sim}{\Lambda}\,[\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t) + \hat{\underset{\sim}{K}}(t)] \tag{2.9}$$

Remarks:

(2.1) The above result indicates that the following implementation of the parameter adjustment law is viable:

$$\dot{\underset{\sim}{R}}(t) = -\alpha\underset{\sim}{R}(t) + \underset{\sim}{H}(t)\underset{\sim}{H}^T(t), \qquad (2.10a)$$

$$0 \leq t \leq t_o \qquad \underset{\sim}{R}(0) = 0$$

$$\frac{d}{dt}[\underset{\sim}{R}(t)]^{-1} = \alpha\underset{\sim}{R}^{-1}(t)$$

$$- \underset{\sim}{R}^{-1}(t)\underset{\sim}{H}(t)\underset{\sim}{H}^T(t)\underset{\sim}{R}^{-1}(t), \forall t > t_o \qquad (2.10b)$$

$$\dot{\underset{\sim}{r}}(t) = -\alpha\underset{\sim}{r}(t) + h_o(t)\underset{\sim}{H}(t) \qquad \underset{\sim}{r}(0) = 0 \qquad (2.11a)$$

$$\dot{\underset{\sim}{k}}(t) = -2\frac{\partial\hat{\underset{\sim}{K}}(t)}{\partial\underset{\sim}{k}(t)}\underset{\sim}{\Lambda}[\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t) + \hat{\underset{\sim}{K}}(t)] \qquad (2.11b)$$

Here the first equation need be implemented only until $\underset{\sim}{R}(t)$ becomes nonsingular at $t = t_o$ when one can initialize $\underset{\sim}{R}^{-1}(t)$ to the appropriate value and thereafter implement the second equation instead.

2.2 Conditions for Implementability

As is evident from the above R(t) must be nonsingular for the algorithm (2.4) to be implementable. As in the two parameter case (Dasgupta, Anderson and Kaye, 1983) this requires the elements of $\underset{\sim}{H}(t)$ to be linearly independent. Following analysis similar to that by Dasgupta, Anderson and Tsoi (1983) this will hold if there exist no collection of scalars θ_r not all zero such that

$$\sum_{r\subset S}\theta_r p_r(s) \equiv \sum_{r\subset S}\theta_r q_r(s) \equiv 0 \qquad (2.12)$$

and the input u(t) is persistently exciting in the sense described below. It turns out that u(t) is p.e. if there exist α_1, α_2 and $\delta > 0$ such that $\forall\ \sigma\in R_+$

$$\alpha_1\underset{\sim}{I} \leq \int_\sigma^{\sigma+\delta}\underset{\sim}{U}(t)\underset{\sim}{U}(t)^T dt \leq \alpha_2\underset{\sim}{I} \qquad (2.13)$$

Here $\underset{\sim}{U}^T(t)=[u(t), u(t)/(s+\bar{\gamma}),\ldots,u(t)/(s+\bar{\gamma})^m]$, u(t) has piecewise bounded and piecewise continuous derivatives, $\gamma > 0$ and m is the degree of the highest order polynomial among $p_r(s)q_{\bar{r}}(s) - q_r(s)p_{\bar{r}}(s)$, $\forall\ r \subset S$ and $\forall\ \bar{r} \subset \{0\} \cup S$. By the definition of $\underset{\sim}{H}(t)$ it is evident that satisfaction of (2.12) precludes the nonsingularity of $\underset{\sim}{R}(t)$.

It should be stressed that the linear dependence of the elements of $\underset{\sim}{H}(t)$ does not always imply the lack of system identifiability. Yet, as we have stated it our algorithm cannot be implementated in such situations. While we do have modifications ot (2.4) which handle linear dependence arising out of $p_r(s) \equiv q_r(s) \equiv 0$ for one or more r, the more general problem remains unsolved.

2.3 Rationale and a Modification

It is clear that $\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t) = -\underset{\sim}{K}$. The first N elements of $\underset{\sim}{K}$ are the true parameter values $k_1,..k_N$. Thus in our algorithm we are apparently computing $k_1,..k_N$ first and then using these to update the parameter estimates. One may well ask: why not simply compute $\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t)$ and dispense with the second part altogether? The answer lies in issues related to robustness. The quantity $-\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t)$ equals $\underset{\sim}{K}$ under ideal settings only and may not do so in the face of departures from ideality. In particular it may not belong to the correct model set, e.g. its (N+1)th element may not equal the product of the first two. The second part of our algorithm forces the estimates to lie in the correct model set and imparts a measure of robustness to the procedure.

In a sense, ours is a constrained minimization problem. For, we have to find an estimate of $\underset{\sim}{K}$ which minimizes the quadratic $L = (\hat{\underset{\sim}{K}}-\underset{\sim}{K})^T\underset{\sim}{\Lambda}(\hat{\underset{\sim}{K}}-\underset{\sim}{K})$ and lies in the correct model set. Our algorithm accomplishes this in two steps.

(i) It computes $-\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t)$ as the unconstrained estimate of $\underset{\sim}{K}$, obtained using the measured quantities $\underset{\sim}{H}(t)$ and $h_o(t)$, by minimizing L.

(ii) It uses (2.11b) to compute the vector in the correct model set which is the nearest to $-\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t)$.

This interpretation makes possible the consideration of a family of other algorithms which offer the prospect of reduced computational burden. Instead of implementing

$$\dot{\underset{\sim}{k}} = -2\frac{\partial\hat{\underset{\sim}{K}}(t)}{\partial\underset{\sim}{k}(t)}\underset{\sim}{\Lambda}[-\underset{\sim}{K}(t) + \hat{\underset{\sim}{K}}(t)] \qquad (2.14)$$

where $\underset{\sim}{K}(t)$ is an unconstrained least squares estimate of the unknown parameter vector (in this case $-\underset{\sim}{R}^{-1}(t)\underset{\sim}{r}(t)$), we could implement

$$\dot{\underset{\sim}{k}} = -2\frac{\partial\hat{\underset{\sim}{K}}(t)}{\partial\underset{\sim}{k}(t)}\underset{\sim}{\Lambda}[-\underset{\sim}{K}_u(t) + \hat{\underset{\sim}{K}}(t)] \qquad (2.15)$$

where $\underset{\sim}{K}_u(t)$ is any estimate of the true parameter vector $\underset{\sim}{K}$ which has the property that in the noise free, exact modelling case, $\underset{\sim}{K}_u(t) \to \underset{\sim}{K}$. Thus $\underset{\sim}{K}_u(t)$ could be an estimate of $\underset{\sim}{K}$ defined using a gradient algorithm, or a discounted least squares algorithm which did not purport to achieve precise identification in finite time in a noise free, exact modelling case.

The possible advantage of using such a $\underset{\sim}{K}_u(t)$ is the potentially simpler computing task. The requirement of suitably fast convergence in the idealized situation will be the subject of later clarification.

3. CONVERGENCE ANALYSIS

In this section, the behaviour of the parameter adjustment algorithm (2.4) and its modifications are analysed. Though the stability results on (2.4) are apparently independent of any p.e. conditions without p.e. (2.4) cannot be implemented, so the p.e. requirement is after all present.

The arguments in this section all refer to the behaviour of the algorithm in the idealized case of no noise, time-invariance of parameters, no nonlinearity, etc. Later, we shall comment on what can happen given departures from the ideal, using our conclusions about behaviour in the ideal case.

3.1 Local and semiglobal stability

The theorem stated below establishes the local and what we call "semiglobal" uniform asymptotic convergence of the algorithm in (2.4). The semiglobal result states that global uniform asymptotic convergence can be claimed if the unknown parameters or some of the elements of the gain matrix $\underset{\sim}{\Lambda}$ fall in a certain region of space. In the sequel we shall assume that

$$\underset{\sim}{\Lambda} = \begin{bmatrix} \underset{\sim}{\Lambda}_1 & | & 0 \\ \hline & | & \\ 0 & | & \underset{\sim}{\Lambda}_2 \end{bmatrix} \qquad (3.1)$$

where

$$\underset{\sim}{\Lambda}_1 = \text{diag}\,\{\lambda_1, \ldots \lambda_N\} \text{ and}$$

$$\underset{\sim}{\Lambda}_2 = \text{diag}\,\{\lambda_{12},\ldots,\lambda_{123\ldots N}\},$$

the diagonal elements of $\underset{\sim}{\Lambda}_1$ being positive and those of $\underset{\sim}{\Lambda}_2$ non-negative.

Theorem 3.1

There exist positive numbers $r(R,\underset{\sim}{\Lambda}), r'(R,\underset{\sim}{k}), r''(\underset{\sim}{k},\Lambda)$ such that the parameter adjustment law

$$\dot{\underset{\sim}{k}} = -\frac{\partial L(\underset{\sim}{x},\underset{\sim}{k})}{\partial \underset{\sim}{x}}$$

is u.a.s. if any of the following hold:

(i) The initial $\underset{\sim}{x}(0)$ lies in a ball around the origin of arbitrarily large radius R, and $\underset{\sim}{k}$ lies in a ball around the origin of radius $r(R,\underset{\sim}{\Lambda})$.

(ii) The initial $\underset{\sim}{x}(0)$ lies in a ball of arbitrarily large radius R and the elements of $\underset{\sim}{\Lambda}_2$, see (3.1), lie in a ball around the origin of radius $r'(R,\underset{\sim}{k})$.

(iii) The initial $\underset{\sim}{x}(0)$ lies in a ball around the origin of radius $r''(\underset{\sim}{k},\underset{\sim}{\Lambda})$.

Remarks:

(3.1) The above theorem fails to specify the exact extent of $\| \underset{\sim}{k} \|$ and $\| \underset{\sim}{\Lambda}_2 \|$ for which global (with respect to $\underset{\sim}{x}$) uniform asymptotic convergence can be claimed, although these are known for the two parameter case, (see Dasgupta, Anderson and Kaye, 1983).

(3.2) Of the above (ii) shows that the algorithm is exponentially stable if $\underset{\sim}{\Lambda}_2 = 0$. But this means that the multilinearities are no longer explicitly accounted for. Thus, intuitively, useful information is being discarded.

3.2 Modified Globally Convergent Algorithm

Using L as the Lyapunov function one can see that $\dot{L}$ is negative semidefinite, equalling zero at the stationary points of (2.4). Unfortunately (2.4) in general has stationary points apart from $\underset{\sim}{x} = 0$ and thus global u.a.s. of (2.4) cannot be claimed. It can be shown, however, that all stationary points apart from $\underset{\sim}{x} = 0$ are unstable with respect to changes in $\underset{\sim}{\Lambda}_2$ and thus in principle one can find perturbations in $\underset{\sim}{\Lambda}_2$ which will induce the parameter estimates to drift away from these "false" points of equilibrium. One way of achieving this is to continuously alter the elements of $\underset{\sim}{\Lambda}_2$ in a manner indicated by Theorem 3.2.

Theorem 3.2:

With L and $\underset{\sim}{\Lambda}_2$ defined as in (2.3) and (3.1) respectively, the parameter update law (2.4) and

$$\dot{\underset{\sim}{\Lambda}}_2(t) = \text{diag}\,\{f_r(\underset{\sim}{\Lambda}_2(t),t)\}_{\substack{r\subset S \\ |r|>1}} \qquad (3.2)$$

is u.a.s. if f_r are continuous, $\lambda_r(0) > 0, \forall \lambda_r(0)$ a diagonal element of $\underset{\sim}{\Lambda}_2(0)$ and $-f_r(\underset{\sim}{\Lambda}_2,t) > \phi_r(\underset{\sim}{\Lambda}_2)$ with ϕ_r obeying

$$\left.\begin{array}{ll} \text{(i)} & \phi_r = 0 \quad \text{iff } \lambda_r = 0 \\ \text{(ii)} & \phi_r > 0 \quad \forall\, \lambda_r > 0 \end{array}\right\} \qquad (3.3)$$

Remarks:

(3.3) A possible $f_r(\underset{\sim}{\Lambda}_2,t)$ is $-a_r\lambda_r^{\ell_r}$, $\ell_r>0$.

(3.4) It is evident, nontheless, that in the limit we could drive $\underset{\sim}{\Lambda}_2$ to zero. An attractive modification of the algorithm would then be the following:

(i) leave $\underset{\sim}{\Lambda}_2$ constant until $\dot{\underset{\sim}{x}}$ slows down;

(ii) then change $\underset{\sim}{\Lambda}_2$ according to $\Delta\underset{\sim}{\Lambda}_2 = -a\underset{\sim}{\Lambda}_2$, until the convergence rate picks up again.

Then if the equilibrium point being approached is $\underset{\sim}{x} = 0$ changing $\underset{\sim}{\Lambda}_2$ will not alter matters. False equilibrium points on the other hand will be driven away.

3.3 Sure Uniform Asymptotic Convergence When $\underset{\sim}{\Lambda}_2$ is Held Constant

In this subsection we demonstrate that even when $\underset{\sim}{\Lambda}_2$ is held constant, initializing the parameter estimates with correct signs guarantees uniform asymptotic convergence. Indeed, in many real situations such knowledge will in fact be available. The unknown parameter may be a moment of inertia, a frictional coefficient and so on. If on the other hand the signs are not known we show that $\hat{\underset{\sim}{k}}(0) = 0$ will suffice. This is because the closed orthant 0, in the $\hat{\underset{\sim}{k}}$ space, which contains the true parameter $\underset{\sim}{k}$ has no false points of equilibrium and is closed with respect to all trajectories.

Theorem 3.3:

Suppose the initial estimate of the parameter vector $\hat{\underset{\sim}{k}}$ be such that

$$k_i\hat{k}_i(0) > 0 \qquad \forall i \in S \qquad (3.4)$$

Then (2.4) is u.a.s. and $\underset{\sim}{x}$ converges to zero at an exponential rate.

Remarks:

(3.5) Adaptive algorithms with exponential rates of convergence are substantially immune to noise and a variety of modelling deficiencies. Thus our algorithm should, as long as the trajectories remain in the Orthant 0, be robust. If the presence of noise, of reasonable magnitudes, does force the trajectories to leave 0, then convergence can still be expected. For, if

$$\hat{k}_j = 0$$

$$\dot{x}_j = -\Big[\sum_{\substack{r \subset S \\ j\in r}} \lambda_r \Big(\prod_{\substack{\alpha\in r \\ \alpha\neq j}} \hat{k}_\alpha k_\alpha\Big)\Big] x_j \qquad (3.5)$$

and if $\hat{\underset{\sim}{k}} = 0$

$$\dot{\underset{\sim}{x}} = -\underset{\sim}{\Lambda}_1 \underset{\sim}{x} \qquad (3.6)$$

It is easy to see from these that in the vicinity of $\hat{\underset{\sim}{k}} = 0$ and/or the hyperplanes bounding 0, the trajectories still point towards the interior of 0. Thus for sufficiently small excursions from 0, they can be expected to re-enter 0.

3.4 Convergence of Alternative Parameter Estimate Update Law

In section 2.3 we had proposed the alternative parameter estimate update law (2.15) where

$$\lim_{t\to\infty} \underset{\sim}{K}_u(t) = \underset{\sim}{K}$$

We now put forward conditions on $\underset{\sim}{K}_u(t)$ which force (2.15) to retain the convergence characteristics of (2.4). First we shall require $\underset{\sim}{K}_u(t)$ to approach $\underset{\sim}{K}$ exponentially fast i.e.

$$\|\underset{\sim}{K}_u(t) - \underset{\sim}{K}\| < \nu_1 \|\underset{\sim}{K}_u(0) - \underset{\sim}{K}\| e^{-\nu_2 t} \qquad (3.7)$$

where ν_1 and $\nu_2 > 0$.

Theorem 3.4:

The parameter estimate update law (2.15) is u.a.s. if:

(i) the parameter estimate $\hat{\underset{\sim}{k}}(t)$ remains in a region where $\|\partial L/\partial x\|$, with L defined in (2.3), equals zero iff $\underset{\sim}{x} = 0$, and

(ii) The adaptive law generating $\underset{\sim}{K}_u(t)$ is such that (3.7) is satisfied in the ideal case.

Remarks:

(3.6) Condition (i) is satisfied if $\hat{\underset{\sim}{k}}(t)$ remains in 0.

(3.7) It is evident from the foregoing that (2.16) will be globally u.a.s. whenever $\underset{\sim}{\Lambda}_2 = 0$ and $\underset{\sim}{K}_u(t)$ satisfies the condition of the above theorem. For nonzero $\underset{\sim}{\Lambda}_2$, however, one can show that $\underset{\sim}{K}_u(t)$ needs to be so generated as to force each of its elements to either have the same sign as the corresponding element in $\underset{\sim}{K}$ or else to equal zero. In cases where the bounds on the magnitude of k_j are known a priori $\underset{\sim}{K}_u$ may be generated through adaptive laws of the form

$$\dot{\underset{\sim}{K}}_u(t) = -\underset{\sim}{H}(t)(h_0(t) + \underset{\sim}{K}_u^T \underset{\sim}{H}(t) - g(\underset{\sim}{K}_u(t))$$

where g is a penalty function of the form described by Kreisselmeier (1983), and is so selected as to ensure that the sign condition holds.

4. CONCLUSION

Adaptive algorithms for identifying linear systems which exhibit a multilinear dependence on the system parameters are proposed. Persistence of excitation conditions on the input, necessary and sufficient for implementability are stated and conditions for uniform asymptotic stability established.

REFERENCES

Anderson, B.D.O. (1977). Exponential stability of equations arising in adaptive identification. IEEE Trans. on Automat. Contr., AC-22, 83-88.

Dasgupta, S., B.D.O. Anderson and R.J. Kaye (1983). Robust identification of partially known systems. Proc. 22nd C.D.C., San Antonio, Texas, 3, 1510-1514.

Dasgupta, S., B.D.O. Anderson and A.C. Tsoi (1983). Input conditions for continuous-time adaptive system problems. Proc. 22nd C.D.C., San Antonio, Texas, 1, 211-216. Also submitted to IEEE Trans. on Auto. Contr.

Kreisselmeier, G. (1977). Adaptive observers with exponential rate of convergence. IEEE Trans. Automat. Contr., AC-22, 2-8.

Kreisselmeier, G. (1983). An approach to stable indirect adaptive control. Submitted for publication.

Morgan, A.P. and K.S. Narendra (1977). On the uniform asymptotic stability of certain linear non-autonomous differential equations. SIAM J. Contr., 15, 5-24.

Sondhi, M.M. and D. Mitra (1976). New results on the performance of a well known class of adaptive filters. Proc. IEEE., 64, 1583-1597.