Proceedings of 23rd Conference
on Decision and Control
Las Vegas, NV, December

A ROBUST, LOCALLY OPTIMAL MODEL REFERENCE ADAPTIVE CONTROLLER

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Abstract

This paper proposes a new model reference adaptive controller. The innovation of this algorithm is that it adds a filtering of the plant inputs and outputs prior to their use in determining the adapted controller parameter vector update. This modification is based on engineering intuition that suggests that this filtering should provide an improvement in the robustness of the adaptive controller to unmodelled plant actions outside the bandwidth of this added filter. Global convergence of this algorithm is easily proven in ideal use, i.e. when there are no unmodelled plant components. In non-ideal use this filtering is interpreted as appropriately influencing the cost function being locally minimized by this adaptive controller, if stable. This optimization interpretation provides a guide to how to choose this added filtering in order to improve algorithm robustness. These results are illustrated with simulations.

I. Introduction

In this paper we consider the development of a novel, robust model-reference adaptive control algorithm. We will utilize concepts of classical robust controller design concerned with specification of frequency regions of control interest. In doing so, we return to the orientation of early research in adaptive control, e.g., [1], by considering algorithms that are, at least locally, optimal after "convergence" in non-ideal use. Our contention is that adaptive controllers designed to achieve local optimization not only provide predictable performance improvement but should also possess improved robustness properties. Global asymptotic stability requires the zeroing of the adaption error, which typically is zero only if the controller performs precisely as desired and, for example, the model-following error is zeroed. Thus any globally, asymptotically convergent adaptive controller has the same ultimately perfect *performance as any other globally convergent adaptive controller.* But, if the assumed plant structure is inadequate to model the actual plant, then any controller predicated on this inadequate structure cannot be expected to regulate *all* modes of the plant in order to zero, e.g., the closed-loop system error in tracking the response of an arbitrary reference model. The stability theory issue then becomes one of the boundedness of the control system states, including the adapted controller parameters. We note that for a particular reference model and operating condition, an "optimal" controller parameterization exists among the class of controller parameterizations stabilizing the closed-loop system. Our concern is whether or not we adaptively select such an optimal restricted complexity controller parameterization, if the adaptive control system remains stable.

In addition to evaluating performance in terms of local cost function minimization rather than just in terms of stability retention, we will consider an algorithm enhancement that is justifiable via engineering intuition fot its potential robustness improvement. Our "fix" is based on the assertion that the non-ideal effects, such as unmodeled dynamics and measurement noise, are principally high frequency phenomena in an approximately linear model (though, in principle, other frequency bands for unstructured uncertainty could be contemplated). This view accepts the high frequency assignment of unstructured plant uncertainty common to robust control studies. Thus, we will modify a "standard" adaptive model-following scheme by low-pass filtering the information vector, which is composed of past plant inputs and outputs, before using it in the adaptive parameter update. Such filtering has a solid practical basis and has been suggested in various forms for inclusion in adaptive controllers, e.g. [2, remarks below (5.2)] and [3]. The purpose is to attenuate unmodeled (and "uncontrollable"), spurious components of the plant behavior prior to extracting information about how to improve the controller parameterization.

This work supported by NSF Grant ECS-8119312, the Radio Research Board of Australia, and the US/Australia Cooperative Science Program.

In section II a particular adaptive model-following problem is formulated. Section III introduces the practically-motivated information vector filtering. In section IV an argument is made for the cost function that is locally minimized in non-ideal use, *if* stability is retained. The designer's ability to influence this cost function for a reasonable improvement in robustness is indicated. Simulations are provided in section V to support these and further observations. These simulations are directed to the study of performance vis-a-vis the misbehaving controllers of [4].

The applicability and robust performance of the locally optimal methods addressed here should not be expected to be a global property pertaining to their use with any class of unmodelled plant effects. Rather, the approach is designed to make the adaptive control robust for a reasonable class of plants whose unmodelled, or out-of-band, modes are seen as inappropriate targets for control. What is being traded here is requirements on system structural prespecification for foreknowledge of frequency ranges of importance. Our paramount objective is to approximate a desired behavior within such important frequency bands. In [5] and [6], a frequency domain measure of the dynamic closeness of transfer functions is formally developed. It is this metric we refer to in our use of terms such as "approximately equal" throughout this paper.

II. An Adaptive Model-Following Problem

We consider an adaptive model-following problem as is familiar from, e.g. [7]-[11]. The control objective is to force the system output to track the response of a desired model transfer function. In ideal situations this tracking error is asymptotically zeroed but in more practical instances the controller structure does not admit exact matching so that a residual error persists [2][4][12]-[15]. A basic question arises: How does the "steady-state controller" chosen by the particular adaptive algorithm perform in terms of its tracking error? In this paper we will consider this question for a model reference adaptive controller that is unique due to its development so as to locally minimize, after effective convergence, a squared, frequency-weighted tracking error in the restricted complexity case.

Full complexity, non-adaptive control

Consider the following model-following problem. The plant is described by

$$A(q^{-1})y(k) = q^{-d} B(q^{-1})u(k), \tag{2.1}$$

where y is the output, u the input,

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_\alpha q^{-\alpha}, \tag{2.2}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_\beta q^{-\beta}, b_0 \neq 0, \tag{2.3}$$

and q^{-1} is the unit delay operator, i.e. $q^{-i}u(k) = u(k-i)$. This plant is to be controlled such that its response y "matches" (given the same initial conditions) the output z of the desired model

$$C(q^{-1})z(k) = q^{-\bar{d}} D(q^{-1})r(k) \tag{2.4}$$

where

$$C(q^{-1}) = 1 + c_1 q^{-1} + c_2 q^{-2} + \cdots + c_\gamma q^{-\gamma}, \tag{2.5}$$

$$D(q^{-1}) = d_0 + d_1 q^{-1} + d_2 q^{-2} + \cdots + d_\delta q^{-\delta}, \tag{2.6}$$

and

$$\bar{d} \geq d \geq 1. \tag{2.7}$$

Formally the desired objective is to (adaptively) select controller parameters that minimize

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$$J = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \{H(q^{-1})[z(k)-y(k)]\}^2, \qquad (2.8)$$

where $H(q^{-1})$ is a stable transfer function that emphasizes (or de-emphasizes) the importance of various frequency components of the tracking error z-y. The problem is complicated by the fact that $A(q^{-1})$ and $B(q^{-1})$ are initially unknown (and possibly slowly time-varying) such that an adaptive controller is suggested.

To formulate the adaptive controller problem in the standard manner as a recursive parameter estimation problem, a controller structure is first chosen that could be adequately parameterized if A and B were time-invariant and fully known. If $A(q^{-1})$ and $B(q^{-1})$ were known *and* $B(q^{-1})$ had all of its roots inside the unit circle the controller structure of Figure 2.1 would prove satisfactory when certain relations, described below, are satisfied by A, B, C, D, M, and N. Note that the cancellation of the plant numerator by the series compensator results in the requirement that $B(q^{-1})$ be stable in order for u to remain bounded. From Figure 2.1

$$\frac{Y(z)}{R(z)} = \frac{z^{-\bar d}D(z^{-1})}{M(z^{-1})A(z^{-1}) - z^{-d}N(z^{-1})}. \qquad (2.9)$$

Thus $e \to 0$, if

$$C(q^{-1}) = M(q^{-1})A(q^{-1}) - q^{-d}N(q^{-1}) \qquad (2.10)$$

can be solved for the m_i and n_i in

$$M(q^{-1}) = 1 + m_1q^{-1} + \cdots + m_\mu q^{-\mu} \qquad (2.11)$$

$$N(q^{-1}) = n_0 + n_1q^{-1} + \cdots + n_\nu q^{-\nu}. \qquad (2.12)$$

For (2.10) to be solvable for an arbitrary stable C it is necessary and sufficient that

$$\mu + \nu + 1 \geq \max(\mu+\alpha, \nu+d) \geq \gamma. \qquad (2.13)$$

The leftmost inequality in (2.13) insures that M and N have enough variable parameters, i.e. the $(\mu+\nu+1)$ n_i and m_i, to adjust all of the coefficients up to the highest power of q^{-1} in $MA + q^{-d}N$, i.e. $\max(\mu+\alpha,\nu+d)$. For a unique solution of (2.10) for the n_i and m_i this leftmost inequality in (2.13) should be an equality. An appropriate choice is $\mu = d-1$ and $\nu = \alpha-1$. Uniqueness is required for exact parameter identification in the exact matching case. The rightmost inequality in (2.13) states that the desired characteristic polynomial C on the left of (2.10) cannot have higher powers of q^{-1} than achievable by solution for M and N on the right of (2.10). Define

$$W(q^{-1}) \equiv M(q^{-1})B(q^{-1})$$

$$= w_0 - w_1q^{-1} - \cdots - w_\omega q^{-\omega}, w_0 \neq 0, \qquad (2.14)$$

where $\omega = \mu + \beta$. Then at the summing junction of Figure 2.1

$$W(q^{-1})u(k) = q^{-(\bar d - d)}D(q^{-1})r(k) + N(q^{-1})y(k), \qquad (2.15)$$

which can be solved for a causal description of u(k) as

$$u(k) = \frac{1}{w_0}\Big[\sum_{i=1}^{\omega} w_i u(k-i) + \sum_{j=0}^{\nu} n_j y(k-j) + \sum_{t=0}^{\delta} d_t r(k-t-\bar d+d)\Big]. \qquad (2.16)$$

The control law implicit in (2.15) and explicit in (2.16) uses a deadbeat observer. A nonunity observer may be introduced as in Fig. 2.1 to decrease the effects of high bandwidth output noise. Adding the observer polynomial

$$P(q^{-1}) = 1 + p_1q^{-1} + p_2^{-2} + \cdots + p_\eta q^{-\eta}, \qquad (2.17)$$

the r to y transfer function of Figure 2.1 using (2.14) becomes

$$\frac{Y(z)}{R(z)} = \frac{z^{-\bar d}P(z^{-1})D(z^{-1})}{M(z^{-1})A(z^{-1}) - z^{-d}N(z^{-1})}. \qquad (2.18)$$

The difference equation for the control signal u in Figure 2.1 is

$$u(k) = \frac{1}{w_0}\Big[\sum_{i=1}^{\omega} w_i \bar u(k-i) + \sum_{j=1}^{\nu} n_j \bar y(k-j) + \sum_{t=0}^{\delta} d_t r(k-t-\bar d+d)\Big] + \sum_{s=1}^{\eta} p_s \bar u(k-s). \qquad (2.19)$$

where

$$P(q^{-1})\bar u(k) \equiv u(k) \qquad (2.20)$$

and

$$P(q^{-1})\bar y(k) \equiv y(k). \qquad (2.21)$$

Adaptive control

The adaptive control problem now becomes one of recursively estimating the w_i and n_j in (2.19). The *hope* is that when J in (2.8) cannot be set to zero at adaptive controller convergence, possibly due to the restricted complexity of W and N such that ω and ν are inadequate to satisfy (2.14) with $\mu = \omega - \beta$ and (2.13) with γ replaced by $\gamma + \nu$, that it is practically minimized by the nearly convergent estimates of w_i and n_i. The next section presents a candidate algorithm for this local minimization objective.

III. Our Information Vector Filtering Innovation

This section will begin with the statement of a "standard" candidate adaptive control algorithm, similar to that in [16], for the model reference adaptive control problem mentioned at the end of the preceding section. It will conclude with the introduction of information vector filtering into this algorithm.

Adaptive control without information vector filtering

We assume that P, C, and D are known and that an identity of the form of (2.1) exists in some approximate sense, with the coefficients of A and B unknown. Thus, for some M and N (2.10) (with $C(q^{-1})$ replaced by $C(q^{-1})P(q^{-1})$) holds (approximately) and with $W \approx MB$ the controller (2.19) leads to approximate model following. Now recognize that with these choices the following equation holds approximately

$$C(q^{-1})y(k) = z^{-\bar d}D(q^{-1})r(k) = W(q^{-1})\bar u(k-d) - N(q^{-1})\bar y(k-d) \quad (3.1)$$

In order to formulate the identification component of the adaptive controller, we proceed as follows. Define

$$X(k) \equiv [\bar u(k)\cdots\bar u(k-\omega)\ \ \bar y(k)\cdots\bar y(k-\nu)]^T \qquad (3.2)$$

and

$$\theta \equiv [w_0\ -w_1\cdots-w_\omega\ \ n_0\ -n_1\cdots-n_\nu]^T. \qquad (3.3)$$

Thus, at least approximately,

$$C(q^{-1})y(k) = X^T(k-d)\theta. \qquad (3.4)$$

Now let $\hat\theta(k-1)$ be an estimate of θ available after measurement of y(k-1) and before time k. Then

$$C(q^{-1})y(k) - X^T(k-d)\hat\theta(k-1) = X^T(k-d)[\theta - \hat\theta(k-1)], \qquad (3.5)$$

approximately. This is the equation error style prediction error useful in adapting the estimates in θ.

The form of (3.5) suggests an adaptive control procedure, similar to that in [16]: (i) measure y(k), (ii) update $\hat\theta(k-1)$ to $\hat\theta(k)$ via

$$\hat\theta(k) = \hat\theta(k-1) + \frac{\lambda(k)X(k-d)}{\epsilon + X^T(k-d)X(k-d)}[C(q^{-1})y(k) - X^T(k-d)\hat\theta(k-1)], \qquad (3.6)$$

where $\epsilon > 0$ and $\lambda(k)$ is chosen as a scalar within (0,2) such that the first entry of $\hat\theta(k)$, i.e. $\hat w_0(k)$, will not be zero, (iii) using the entries of

$$\hat\theta(k) \equiv [\hat w_0(k)\cdots-\hat w_\omega(k)\ -\hat n_0(k)\cdots-\hat n_\nu(k)]^T \qquad (3.7)$$

solve (2.19) rewritten as

$$u(k) = \frac{1}{\hat w_0(k)}\Big[\sum_{i=1}^{\omega}\hat w_i(k)\bar u(k-i) + \sum_{j=1}^{\nu}\hat n_j(k)\bar y(k-j) + \sum_{t=0}^{\delta} d_t r(k-t-\bar d+d)\Big] + \sum_{s=1}^{\eta} p_s \bar u(k-s), \qquad (3.8)$$

which is equivalent to solving $q^{-\bar d}D(q^{-1})r(k) = \hat\theta^T(k)X(k)$ for $\bar u(k)$ and solving (2.20) for u(k), and (iv) form $\bar u(k)$ and $\bar y(k)$ from (2.20) and (2.21), respectively. Note that, defining

$$\Lambda(k) = \frac{\lambda(k)}{\epsilon + X^T(k-d)X(k-d)} \qquad (3.9)$$

and

$$\tilde\theta(k) \equiv \theta - \hat\theta(k) \qquad (3.10)$$

and using (3.5), (3.6) can be written compactly as

$$\hat\theta(k) = \hat\theta(k-1) + \Lambda(k)X(k-d)X^T(k-d)\tilde\theta(k-1). \qquad (3.11)$$

For this algorithm to work satisfactorily in a non-ideal situation, it is virtually essential that the vector X(k) satisfy a persistency of excitation condition [17],[18].

Adaptive control with information vector filtering

In an attempt to provide a more robust adaptive controller retaining the update form of (3.11), which is provably convergent when exact matching occurs, consider

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Lambda(k)\bar{X}(k-d)\bar{X}^T(k-d)\tilde{\theta}(k-1) \qquad (3.12)$$

where $\Lambda(k)$ is redefined as

$$\Lambda(k) = \frac{\lambda(k)}{\epsilon + X^T(k-d)X(k-d)} \qquad (3.13)$$

and $\bar{X}(k)$ is a filtered version of X(k)

$$[F(q^{-1})I]\bar{X}(k) = [G(q^{-1})I]X(k), \qquad (3.14)$$

where F and G both have stable roots (within $|q| < 1$),

$$F(q^{-1}) = 1 + f_1 q^{-1} + \cdots + f_p q^{-p}, \qquad (3.15)$$

and

$$G(q^{-1}) = g_0 + g_1 q^{-1} + + \cdots + g_p q^{-p}, \; g_0 \neq 0. \qquad (3.16)$$

The filtering in (3.14) is the pragmatic information vector filtering mentioned in the introduction. As we shall see in section IV, choosing nonunity F and G will affect the performance index (2.8), which is in effect being minimized, in a reasonable manner.

As (3.11) is a compacted, implicit version of the implemented form in (3.6), (3.12) is also implicit. Developing an implementable form of (3.12) requires construction of $\bar{X}^T(k-d)\theta(k-1)$ from available entities such as $X^T(k-d-i)\tilde{\theta}(k-1-i)$, X(k-d-i), $\bar{X}(k-d-i)$, and $\hat{\theta}(k-1-i)$ for i = 0,1,2,... . This is possible as described in the following lemma.

Lemma 3.1: With quantities as defined above, $\bar{X}^T(k-d)\tilde{\theta}(k-1)$ is (approximately) obtainable from measurable quantities by

$$Z^T(k-d)\tilde{\theta}(k-1) = g_0[C(q^{-1})y(k) - X^T(k-d)\hat{\theta}(k-1)]$$

$$+ \sum_{i=1}^{p} [g_i - g_0 f_i][r(k-i) - Y^T(k-d-i)\hat{\theta}(k-1)]. \qquad (3.17)$$

where

$$F(q^{-1})r(k) = C(q^{-1})y(k) \qquad (3.18)$$

and

$$G^{-1}(q^{-1})Z(k) = Y(k) = F^{-1}(q^{-1})X(k). \qquad (3.19)$$

As may be readily shown, (3.17), which defines the causal implementation of $\bar{X}^T(k-d)\tilde{\theta}(k-1)$, becomes exact when (3.4) is exact, i.e. in the exact matching case. Note that for robustness of the adaptive controller associated with (3.12) the new information vector $\bar{X}(k)$ needs to be persistently exciting, just like X(k) should be for (3.6).

IV. Local Optimality and Robustness Features

The information vector filtering in (3.14) was introduced with the hope that adaptive algorithm robustness would be enhanced in non-ideal use. Note that the filtered information vector $\bar{X}$ is used only in the adaptive algorithm of (3.12) but not in the control law of (3.8), which still uses entries from X in (3.2). If the filtering in (3.14) is appropriately chosen, e.g. as low-pass, then high-frequency, unmodelable "perturbations" in X will be attenuated in their deleterious effect on the adaption with an anticipated improvement in robustness. As indicated in the introduction, this should help insulate the adaption mechanism from overreacting to broadband measurement noise or to well-damped, high frequency, unmodeled plant modes that are intentionally imprecisely controlled due to restricted controller order. The cutoff frequency of this low-pass information vector filtering should not be too low. If it were, the persistent excitation engendered by the reference signal r might be effectively lost. It would be reasonable to require the low-pass information vector filter bandwidth to be comparable to or greater than that desired for the control system.

The source of adaptive controller robustness improvement due to information vector filtering can be more formally supported in the following manner, as will be more fully developed in a subsequent work. Evoking the persistence of excitation of the filtered information vector $\bar{X}$ due to suitable reference signal persistence as in [17] and [19], results in proof of the exponential asymptotic stability (EAS) of (3.12) in the exact matching case. Note that the persistent excitation in the reference signal r should be within the bandwidth of the information vector filter, which is true if the spectrum of r exists primarily within the desired control system bandwidth of (2.4), as is reasonable and has been exploited by others in this context [13]. This previous idea is important or two reasons. It is crucial that the signals remain persistently exciting after filtering is used and that the signals involved should not unduly excite the unmodelled parasitic modes of the system. Then we can apply local bounded-input, bounded-state stability results, arising from EAS in the ideal homogeneous case as in [20] and [21], to the forced adaptive controller error system. This nonlinear, time-varying, forced error system has an input that is essentially the high frequency nonremovable tracking error passed through the low pass information vector. Thus, the magnitude (and energy) of the error system forcing function is reduced relative to the case without information vector filtering. Since the error system is nonlinear, its local stability relies on a tradeoff between the magnitude of the initial condition, actually the parameter error in $\hat{w}_i$ and $\hat{n}_i$ from their locally optimal values, and the forcing function size. Therefore, we would expect appropriate information vector filtering to increase the size of the region of attraction in the parameter estimate space. This robustness improvement will be verified via simulation in the next section.

As a bonus, the information vector filtering is susceptible to an analytical interpretation in terms of the optimization problem statement of section II. This interpretation establishes a simple guide for selecting the information vector filtering polynomials F and G as will be demonstrated by simulations in the following section. From a projection interpretation, as in [22], the equation error form algorithm of (3.12), if convergent, locally minimizes the mean of the square of $Z^T(k-d)\theta$ as implemented on the right of (3.17), where the time index of θ has been removed to indicate the convergence of $\hat{\theta}$. Our argument is that, if (3.12) remains stable such that θ and X remain bounded, despite the inability to zero $Z^T(k-d)\tilde{\theta}$ due to nonideal use, *and* λ is small enough that the jitter in $\hat{\theta}$ near convergence is effectively imperceptible, then upon "convergence" J in (2.8) is minimized for the $H(q^{-1})$ that equates $Z^T(k-d)\theta$ as implemented in (3.17) and $H(q^{-1})[z(k)-y(k)]$. Assuming that the "convergent" $\hat{\theta}$ is effectively time-invariant, permits the derivation, similar to that of (4.10) from (3.17), of

$$Z^T(k-d)\tilde{\theta} = \frac{G(q^{-1})C(q^{-1})}{F(q^{-1})}[y(k)-z(k)]. \qquad (4.1)$$

Thus, our algorithm if stably convergent, is expected to locally minimize (2.8) with H set via

$$H(q^{-1}) = \frac{G(q^{-1})C(q^{-1})}{F(q^{-1})} \qquad (4.2)$$

One might be tempted to suggest using $\{F^{-1}(q^{-1})G(q^{-1})[X(k-d)]\}\{F^{-1}(q^{-1})G(q^{-1})[X^T(k-d)\tilde{\theta}(k-1)]\}$ rather than $\{F^{-1}(q^{-1})G(q^{-1})[X(k-d)]\}\{F^{-1}(q^{-1})G(q^{-1})[X^T(k-d)]\}\tilde{\theta}(k-1)$ in (3.12), since the convergent performance, with θ effectively convergent to a constant value, is the same. The difficulty with this alternate choice is that the resulting algorithm is no longer an equation error scheme and undesirable restrictions are likely to be required on $F^{-1}G$ to establish global asymptotic stability and (thus EAS and BIBS stability) in the exact matching case. Clearly this alternate choice is readily implemented, which accounts for the appeal of its use. That the equation error form can be retained with an implementable adaptive algorithm kernel is the innovation of Lemma 3.1.

Note that if the filtering of X in (3.14) is not utilized, i.e. $F \equiv G \equiv 1$, then (3.12) reduces to (3.11) or equivalently (3.6) and $H \equiv C$ in (4.2). This implies that the mean of $[C(q^{-1})(z(k) - y(k))]^2$ is being minimized. This would be inappropriate in a typical restricted complexity setting by the following reasoning. Since C represents the desired characteristic equation, C^{-1} will typically be low-pass. Thus, with H = C, J in (2.8) inappropriately emphasizes the high frequency mismatch of z and y that is assumed to be "uncontrollable" due to the restricted complexity of W and N. Assuming that the low frequency portion of z-y will generate the error we most wish to reduce suggests using F and G in (4.2) to correct the high-pass tendency of C in a more appropriate H. One such choice is F = C and G = 1, which reduces (4.2) to H = 1 so that the mean of $[z - y]^2$ is being minimized. This choice does not emphasize the high frequency content of the model-following error as does the absence of information vector filtering with F = G = 1. To further attenuate the high frequency model-following error additional roots could be added to F beyond those of C. Note that it is reasonable to require that the cutoff frequency of these additional roots should not be less than the bandwidth of the low-pass C.

V. Simulations

The proposed information vector filtering appears to offer practical performance improvements with appropriate selection of F and G guided by a minimization interpretation. In this section we will briefly support the realizability of these anticipated robustness benefits via simulations.

To summarize succinctly key observations from our current simulation experience we will focus on an example of reduced-order adaptive *model-following* behavior *described* in [4] and interpreted in [2]. The actual, continuous-time, third order plant has the transfer function

$$\frac{Y(s)}{U(s)} = \frac{458}{(s+1)(s^2+30s+229)}. \tag{5.1}$$

A first order model is assumed, i.e. $\alpha = 1$, $\beta = 0$, and $d = 1$ is presumed in (2.1)-(2.3), and the *desired discrete-time* (sample *period* = 0.04 seconds) reference model transfer function is chosen as

$$\frac{Z(z)}{R(z)} = \frac{1}{z-0.8}, \tag{5.2}$$

where $\bar{d} = \gamma = 1$ and $\delta = 0$. (This corresponds to a continuous-time reference model with transfer function 27.9/(s-5.58).) With $\mu = d-1 = 0$ and $\nu = \alpha-1 = 0$ in (2.10)-(2.12) so $\omega = 0$ in (2.14), (2.13) is satisfied with $1 \geqslant \max(1,1) \geqslant 1$.

Simulation 1 (Figures 5.1, 5.2, and 5.3)

With the parameter estimates initialized near zero ($\hat{n}_0(0)=0$ and $\hat{w}_0(0)=0.001$, where $\hat{w}_0(0)\neq 0$ to avoid division by zero in (3.8)), r the persistently exciting sum of a unit step and a unit magnitude sinusoid of frequency 5 rad/sec, and $\lambda = 0.1$ and $\epsilon = 1$ in (3.9) (and (3.13)), the adaptive control algorithm of (3.8) and (3.11) without information vector filtering exhibited instability with the plant output exceeding machine limits (10^{38}) in less than 3 seconds. With appropriate information vector filtering of G=1 and $F=(1-0.6q^{-1})C$, suggested by the minimization interpretation of the preceding section, stability is retained as shown in Figure 5.1. The parameter estimate trajectories in the parameter estimate plane are shown for these two cases in Figures 5.2 and 5.3. Note how with information vector filtering the parameter estimates converge to within a small elliptical region of stabilizing controller parameterizations. (Note that for this example fixed $\hat{w}_0$ and $\hat{n}_0$ must satisfy $-10.9 < \hat{n}_0/\hat{w}_0 < 0.5$ for closed-loop stability.) Without information vector filtering the parameter trajectories wander until they unfortunately lock-onto a destabilizing parameterization. Thus, adaption without information vector filtering has destabilized an initially stable control system.

Simulation 2 (Figures 5.4 and 5.5)

With the addition of plant output measurement noise (white, gaussian, standard deviation = 0.3) and the use of an underexciting unit step reference input, we noted the parameter drift cited in [4] and discussed in [2]. As is expected, the error system forcing function reduction from information vector filtering substantially reduces the rate of this drift. Compare Figures 5.4 and 5.5. Note that with the controller parameterization in [4], where the feedback parameter is estimated directly, such parameter estimate drift toward infinity will inevitably lead to instability. The parameterization of our controller in (3.8), with its division of other estimated parameters by $\hat{w}_0$ to compute the controller parameters, mitigates this effect. In this example, the added loop gain is $\hat{n}_0/\hat{w}_0$. Thus, drift of both $\hat{n}_0$ and $\hat{w}_0$ toward positive infinity along the manifold offering exact step tracking, i.e. correct DC gain, constrains $\hat{n}_0/\hat{w}_0$ such that closed-loop stability is retained indefinitely. Refer again to Figures 5.4 and 5.5. This key observation suggests consideration of indirect adaptive controller parameterizations from the estimated parameters that offer robustness against known mechanisms of adaptive controller instability in the absence of persistent excitation.

VI. Conclusion

In this paper we were concerned with two adaptive controller performance issues: robust stability retention and convergent local optimality, that can both be enhanced with the addition of information vector filtering in the adaption algorithm. A causal candidate algorithm was proposed. Simulations were used to verify its performance improvements.

Certainly extensive study is required before this modification can be suggested for practical use. Of theoretical interest are proofs of the local stability and optimization properties that compliment our brusque derivations and simulated verifications. However, our current results provide ample justification for these efforts.

VII. Acknowledgement

The authors thank M. L. Cauldwell of Cornell University for performing the simulations discussed in section V of this paper. These simulations are written in SIMNON. Listings are available on request.

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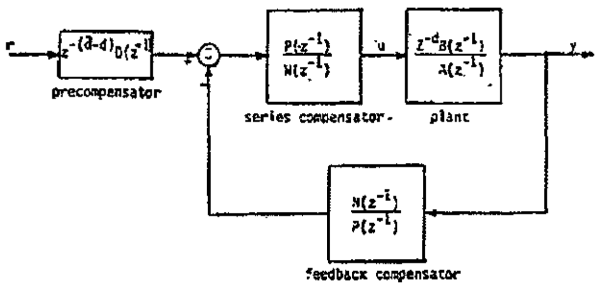

Figure 2.1: Control System Configuration

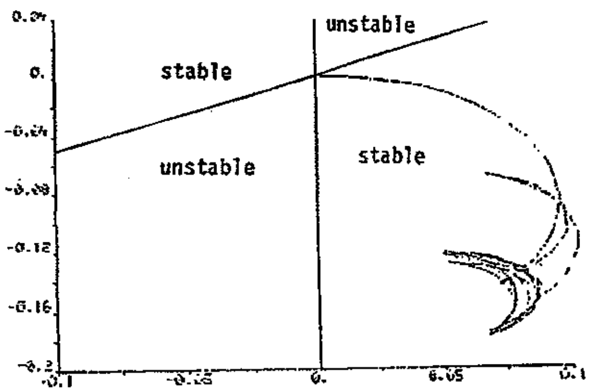

Figure 5.3: Parameter Space Trajectory (With Filtering)

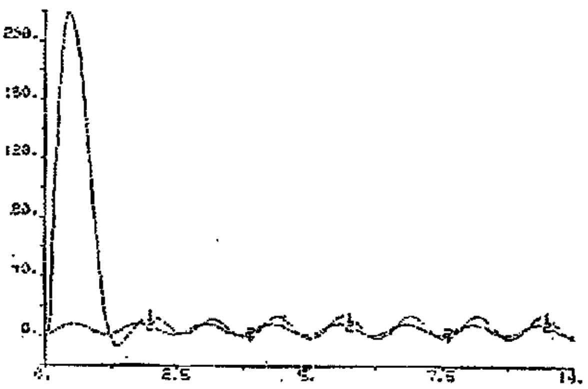

Figure 5.1: Stable Output Tracking of a Step Plus Sinusoid by Adaptive Algorithm Using Filtering (Note: Same example unstable without filtering)

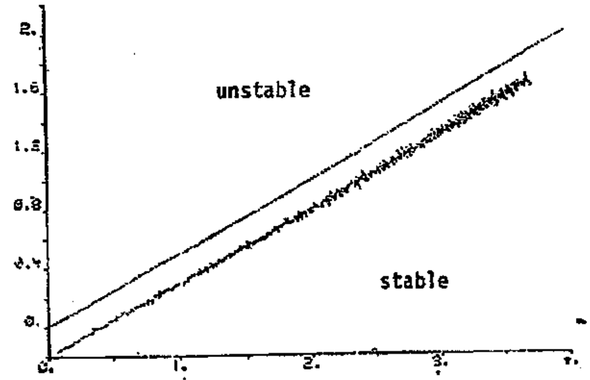

Figure 5.4: 500,000 Iterations Parameter Drift (Without Filtering)

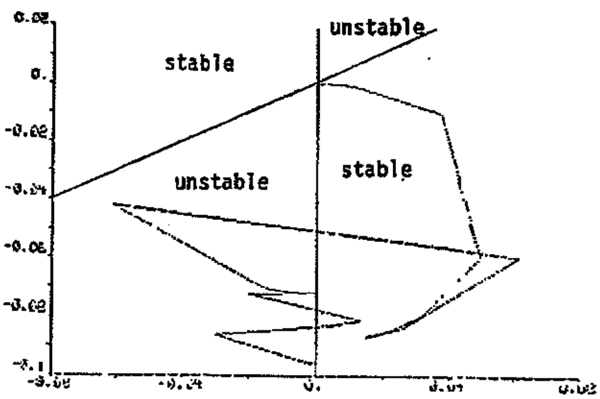

Figure 5.2: Parameter Space Trajectory (Without Filtering)

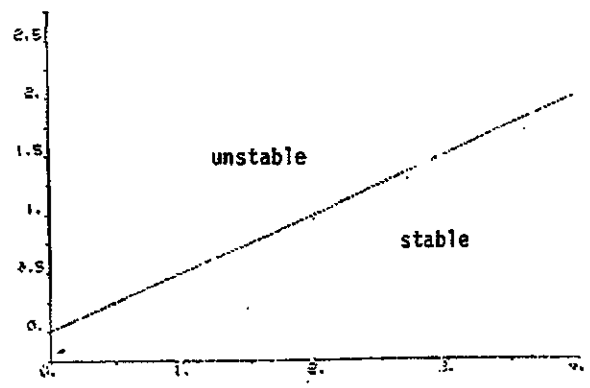

Figure 5.5: 500,000 Iterations Parameter Drift (With Filtering)

"If Charlie Manson checked into the Sahara tomorrow morning, nobody would hassle him as long as he tipped big."
H.S.T.