The author also makes interesting suggestions in Chapter 10 and elsewhere on coordinating long-run (growth) policy with short-run (stabilization) policy. The separation of short-run policies from long-run policies suggests using two time-aggregated models of different time units. These need to be worked out more fully to really explore the implication of Level III optimization.

The theoretical material on these generalizations is given in Chapters 5–8. He illustrates his approaches using a version of the Wharton model which is described in Chapter 2, and its linearization, described in Chapter 3.

As with any books, there are some flaws. The book does not use state space representation, but stacked vectors which is the usual notation in econometrics literature. Consequently, some derivations and expositions are cumbersome or involved. This is true especially in Chapter 5, where dynamic multipliers are derived. Dynamic multipliers summarize the relationships over time between specific exogenous and endogenous variables. In the state space representation, the dynamic multipliers are especially transparent since they are weighting functions.

It is also unfortunate that, for illustrative purposes, the linearized Wharton model turned out to be unstable (although the nonlinear Wharton model is stable). In spite of the author's comments, one feels uneasy in seeing the suggested procedure applied to unstable models. Actually, as Friedman mentions in Chapter 5, in applications described in Chapter 9, some of the unstable multipliers were arbitrarily made stable or replaced by zero. Thus, the linearized models do not exactly represent the correct linearized Wharton model. The author recognizes and discusses somewhat, in nonmathematical terms, various potential pitfalls of his proposed algorithms such as nonconvergence of the sequence of proposed iterations or nonexistence of solutions, but they await much more rigorous treatment. There are some occasional slips. For example, what the control engineers call the open-loop feedback is called closed-loop by Friedman (p. 161). On the whole, however, the book is remarkably free of errors, typographical or otherwise.

Overall, we feel that he has succeeded in large measure to fulfill his stated objective of advancing an integrated set of techniques for applying econometric models to problems of stabilization policy.

Singular Optimal Control Problems—D. J. Bell and D. H. Jacobson (New York: Academic, 1976, 190 pp., $15.00). *Reviewed by Brian D. O. Anderson.*

Brian D. O. Anderson was educated at the University of Sydney, Sydney, Australia, and Stanford University, Stanford, CA, and is now a professor in the Department of Electrical Engineering at the University of Newcastle, Australia. He is the author of several books and a number of papers, which include material on linear-quadratic control problems.

"We feel that the time is now right for the theory of singular problems to be collected together, scattered as it is in numerous different journals, and presented under one cover. That is the purpose of the present volume". At the time of writing this review, that quote is two years old, and the book in which it appears at least eighteen months old. At the time of this review becoming available to Transactions readers, the quote will be nearer two and a half years. Therefore, if the reader wants to learn something about singular control, he must go further than this book, since much has happened in the interim. The reader might even have a mild feeling of irritation that the chain of delays between book publication and subsequent review is about two and a half years, as the reviewer certainly does. But obviously, the last people who can be held accountable for these delays are the authors. What then of their accountability in the task of authorship? Do contents match prefatory claims?

In one sense, yes; perhaps in another, no. The preface suggests a comprehensiveness of coverage, and there is almost an obligation resting on authors of the first book in a field to present a treatment which does not dwell too much on their own contributions to the exclusion of others. In this book, I doubt that many contributors would feel excluded, for the authors indeed mention most contributions. But what is also true is that very frequently the mentions are titillatingly brief. For a real understanding of many of the contributions, one is inevitably driven to the original papers; on the other hand, the work in which the authors have been involved is fairly fully exposited in the book.

There is though one quite strong defense to the implied criticism; at least in the reviewer's opinion, the work of the authors has been unusually significant, displacing or unifying many earlier ideas.

What now can one say about the individual chapters? The first chapter is an historical survey; there is a most extensive bibliography of almost 95 references, and there are 28 pages (of reproduced typewriting, not set by book printer and therefore containing perhaps a little more than 200 words per page). The result is a rather rushed history, not quite at the same speed as a Cecil B. de Mille presentation of the Bible in 3 hours of course, but not leisurely enough for the reviewer's taste, nor, he would surmise, for many readers.

The purpose of Chapter 2 is to set up first and second variations. The calculations are standard, and respectably done. One point started nagging at the reviewer in this chapter, and persisted through later chapters. "...we shall assume...that a control vector u belongs to the interior of the space V (of allowable controls) so that $a_i < u_i < b_i$, $i = 1,2,\cdots,m$." The point is that openness of the set of allowable control values is assumed. Naturally this makes life much easier. But as politicians sometimes say in times of economic distress, real life may not always be easy. And I suspect that a number of interesting problems may be being swept aside from considerations here. Be that as it may, it permits almost exclusive focus in the remainder of the book on linear-quadratic singular problems. (A linear-quadratic problem is singular if and only if the weighting matrix for the square control term in the loss function is singular.)

Chapter 3 reviews necessary conditions for singular controls; the advantage of necessary conditions is that they often allow some rapid sorting out of candidate controls. In this chapter, two classes of conditions are reviewed: the generalized Legendre-Clebsch conditions and an important condition (not equivalent) due to Jacobson. It may not be a particularly important point but the case of vector controls with entries of different orders of singularity is not really dealt with here, or later in the book. This is a fairly technical issue to resolve, which the reviewer has studied.

In Chapter 4, we turn to sufficient conditions and conditions which are necessary and not quite sufficient, and sufficient and not quite necessary. (That a condition can be labeled "not quite sufficient" or "not quite necessary" sounds odd, if not ridiculous; what is meant here is that a modification to the condition which a panel of experts would agree on as being intuitively minor will remove the "not quite" qualification. For the purposes of the review, the reviewer has empanelled himself). The conditions (other than perhaps for their slight asymmetry) are aesthetically beautiful. To the extent that they are often only existence conditions, they must be regarded as limited, with the limit in part removed by a computational scheme of the author outlined in Chapter 5 and in other more recent work by the reviewer. Actually, the slight asymmetry of the conditions can also be removed (though this is not made clear in the text) so that conditions which are simultaneously necessary and sufficient are obtained. This is done by a slight trick: changing the definition of the problem slightly, by asking of a given problem what are the conditions for it to have a *robust* solution.

Chapter 5 has, as its main concern, the use of regularization to solve singular problems. By adding ϵu^2 into the loss function for arbitrary small positive ϵ, one usually converts a singular problem to a nonsingular one. The theory is quite solid, and it is claimed that the idea is computationally feasible. In view of the recent work of the Illinois school on singular perturbations, my guess would be that great computational care would be needed. The control, gain, and performance index in the regularized problem will almost certainly have very rapidly varying behavior near the endpoints, and may require special techniques for their study.

In a field which has not had close pedagogical scrutiny, it is often the case that subjective opinion masquerades as objective validity, as much in reviewer as in author. With this proviso, I would claim that as a first book in its field, it is most welcome and certainly good value—at least in a nonmonetary sense.