the topics considered which are usually not seen except in more advanced references[1]: fairly general initial and terminal manifolds, end point inequality constraints, control constraints dependent on system state, state space constraints, systems with parameters. Naturally in such a brief volume many items of interest must be slighted or completely omitted. Thus nothing is said about sufficient conditions and the Hamilton–Jacobi theory, about linear systems with quadratic criteria, or about computational methods. Moreover, while the examples and exercises are pertinent, they are not as diverse or completely developed as one might desire. The existence of solutions, relations to the classical calculus of variations, the concept of feedback control, and the notion of normality are all mentioned but not explored in depth. Still the overall result is a remarkably comprehensive view of the subject.

The author does not pretend that the treatment has a strong mathematical orientation. However, the control problems and necessary conditions are stated in reasonably precise mathematical language. In most cases the format of the theorems corresponds closely to that of Pontryagin *et al.*,[1] although in certain cases the necessary conditions are simplified by introducing stronger hypotheses. The justification of the necessary conditions is certainly heuristic. It is based on a novel method of attack due to the author and A. Blaquiere. In this method of attack, trajectories of the controlled system are examined in state-cost space. By working with the optimal cost function it is possible to define "limiting surfaces" in this space. Optimal trajectories must be in a limiting surface, while nonoptimal trajectories must lie at least in part "above" a limiting surface. This result leads directly to the notion of adjoint differential equations and a geometric interpretation of the maximum condition. The general idea is closely allied to the dynamic programming-principle of optimality approach but places a much stronger emphasis on the trajectories of the system. The effect is a particularly good "geometric feeling" for what the maximum principle means. Because the more interesting mathematical subtleties are glossed over, the development will probably not appeal to the mathematically sophisticated reader. A deficiency of the method of attack is its lack of reference to the "attainable set" of motions. This notion occurs in much of the current literature and provides an alternative geometric interpretation of optimality.

The quality of exposition is generally acceptable, but its conciseness leads to some difficulties. The overall organization, logical structure, and balance of subject matter are good. The principal weaknesses are occasional omission of key statements and some trouble with terminology and notation. For instance, in Section 1.1 it is not stated whether or not the motion path is a continuous function of t. The following are examples of terms which are introduced with essentially no explanation: tangent plane of $(n-q)$-dimensional manifold, orientation of plane, normal to surface, and homogeneous of degree one. The notation $\pi(x^*(t))$ for a plane is unfortunate because it implies that a plane is determined solely by a point $(x^*(t))$ in the plane. For the experienced reader these and similar shortcomings are not major obstacles, but for the novice they may be a real source of confusion. The rather spotty reference to supporting texts does not ease this situation. Finally, the vocabulary of the subject is somewhat lacking: Lemma 1 is the principle of optimality, equations (1.33) are Hamilton's equations, $A(t_0, t)$ is a fundamental matrix.

While the presentation is essentially self-contained, it does assume reader maturity. The required background includes the theory of matrices and differential equations and the elements of mathematical analysis. This background, which need not be in great depth, may be supplied by any of a number of sources, preferably by an introductory text on modern systems theory such as DeRusso, Roy, and Close[2] or Zadeh and Desoer.[3] The author indicates that parts of the text may be used by readers with less preparation, say a senior student in engineering or a chemical engineer with little experience in systems theory. Due to the closely knit character of the text, the reviewer doubts that this is possible except in unusual cases.

The following are some specific comments on the contents of the book. On page 11 the discussion of existence of solutions of differential equations includes no reference to the possibility of finite escape time. The isochrones shown in Figure 2.3 and related equations are in error. In Section 3.6 it is not mentioned that inequality constraints, which are satisfied as equalities, may not be satisfied as equalities for all $t \in [t_0, t_1]$. This means that the equations given may apply only over a subinterval of $[t_0, t_1]$, and that different equations must be written for different subintervals. This remark also applies to some of the equations given in Chapter 4. Theorem 3 on problems with parameters is correctly stated (in Reference 1 it is not). The treatment of endpoint inequality constraints overlooks certain necessary conditions, e.g., if $x_\nu^*(t_1) = x'_{\nu,\min}$, then $\lambda_\nu(t_1) \leq 0$. The example on a system with a bounded parameter (Section 3.11) contains several errors. Equation (3.119) holds only "above" BOA and a similar difficulty exists with (3.121). Figures 3.11, 3.12, and 3.13 show segments of $B'OA$ and BOA' as straight lines; in all cases this is not so. In Chapter 4, on state space constraints, it is observed that $\nu_0(t) \leq 0$, $t_e \leq t \leq t_l$. Actually more is known[1]: $\nu_0(t)$ is a nonincreasing function on $[t_e, t_l]$. This additional necessary condition could probably be obtained by an extension of the arguments in Chapter 4. Lastly, if $\lambda(t_l - 0) = c\nabla g(x^*(t_l - 0))$, then $\lambda(t)$ must be discontinuous at $t = t_l$ (equation (4.35) not valid). Otherwise, the solution of (4.15) is a trivial one.

The layout of the chapters and sections is clear and pleasing. The typography is excellent, and the figures are well done. Typographical errors are rare.

To summarize, the book is a concise introduction to the theory of optimal control. It features an effective presentation of a large number of results, with a heuristic development of these results. As such it seems best suited for two audiences: the students of a first-year graduate course in engineering, organized around the book, and mature researchers in specialty fields who desire through self-study to learn how optimal control might be applied to their work. Those who intend to study optimal control in depth will desire a more thorough and rigorous introduction, say the text by Athans and Falb,[4] or the book by Lee and Markus.[5] For them the present text might serve as a useful supplementary reference.

[1] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze and E. F. Mishchenko, *The Mathematical Theory of Optimal Processes*, Translation. New York: Interscience, 1962.
[2] P. M. De Russo, R. J. Roy, and C. M. Close, *State Variables for Engineers*. New York: Wiley, 1965.
[3] L. A. Zadeh and C. A. Desoer, *Linear System Theory*. New York: McGraw-Hill, 1963.
[4] M. Athans and P. L. Falb, *Optimal Control*. New York: McGraw-Hill, 1966.
[5] E. B. Lee and L. Markus, *Foundations of Optimal Control*. New York: Wiley, 1967.

Analysis of Discrete Physical Systems—H. E. Koenig, Y. Tokad, and H. K. Kesavan. (New York: McGraw-Hill, 1967, 440 pp.) *Reviewed by Brian D. O. Anderson.*

Dr. Anderson did graduate work at the University of Sydney, Australia, and Stanford University, Stanford, Calif., where he received the Ph.D. degree in electrical engineering in 1964. He has held teaching positions at Stanford University and the University of Sydney, and until recently he was a Staff Consultant with Vidar Corporation, Mountain View, Calif. In February 1967 he became Chairman of the Department of Electrical Engineering at the University of Newcastle, New South Wales, Australia. His research interests are in control and network theory.

The book draws from several areas of systems: network theory, linear graph theory, and what has become known as linear system theory. It attempts to present the concept of a linear system in terms of ideas normally peculiar to each of the afore-mentioned areas and is aimed at the senior undergraduate or beginning graduate.

The thesis of the authors is that any linear system is an interconnection of smaller blocks which are normally two-terminal elements but may be n-terminal ($n > 2$). The two-terminal elements normally can be described by the same type of equations that describe inductances, capacitances, and resistances, while the n-terminal elements can normally be described by the same type of equations that describe gyrators and transformers. The interconnection of such elements can normally be described in graph theoretic terms, i.e., in the language of branches, trees, circuits, and cutsets. Finally, the behavior of this interconnection can be described not only via Laplace transforms but also by simultaneous first-order linear differential equations. Moreover, the solution of these equations can be studied, with particular attention being paid to the evaluation of e^{Pt} for constant P.

An initial and possibly naive criticism of such an approach is that the structure the authors impose on systems, i.e., the view of the system as an interconnection of components with the interconnection spelled out with graph theory ideas, is not the same as the sorts of structure which systems theorists have imposed on linear systems, i.e., the decomposition of the system into, for example, a controllable and an uncontrollable part, characteristic modes, and so forth. To invalidate this criticism, it seems reasonable to expect that the authors' approach should, at the very least, illuminate more clearly those facts obtainable from more standard approaches, but apparently the authors do not completely succeed in this endeavour. Complete controllability is not mentioned in the book. Also, the derivation of state-space equations for electromechanical systems, rather than for mere electrical systems, is claimed to be treated, but in attempting to solve a problem where electromechanical transducers are present, some students would find the book insufficient background for developing state-space equations and for assessing the presence of circuits containing mixtures of capacitors and their mechanical equivalents.

The first four of the eight chapters are essentially concerned with the network-cum-graph theory of linear systems. In Chapter 1 the authors introduce the concept of *through* and *across* variables, corresponding to current and voltage in an electrical network. Using these variables they define notions of power and energy. These notions are not vitally important to linear systems, unless they are connected with establishing the stability of interconnections of passive components, and it is curious that they should be given so much prominence. In view of this prominence, it is rather disturbing to note that a definition of activity does not appear until Chapter 3, even though an example in Chapter 2 requires a knowledge of the concept, and that an interpretation of the authors' definitions implies that a negative capacitor or inductor is passive.

In the first four chapters discrete-time systems (termed discrete-state systems by the authors, probably to the consternation of automata theorists) and time-varying systems are not treated. It seems advisable these days to introduce students as early as possible to the notion that systems may be discrete-time, time-varying, or for that matter nonlinear (a concept which is briefly mentioned in the first four chapters), and one wonders why time-varying systems at least could not have received some attention prior to their very brief mention in Chapter 5.

The description of multiterminal components takes up Chapter 3 entirely, and it is evident that the authors have been anxious to indicate methods of coping with such subsystems of complete systems. Open-circuit, short-circuit, and hybrid parameters are all introduced, and the authors very properly point out that in numerous cases the selection of independent *through* and *across* variables for describing the behavior of a multiterminal component is difficult. What they fail to point out, perhaps because of their preference for the *through* and *across* variables, is that use of scattering variables completely resolves this difficulty. Of course, the use of these variables makes more difficult the formulation of state equations via linear graph theory, but it certainly does not make it impossible, nor does it necessarily require the adoption of any ad hoc procedures.

In the treatment of multiterminal devices, transformers and gyrators are postulated. One might have hoped to see more made of electromechanical transducers such as a loudspeaker, which is essentially a transformer with the primary side electrically driven and the secondary side mechanically driven, for in this reviewer's opinion, the greatest scope now for applying linear graph theory to problems of interconnected two- or n-terminal devices lies in those problems involving mixtures of broad classes of elements, i.e., mixtures of electric, hydraulic, mechanical, and other types of elements. Such problems have received little attention by graph theorists, and the subsequent application, later in the book, of graph theory to the development of system equations shows up this lack of an earlier investigation, for the authors shy away from these "mixed" systems with "mixed" multiterminal components.

That part of the book embracing true linear system theory comprises the hundred or so pages of the fifth and sixth chapters. These two chapters write down the system equations. The transition matrix is introduced for linear systems, and standard solutions of the equations are developed. A great deal of attention is paid to evaluating e^{Pt} or, equivalently, $(sI-P)^{-1}$.

It is interesting to note that the authors treat as the fundamental version of forced state-space equations

$$\dot{\psi} = P\psi + Q_1E + Q_2\dot{E} \qquad (1)$$

where ψ is the state vector and E the system input. Apart from quibbling for little apparent reason about the introduction of yet further symbols in place of the more usual x and u, the formulation of (1) introduces obvious, though not necessarily insurmountable, difficulties in the case of piecewise continuous inputs.

The attention given to the evaluation of e^{Pt} is immense. Idempotent decompositions and spectral theory of matrices are developed, Fadeeva's algorithm for evaluating $(sI-P)^{-1}$ is used, and the method of resolving polynomials is demonstrated. These techniques are only really necessary when the eigenvalues of P are not distinct. Thus, it is a shame that the more common case of P with distinct eigenvalues is not investigated with the same amount of detail, at least to the point of exhibiting characteristic modes. As the authors' treatment now stands, one could expect the average student to get so weighted down in the matrix algebra as to lose sight of the physical nature of the solution of $\dot{x}=Px$ as the sum of such characteristic modes.

An appendage to these two chapters is a discussion of linear system stability, using either the standard Liapunov lemma or the Routh criterion; these two concepts do not seem to be presented as related techniques. In the light of the absence of any discussion of controllability, observability, or other system structural properties, some might question the wisdom of including the stability discussion. It seems that the authors might well have been guided by the material found in much earlier (pre state space) textbooks on linear systems, where stability was given a great deal of attention.

The penultimate chapter of the book is entitled "Response Characteristics of Linear System." The authors' judgement in selecting the material is open to some question. The chapter opens with a discussion of the conversion of forced linear equations to equivalent unforced ones, a procedure that is valid when the inputs are sinusoidal or exponential. Subsequently, it considers the steady-state response of systems with e^{st} type inputs, the concept of Q (quality factor) of tuned circuits, and Bode diagrams. Fourier transforms are then covered in two pages, Laplace transforms in three, and then, before expanding some earlier concepts, an all too brief section on z transforms is included. There tends to be a lack of continuity in this material, and some of it does seem irrelevant to the central theme of the book.

The final chapter deals with interconnection of systems. Typical of those interconnections, to which a great deal of attention is given, are generalized versions of the standard sorts of two-port network interconnections and general feedback connections. The application of the topological concepts in treating some of the ideas is interesting.

The book is billed as an undergraduate or beginning graduate text, and this seems reasonable. Students will find less explanation but rather more worked examples and problems than is usual in a book of this type. Recognizing that these things are a matter of personal preference, the reviewer feels that a little more attention to the development of ideas in the text would have been helpful, as would have been a more careful logical structuring of the theoretical ideas presented. In some places an outline or a heuristic proof is used, and an instructor should be aware of the danger of students forming the belief that such proofs are always satisfactory.