Book Reviews

In this section, the IEEE Automatic Control Group publishes reviews of books in the control field and related areas. Readers are invited to send comments on these reviews for possible publication in the Correspondence section of this TRANSACTIONS. The G-A-C dces not necessarily endorse the opinions of the reviewers.

Stephen J. Kahne
Associate Editor—Book Reviews
Dept. of Electrical Engineering
University of Minnesota
Minneapolis, Minn.

Modern Foundations of Systems Engineering—William A. Porter. (New York: Macmillan, 1966, 493 pp.) *Reviewed by Brian D. Anderson.*

Dr. Anderson did graduate work at the University of Sydney, Australia, and Stanford University, Calif., where he received the Ph.D. degree in electrical engineering in 1964. He has held teaching positions at Stanford University and the University of Sydney, and until recently he was a Staff Consultant with Vidar Corporation, Mountain View, Calif. He is presently Chairman of the Department of Electrical Engineering at the University of Newcastle, New South Wales, Australia. His research interests are in control and network theory.

This is clearly a scholarly book. While it is perhaps true that the long bibliographies in some textbooks today reflect the diligence of an abstracting service, of graduate students, or of a secretary, Porter's bibliography reflects a painstaking approach throughout the book, with attention to both pedagogy and the presentation of the most recent results.

The central theme in the book is the application of functional analysis to linear systems problems, culminating in a consideration of minimum-norm type problems in Banach space. In the early part, the author seeks to develop the necessary purely mathematical tools.

Much of the mathematics itself is old hat to mathematicians, but Porter's presentation is different and no doubt refreshing to many. The author has dissected from what might be considered the core of functional analysis and topology, only that material which will usefully serve his end purpose. At the same time, the mathematical flow of ideas is punctuated by ideas more properly termed system theoretic, as, for example, the controllability concept discussed in Chapter 3. The result is a novel book. Yet one cannot help wondering if, on the one hand, the exclusion of mathematical topics has been wise (Hausdorff spaces are not indexed, while compactness, total boundedness, separability, and Banach spaces spring from every page), and if, on the other hand, the inclusion of engineering topics has been too sparse, except in the final chapter. Students using the book will not obtain the full mathematical picture, though they will surely obtain a good one, and some will undoubtedly be left wondering (at least prior to the final chapter) how the sophisticated tools they are learning can be applied in systems theory.

But what should one think of the final chapter which is entitled "Geometric Methods in Problems of Optimal Control"? The omission of discussion or application of the Maximum Principle may annoy those who are conditioned to accept the appearance of the Principle in the first lines of the solution of any optimal control problem. Yet the omission of the Principle turns out to be the very measure which the author proposes for the success of the methods. To those who have mastered the preliminary mathematical detail, the applications of this chapter are truly elegant.

Problems considered in the optimal control chapter deal with linear systems but not necessarily finite dimensional ones, with performance indices involving norms (in Banach space) or linear transformations of the control. True, the emphasis is on linearity, but it is not on finite dimensionality, or time invariance. At the same time, as the author shows, the performances indices that may be considered include most of those for which analytic solutions have been available and, presumably, some new ones. It would, of course, be unfair to compare functional analysis methods with the more classical methods within the framework of the problems to which classical methods have been applied. Rather a broader picture must be looked at in order to observe that the functional analysis methods apparently illuminate areas unassailable with the classical methods. Certainly Porter's book performs such an illumination.

The technical difficulties involved in solving the problems will come as a surprise to many readers. These stem particularly from the necessity to work in Banach space as distinct from Hilbert space for all but a few problems, e.g., minimum energy. The natural orthogonality and projection properties of the Hilbert space are sacrificed. Thus, if B_1 and B_2 are two Banach spaces and T is a bounded linear transformation $T=B_1\rightarrow B_2$, and if for each ξ in the range of T one seeks to find the u of minimum norm such that $Tu=\xi$, then the comparatively simple procedures of the Hilbert space situation (relying on projection properties) must be replaced by far more complex procedures, which are preceded by fundamental questions involving the existence and uniqueness of a solution.

Whether or not the results will have much practical significance remains to be seen, but it would certainly seem that very severe requirements will be placed on computers used for any numerical solutions. In essence this is because finite dimensional state space, finite parameter systems are supplanted by infinite dimensional state space, infinite parameter systems. Those practical situations to which Porter's theory can at present be satisfactorily applied appear also to be those situations to which other existing methods of optimal control can be applied. It would be fair to claim that the theoretical significance of the results is far greater than their (present) practical significance. It would seem that both the problem formulations and the results may stimulate other workers to investigate unexplored paths. Where these paths may lead is a subject for interesting speculation, but one might hope that practically significant results will appear. To the extent that the optimal control of distributed systems is in the prenatal stage, at least in comparison with the optimal control of finite dimensional systems, it would also seem highly appropriate to prosecute such investigations.

We would disagree with some of the author's own prefatory comments about his work. It seems hard to envisage satisfactory use of the book in a first-year course, other than with students having a very strong mathematical background. Despite the careful presentation, the succession of theorems, lemmas, definitions, and examples, the general mathematical level is undoubtedly sophisticated. Even with the disclaimer that theory and applications are present to some degree in all chapters, the claims that "Chapter 1 is intended to establish a uniform level of mathematical proficiency: chapter 4 is devoted to solving a specific class of problems in Optimal control," may mislead some; for the whole book is replete with mathematical developments or ideas, and chapter 4 contains some of the most advanced, e.g., rotundity and separating hyperplanes.

It is interesting to speculate whether or not the publishers may have chosen the title of the book. Some might even call the title misinformative; it is certainly uninformative. If such a title is appropriate for Porter's book, it is probably appropriate for half the books currently appearing that deal with any aspect of control.

Stability Theorems for Linear Motions with an Introduction to Liapunov's Direct Method—Siegfried H. Lehnigk. (Englewood Cliffs, N. J.: Prentice-Hall, 1966, 251 pp.) *Reviewed by Nam P. Bhatia.*

Dr. Bhatia received the degree of Dr.rer.nat. from the Technische Hochschule, Dresden, Germany, in 1961. Presently he is Associate Professor of Mathematics at Western Reserve University, Cleveland, Ohio, and Editor of Mathematical Systems Theory. *His research is in the theory and application of Liapunov's direct method, and the theory of dynamical systems.*

The book is principally devoted to the stability theory of linear differential systems

$$\dot{x} = Ax \tag{1}$$

and its applications. Here x is an n-dimensional column vector, A an $n \times n$ constant matrix (with real or complex entries), and $(\cdot)$ denotes differentiation with respect to the independent variable t. We will briefly describe the contents before offering our comments. The first three chapters are devoted to the introduction of Liapunov's notion of stability, asymptotic stability, and instability, and to the basic theorems of Liapunov's direct method. These chapters comprise 71 pages of the book. The remaining three chapters, comprising 176 pages, are devoted to algebraic and geometric criteria for the stability of linear motions and to structurally Hurwitzian polynomials.

In Chapter 1 Liapunov's notion of stability, asymptotic stability, and instability of the equilibrium of

$$\dot{x} = f(x, t), \qquad f(0, t) \equiv 0 \tag{2}$$

is introduced, and examples are given to illustrate the concepts.

Chapter 2 deals with the explicit form of the solutions of the linear system (1) and establishes necessary and sufficient conditions for stability, asymptotic stability, and instability of the equilibrium of (1).

In Chapter 3 the three basic theorems of Liapunov's direct method are given, which provide sufficient conditions for stability, asymptotic stability, and instability of the equilibrium of (2) in terms of the existence of certain scalar functions. For the case of linear systems (1), it is shown that in the noncritical case, i.e., when either all characteristic roots of A have negative real parts or there is a root with a positive real part, there exist quadratic forms satisfying the theorems of asymptotic stability and instability. Notice that in the noncritical case the presence of stability alone is ruled out. Lastly, the existence of these quadratic forms as Liapunov functions, i.e., functions satisfying conditions of a theorem of Liapunov's direct method, is used to demonstrate that the equilibrium of certain differential systems of the form

$$\dot{x} = Ax + f(x, t), \qquad f(0, t) \equiv 0 \tag{3}$$

is asymptotically stable or unstable whenever the equilibrium of (1), the so-called equation of the first approximation of (3), has the corresponding property.

In chapter 4 the author returns to his main theme: stability in autonomous linear systems (1). A fundamental theorem about the stability of the system (1) states that its equilibrium is asymptotically stable if and only if all zeros of the characteristic polynomial of (1) have negative real parts, i.e.,

$$f(s) = a_0 \det (sI - A) = \sum_{i=0}^{n} a_i s^i, \qquad a_0 \neq 0 \tag{4}$$

where I is a unit matrix. A polynomial with this property is called a Hurwitz polynomial, or simply Hurwitzian, and the first four sections in Chapter 4 are devoted to criteria for a polynomial to be Hurwitzian. Such criteria are generally called stability criteria. The author discusses in some detail four stability criteria usually associated with the names of Hermite, Bilharz, Hurwitz, and Routh. Whereas Hermite and Bilharz criteria are for complex polynomials (and thus as a particular case for real polynomials), those of Hurwitz and Routh consider only real polynomials and are therefore less general. However, the latter are more useful in practice and, thus, more widely known. Each criterion gives necessary and sufficient conditions for a polynomial of degree n to be Hurwitzian in terms of n real functions of its coefficients which are required to be positive. The differences lie mainly in the facility of calculation of these functions. The fifth section deals with the Euclid chain of two polynomials and a theorem of Sturm on the number of zeros of a polynomial in a given interval. Given two polynomials $r_1(s)$ and $r_2(s)$ of degrees n_1 and n_2, respectively, $n_1 \geqq n_2 > 0$, there exist unique polynomials $q_1(s)$ and $r_3(s)$ such that $r_1 \equiv q_1 r_2 - r_3$. Here the degree n_3 of the remainder r_3 satisfies $0 \leq n_3 < n_2$. The repetition of this process leads to a sequence of polynomials $r_1, r_2, r_3, \cdots$ where any three consecutive ones are connected by an identity $r_i \equiv q_i r_{i+1} - r_{i+2}$, $i = 1, 2, 3, \cdots$, with unique q_i, and their degrees $n_1, n_2, n_3, \cdots$ satisfy $n_1 \geqq n_2 > n_3 > n_4 \cdots \geqq 0$. The last relation indicates that there is an integer $k \geqq 2$ such that $r_k(s) \not\equiv 0$, whereas $r_p(s) \equiv 0$ for $p > k$. The sequence $r_1(s), r_2(s), \cdots, r_k(s)$ thus obtained is called the Euclid chain of $r_1(s)$ and $r_2(s)$, and the process of obtaining this chain as the Euclid algorithm. The author describes various properties of this chain and establishes relations between the zeros of r_1 and r_2 in terms of properties of the chain. The sixth section describes methods of counting zeros of a polynomial having negative, positive, or zero real parts. This information is sometimes essential in applications to control theory.

Chapter 5 deals with geometric interpretations of algebraic criteria established in Chapter 4 which lead to the geometric methods of Leonhard, Nyquist, and the root-locus method, and with their applications in control engineering. More particularly, in the first section the Cremer-Leonhard separation criterion, i.e., the polynomial (4) is Hurwitzian if and only if the zeros of the polynomials

$$\begin{aligned} r_1(\omega) &= a_0\omega^n - a_2\omega^{n-2} + \cdots \\ r_2(\omega) &= a_1\omega^{n-1} - a_3\omega^{n-3} + \cdots \end{aligned} \tag{5}$$

are interlacing, is established via the Routh criterion and properties of the Euclid chain. The Leonhard locus of a polynomial $f(s)$ is discussed in which one considers the locus of $f(i\omega)$ in the complex plane for all non-negative values of ω. This leads to the Leonhard criterion for a real polynomial to be Hurwitzian in terms of the properties of the Leonhard locus. The second section gives the Nyquist stability criterion and its application to control engineering. Kalman's concepts of controllability and observability and Popov's