Roberto Tempo · Stephen Yurkovich
Pradeep Misra
Editors

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Editors
Roberto Tempo (deceased)
IEIIT—CNR
Politecnico di Torino
Turin
Italy

Pradeep Misra
Russ Engineering Center
Wright State University
Dayton, OH
USA

Stephen Yurkovich
Department of Systems Engineering
University of Texas at Dallas
Richardson, TX
USA

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Chapter 2
Collective Formation Control of Multiple Constant-Speed UAVs with Limited Interactions

Brian D. O. Anderson, Zhiyong Sun, Georg S. Seyboth and Changbin Yu

Abstract In this chapter, we consider coordination control of a group of UAV agents with constant and in general nonidentical speeds. The control input is designed to steer their orientations and the control objective is to achieve a desired formation configuration for all the agents subject to constant-speed constraints. Through a formation feasibility analysis by a three-agent example, we show that it is generally impossible to control and maintain a formation by constant-speed agents if target formation shapes are defined by agents' actual positions. We then adopt a circular motion center as a *virtual position* for each agent to define the target formation shape. Two different formation design approaches, namely, a displacement-based approach and a distance-based approach, are discussed in detail to coordinate a group of constant-speed agents in achieving a target formation with stable circular motions via limited interactions.

G. S. Seyboth is now with Robert Bosch Automotive Steering GmbH.
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B. D. O. Anderson · Z. Sun (✉) · C. Yu
Research School of Engineering, The Australian National University,
Canberra, ACT 2601, Australia
e-mail: zhiyong.sun@anu.edu.au

B. D. O. Anderson
e-mail: brian.anderson@anu.edu.au

C. Yu
e-mail: brad.yu@anu.edu.au

B. D. O. Anderson · C. Yu
School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China

G. S. Seyboth
Institute for Systems Theory and Automatic Control, University of Stuttgart,
Pfaffenwaldring 9, 70550 Stuttgart, Germany
e-mail: georg.seyboth@googlemail.com

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2.1 Introduction

Collective coordination control of networked multi-agent systems has received considerable attention in recent years, partly motivated by its applications in many areas. In this chapter, we consider a particular class of cooperative tasks in multi-agent coordination, namely formation control, in which the control objective is to form or maintain a prescribed geometric shape for a group of agents. One of the most active and important challenges in this area is to control and coordinate a group of Unmanned Aerial Vehicles (UAVs) in a formation [1, 4, 14, 18]. A more realistic and complicated model other than single- or double-integrators that can describe the nonholonomic constraints of such vehicles is the unicycle model [7]. Early contributions on coordination and formation control of unicycle-type agents include the consensus-based formation control [10], the pursuit formation design [12], the rendezvous control [5], etc. Other recent papers include e.g. [3, 6, 11] with different control constraints, but all assume that not only the orientation but also the speed of individual agents are controllable.

A particular constraint in the cooperative control design for UAV agents is that on occasions the UAVs used in the control task (e.g. Aerosonde UAVs or other types of *fixed-wing* aircraft) usually fly most efficiently at fixed, nominal speeds [1, 25]. Further, agents within one formation may have different (but similar) speeds [19]. For collective control of unicycle-like agents with constant speed, the problem becomes more challenging. The two seminal papers [16, 17] provide comprehensive studies on how to coordinate different collective planar motions (e.g. parallel motions or circular motions) for such multi-unicycle systems with constant unit-speed constraints. More recently, the paper [19] has extended the results in [16] to control collective circular motions of heterogeneous unicycle-like agents with *nonidentical* constant speeds. It is shown in [19] that two kinds of circular motions are possible: a circular motion with a common angular frequency and different radii for each agent, or a circular motion with a common radius but different angular frequencies for each agent.

In this chapter, we are particularly interested in how to design controllers to achieve a target formation shape for a group of unicycle-type agents with constant and nonidentical speeds (the trajectory tracking control problem involving constant-speed agents has been discussed in a companion paper [22]). The results build on these previous papers including [16, 17, 19], but here we focus on formation shape control, instead of the circular motion stabilization problem as discussed in [16, 17, 19]. The main challenge for formation controller design comes from agent kinematic constraints, i.e., how to define the desired formations and how to design control laws which comply with the constraint of constant speeds. Of course, the constant-speed constraint indicates that all the agents are always moving, which significantly affects the formation maintenance task. To address this issue, the main idea on formation specification and control adopted in this chapter is to use circular motion center positions as *virtual positions* instead of agents' actual positions for defining a desired formation shape. To this end, the controller aims to drive each agent to reach a *stable circular motion* while achieving a target formation shape. We also note that circular motions are particularly useful in several real-life applications, including surveillance, circumnavigation, target circling and area monitoring [23, 26].

2.2 Agent Models

Before presenting model equations, we first introduce some special notation to be used in this chapter. The set $\mathbb{S}^1$ denotes the unit circle and an angle θ_i is a point in the unit circle space, i.e., $\theta_i \in \mathbb{S}^1$. The n-torus is the Cartesian product $\mathbb{T}^n = \mathbb{S}^1 \times \ldots \times \mathbb{S}^1$. For a complex variable $z \in \mathbb{C}$, we use $\bar{z}$ to denote its complex conjugate. For $z_1, z_2 \in \mathbb{C}^n$, the scalar product is defined by $\langle z_1, z_2 \rangle = \mathrm{Re}(\bar{z}_1^T z_2)$, i.e., the real part of the standard scalar product over $\mathbb{C}^n$.

In this chapter, we consider a group of n agents modeled by unicycle-like kinematics subject to nonholonomic dynamics and constant-speed constraint. The kinematic equations of agent k are described by

$$\begin{aligned} \dot{x}_k &= v_k \cos(\theta_k) \\ \dot{y}_k &= v_k \sin(\theta_k) \\ \dot{\theta}_k &= u_k \end{aligned} \tag{2.1}$$

where $x_k \in \mathbb{R}$, $y_k \in \mathbb{R}$ are the coordinates of agent k in the real plane and θ_k is the heading angle. The agents have fixed cruising speeds $v_k > 0$ which in general are distinct for different agents; u_k is the control input to be designed for steering the orientation of agent k.

With complex number notation, the complex variable $r_k(t) = x_k(t) + i y_k(t) := |r_k| e^{i\phi_k(t)} \in \mathbb{C}$ denotes the position of agent k in the complex plane. We also define the vectors $r = [r_1, r_2, \ldots, r_n]^T \in \mathbb{C}^n$, $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{T}^n$ and $e^{i\theta} = [e^{i\theta_1}, e^{i\theta_2}, \ldots, e^{i\theta_n}]^T \in \mathbb{C}^n$ to collect the positions and headings of all the agents. Then the above model (2.1) for agent k can be rewritten as

$$\begin{aligned} \dot{r}_k &= v_k e^{i\theta_k} \\ \dot{\theta}_k &= u_k(r, \theta). \end{aligned} \tag{2.2}$$

2.3 Formation Feasibility Analysis: An Example of Rigid Formation Maintenance by Constant-Speed Agents

In this section, we present a brief discussion on formation feasibility analysis for a group of constant-speed agents, based on an illustrative example. In this example, we suppose the formation is defined by a certain set of inter-agent distances between

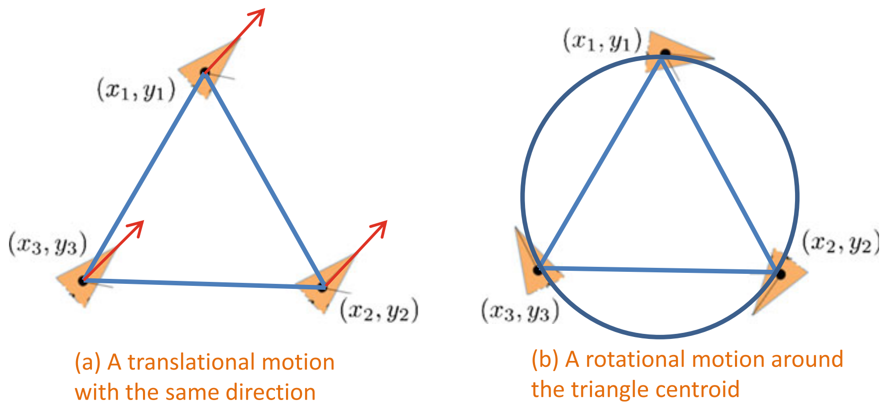

Fig. 2.1 Two feasible formations with a group of constant-speed agents in a triangular formation. (Reproduced with permission from © IEEE 2016, Z. Sun and B.D.O. Anderson [20])

agents' actual positions. This control task[1] is termed *rigid formation control*, which has received increasing attention in the research field of multi-agent coordination, in particular since the publication of [9].

Consider a group of three constant-speed agents in maintaining a triangular formation. The inter-agent distances are denoted by d_{ij} with $i, j = 1, 2, 3, i \neq j$, for which the three agents aim to achieve. If the distance error $e_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 - d_{ij}^2$ is zero for all the three edges, then the target formation is achieved and maintained. For the aim of demonstration, in the following analysis we assume that $d_{12} = d_{23} = d_{31} = d$, i.e., the target rigid formation is an equilateral triangle. In determining whether there exist feasible trajectories for all the agents which respect both the formation constraint and the kinematics constraint (i.e., constant-speed constraints), one needs to formulate a formation feasibility equation and to determine whether such an equation has solutions. We refer the readers to [20, 24] for the development of a formation feasibility theory under different constraints, while we omit the detailed calculations here.

In the case that all agents have identical cruising speeds, i.e., $v_1 = v_2 = v_3$, a simple calculation from the formation feasibility theory [20] shows that the motion solution is that either $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0, \theta_1 = \theta_2 = \theta_3$, or $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3, \theta_1 = \theta_2 + \frac{2\pi}{3} = \theta_3 + \frac{4\pi}{3}$, which correspond to a translational motion with the same direction or a rotational motion around the triangle centroid (see Fig. 2.1). In the case that $v_1 = 0, v_2 = v_3$, a feasible motion exists in which agents 2 and 3 rotate around agent 1 with the same angular velocity. In the case that all agents have nonidentical cruising speeds, there usually does not exist a solution to the feasibility condition except for some special cases, which agrees with our intuition that maintaining a

[1]Section 2.3 of this chapter includes material reproduced with permission from Sun, Z., Anderson, B.D.O.: Formation feasibility on coordination control of networked heterogeneous systems with drift terms. In: Proc. of the 55th Conference on Decision and Control, pp. 3462–3467. IEEE 2016.

rigid shape by a group of UAV agents with nonidentical constant speeds is generally impossible. This suggests that one needs to find an alternative way to define a target formation and to formulate different formation control approaches in coordinating multiple constant-speed agents.

2.4 Formulation of Target Formations for a Constant-Speed Agent Group

In formation shape control, the target formation shape is usually defined by some geometrical relationships between neighboring agents' positions among the group. However, as discussed in the preceding section, in the control problem with constant-speed unicycle-like agents the usual way of defining formation shapes in terms of agents' actual positions does not work in this context, since all the agents will always have motions due to the constant-speed constraints. Hence, we need to find alternative variables that are some kind of surrogate of the actual positions to define the desired formation shape.

Before presenting the formation control design, it is helpful to review the following observations ([16, 19]) on motion properties of constant-speed agents:

- If the control $u_k,\ k = 1, 2, \ldots, n$ is identically zero, then each agent travels in a straight line (with the orientation determined by its initial heading $\theta_k(0)$);
- If the control $u_k = \omega_0,\ k = 1, 2, \ldots, n$ where ω_0 is a nonzero constant, then all the agents travel in a circle of radius $v_k/|\omega_0|$, with the rotation direction determined by the sign of ω_0.
- For the case of circular motion generated by a constant control input $u_k = \omega_0 \neq 0$, the center of the circular motion for the k-th agent is described by

$$c_k = r_k + \frac{v_k}{\omega_0} i e^{i\theta_k} \tag{2.3}$$

  which could be regarded as the "state"(or "virtual position") of agent k in the formation shape control design.

In the following sections, we will describe the desired formation shapes by agents' virtual positions c_k. Thus, the control aim is to drive each agent to reach a *stable circular motion* and also a predefined formation shape specified by their circular motion centers. In the next two sections, we will present two different approaches to achieve a multi-agent formation for constant-speed agents with limited interactions, in contrast to the all-to-all interaction as assumed in e.g. [16, 19].

2.5 Formation Control Design with Limited Interaction: Displacement-Based Approach

2.5.1 Controller Design and Convergence Analysis

In this section, as well as the next section we assume the underlying interaction graph is undirected and connected but *not necessarily complete*. This implies each agent in the formation has *limited interaction* only to its neighboring agents, as opposed to the *all-to-all interaction* in which the underlying graph topology is complete.

In the displacement-based approach, the desired formation is described by a set of relative position vectors $\hat{c}_{kj}$ for each $(j, k) \in \mathscr{E}$ where $\mathscr{E}$ is the edge set of the underlying interaction graph. The control task now is to drive each relative virtual position $c_{kj} = c_k - c_j$ to converge to the desired formation shape described by $\hat{c}_{kj}$ where $(j, k) \in \mathscr{E}$. To achieve this formation control objective, we design the control law as

$$u_k = \omega_0 + \gamma\omega_0 \left\langle \sum_{j \in \mathscr{N}_k} \left(c_{kj} - \hat{c}_{kj}\right), v_k e^{i\theta_k} \right\rangle \tag{2.4}$$

where γ is a positive control gain and $\mathscr{N}_k$ denotes agent k's neighboring set.

The main result in this section is summarized in the following theorem.

Theorem 2.1 *For the designed controller* (2.4) *with an underlying undirected and connected graph, agent k's trajectory $r_k(t)$ of the closed-loop system* (2.2) *converges to a stable circular motion with angular velocity ω_0 and radius $v_k/|\omega_0|$ and all the agents form a desired formation shape defined by the desired relative center positions $\hat{c}_{kj}$ where $(j, k) \in \mathscr{E}$.*

Proof For the purposes of proof and analysis it is convenient to postulate that there are underlying variables $\hat{c}_j$, for all j with the property that $\hat{c}_{kj} = \hat{c}_k - \hat{c}_j$ with $(j, k) \in \mathscr{E}$ Further define $\tilde{c}_k = c_k - \hat{c}_k$ and a vector $\tilde{c} = [\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_m]^T$. Note that $\tilde{c}_k$ is not an available control term in (2.4) since $\hat{c}_k$ is not available for agent k. That is, the control input available in each agent's control term depends on *relative* vectors instead of *absolute* vectors, which are made clear in the expression of (2.4). The introduction of $\hat{c}_k$ and $\tilde{c}$ is for the convenience of proof and analysis. To this end, the control function (2.4) can be rewritten as

$$\begin{aligned} u_k &= \omega_0 + \gamma\omega_0 \left\langle \sum_{j \in \mathscr{N}_k} \left((c_k - c_j) - (\hat{c}_k - \hat{c}_j)\right), v_k e^{i\theta_k} \right\rangle \\ &= \omega_0 + \gamma\omega_0 \left\langle L_k(c - \hat{c}), v_k e^{i\theta_k} \right\rangle \end{aligned} \tag{2.5}$$

where L_k denotes the k-th row of the Laplacian matrix L for the underlying interaction graph which is assumed to be connected but not necessarily complete.

By the definition of $\tilde{c}_k$ and the control (2.4), one has

$$\dot{\tilde{c}}_k = \dot{c}_k - \dot{\hat{c}}_k = \frac{v_k}{\omega_0} e^{i\theta_k}(\omega_0 - u_k) = -\gamma v_k e^{i\theta_k} \left\langle L_k \tilde{c}, v_k e^{i\theta_k} \right\rangle. \tag{2.6}$$

Construct the following Lyapunov function candidate

$$V(\tilde{c}) = \frac{1}{2}\langle L\tilde{c}, \tilde{c}\rangle = \frac{1}{2}\langle H\tilde{c}, H\tilde{c}\rangle \tag{2.7}$$

where H is the incidence matrix for the underlying interaction graph. Note that for an undirected graph there holds $L = H^T H$ (see e.g. [13, Chap. 2]). The function $V(\tilde{c})$ is positive semi-definite in $\tilde{c}$ and positive definite in $H\tilde{c}$. Furthermore, there holds: (i) $V \geq 0$ for all $\tilde{c} \in \mathbb{C}^n$, (ii) $V = 0$ if and only if $H\tilde{c} = 0$, and (iii) $V \to \infty$ for $\|H\tilde{c}\| \to \infty$. Hence, the function V defined in (2.7) is a suitable Lyapunov function to assess the stability and convergence of the formation system consisting of constant-speed agents by the proposed control law (2.4). Note that $\tilde{c}$ is a vector function of (r, θ) and we may also rewrite $V(\tilde{c})$ as $V(r, \theta)$.

The time derivative of V along the solution of the formation system (2.2) with the control (2.4) can be computed as

$$\begin{aligned} \dot{V}(r,\theta) &= \langle L\tilde{c}, \dot{\tilde{c}}\rangle = \sum_{k=1}^{n} \left\langle L_k\tilde{c}, \dot{\tilde{c}}_k \right\rangle = \sum_{k=1}^{n} \left\langle L_k\tilde{c}, -\gamma v_k e^{i\theta_k} \left\langle L_k\tilde{c}, v_k e^{i\theta_k}\right\rangle\right\rangle \\ &= -\sum_{k=1}^{n} \left\langle L_k\tilde{c}, \gamma v_k e^{i\theta_k}\right\rangle \left\langle L_k\tilde{c}, v_k e^{i\theta_k}\right\rangle = -\gamma \sum_{k=1}^{n} \left\langle L_k\tilde{c}, v_k e^{i\theta_k}\right\rangle^2 \leq 0 \end{aligned} \tag{2.8}$$

The set on which $\dot{V} = 0$ is characterized by

$$\mathscr{O}(r,\theta) = \{(r,\theta) : u_k = \omega_0, \left\langle L_k\tilde{c}, v_k e^{i\theta_k}\right\rangle = 0, \forall k\}. \tag{2.9}$$

In the set $\mathscr{O}$ there holds either (i) $L_k\tilde{c} = 0$ or (ii) $L_k\tilde{c}$ and $v_k e^{i\theta_k}$ are perpendicular. Note that in $\mathscr{O}$, $u_k = \omega_0$ and hence $\dot{\theta}_k = \omega_0, \dot{c}_k = 0$. Consequently, because $v_k e^{i\theta_k}$ is not constant, $\langle L_k\tilde{c}, v_k e^{i\theta_k}\rangle = 0$ can be satisfied only if $L_k\tilde{c} = 0$ for all $k = 1, 2, \ldots, n$. Thus, by LaSalle's Invariance Principle, all trajectories converge to the largest invariant set contained in $\bar{\mathscr{O}}$ described as

$$\begin{aligned} \bar{\mathscr{O}}(r,\theta) &= \{(r,\theta) : u_k = \omega_0, L_k\tilde{c} = 0, \forall k\} \\ &= \{(r,\theta) : u_k = \omega_0, \tilde{c} \in \text{span}\{\mathbf{1}\}, \forall k\} \\ &= \{(r,\theta) : u_k = \omega_0, c_{kj} = \hat{c}_{kj}, (j,k) \in \mathscr{E}\}. \end{aligned} \tag{2.10}$$

In the set $\bar{\mathscr{O}}$ each agent reaches a stable circular motion with angular velocity ω_0 and radius $v_k/|\omega_0|$ (for agent k) which forms a stable formation described by $\hat{c}_{kj}$ as desired. The proof is completed. □

Remark 2.1 The above control law is a generalization of Theorem 2 (circular motion stabilization) and Corollary 2 (*SE*(2) symmetry breaking) in [16] which stabilize a group of *unit-speed* unicycles to a circular motion around a *single* and fixed beacon point. The main idea in the controller design is inspired by the consensus-based linear formation control [13, 15]. Note that in the above controller (2.4), the control input term involves the relative information of neighboring agents, i.e., the current displacement $c_k - c_j$ of virtual positions and the desired center displacement $\hat{c}_j - \hat{c}_k$ with respect to its neighbors, which can be calculated by using the formula (2.3).

2.5.2 Implementation Analysis

In this subsection, we discuss how the proposed controller (2.4) can be implemented. By denoting the relative virtual position vector as $c_{kj} = c_k - c_j := r_k - r_j + \frac{v_k}{\omega_0} i e^{i\theta_k} - \frac{v_j}{\omega_0} i e^{i\theta_j}$ with $r_{kj} := r_k - r_j = |r_{kj}| e^{i\phi_{r_{kj}}}$ and the desired relative position vector as $\hat{c}_{kj} = \hat{c}_k - \hat{c}_j = |\hat{c}_{kj}| e^{i\phi_{\hat{c}_{kj}}}$, we can obtain the following control term in real variables

$$
\begin{aligned}
&\left\langle \sum_{j \in \mathcal{N}_k} \left((c_k - c_j) - (\hat{c}_k - \hat{c}_j)\right), v_k e^{i\theta_k} \right\rangle \\
&= \left\langle \sum_{j \in \mathcal{N}_k} (r_k - r_j + \frac{v_k}{\omega_0} i e^{i\theta_k} - \frac{v_j}{\omega_0} i e^{i\theta_j} - \hat{c}_{kj}), v_k e^{i\theta_k} \right\rangle \\
&= \left\langle \sum_{j \in \mathcal{N}_k} (r_{kj} - \frac{v_j}{\omega_0} i e^{i\theta_j}), v_k e^{i\theta_k} \right\rangle - \sum_{j \in \mathcal{N}_k} \mathrm{Re}(\overline{\hat{c}_{kj}} v_k e^{i\theta_k}) \\
&= \sum_{j \in \mathcal{N}_k} \left(|r_{kj}| v_k \cos(\phi_{r_{kj}} - \theta_k) + \frac{v_j v_k}{\omega_0} \sin(\theta_k - \theta_j) \right) - \sum_{j \in \mathcal{N}_k} \left(|\hat{c}_{kj}| v_k \cos(\phi_{\hat{c}_{kj}} - \theta_k) \right).
\end{aligned}
\tag{2.11}
$$

Note that in the third line of the above derivation we have used the equality $\left\langle \frac{v_k}{\omega_0} i e^{i\theta_k}, v_k e^{i\theta_k} \right\rangle = 0$. As noted in Remark 2.1, only relative information is required for the control implementation.

Remark 2.2 (**Communication and measurement requirement**) It can be seen from the designed controller (2.4) and its real variable version (2.11) that each agent needs to measure the relative positions r_{kj} and relative headings $\theta_k - \theta_j$ with respect to its neighbors. We note that in practice information of relative headings $\theta_k - \theta_j$ can be obtained either by bearing sensors or by communication. Thus, the controller (2.4) is distributed since only neighboring information is involved.

We also note that in the control function (2.11) there is one term $\hat{c}_{kj}$ for defining the target formation shape, which implies each agent needs global knowledge of the orientation of a *common* coordinate frame such that these vectors can be correctly interpreted.

2.6 Formation Control Design with Limited Interaction: Distance-Based Approach

In this section, we aim to propose an alternative formation controller based on distance specifications. Suppose that the desired formation is specified by a set of m inter-agent distances d_{kj} for each $(j, k) \in \mathscr{E}$, which describes a desired *rigid* formation shape (for the definition of rigid formation, the reader is referred to [2]). We define the distance error as

$$\epsilon_{l_{kj}} = \|c_k - c_j\|^2 - d_{kj}^2$$

and the distance error vector for all the edges is constructed as $\epsilon = [\epsilon_1, \epsilon_2, \ldots, \epsilon_m]^T$. We also assume that any target formation that realizes the distances d_{kj} via the virtual position c is infinitesimally and minimally rigid [8]. By respecting the constant-speed constraint, the control in such a distance-based framework aims to drive all the agents to reach stable circular motions and to achieve the desired distances between their circular motion centers.

For distance-based rigid formation control, the rigidity matrix, denoted by R, plays a central role in shape specification and convergence analysis [2]. We show a nice structure of R with complex variable entries, regarding it as an extension of the standard rigidity matrix defined in real spaces. Denote the incidence matrix of the graph as H. Construct a block diagonal matrix $Z = \text{diag}\{z_1, z_2, \ldots, z_l, \ldots, z_m\}$, where z_l is the relative virtual position vector $z_l = c_k - c_j$ which relates to the relative centers of agent k and agent j with $(j, k) \in \mathscr{E}$. The construction of Z should be consistent with the direction specification of the incidence matrix H. Then the complex rigidity matrix R can be written as $R = ZH \in \mathbb{C}^{m \times n}$.

In order to achieve the target formation defined by prescribed distances d_{kj} between adjacent agents involving their virtual positions and to drive all agents to reach stable circular motions, we design the following distributed formation controller

$$\begin{aligned} u_k &= \omega_0 + \gamma\omega_0 \left\langle \sum_{j \in \mathscr{N}_k} \left(\epsilon_{l_{kj}}(c_k - c_j)\right), v_k e^{i\theta_k} \right\rangle \\ &= \omega_0 + \gamma\omega_0 \left\langle R_k^T \epsilon, v_k e^{i\theta_k} \right\rangle \end{aligned} \tag{2.12}$$

where R_k^T is the k-th row of the transposed rigidity matrix which involves complex variables. The main result in this section is summarized in the following theorem.

Theorem 2.2 *For the designed controller* (2.12)*, agent k's trajectory $r_k(t)$ of the closed-loop system* (2.2) *converges to a stable circular motion with angular velocity ω_0 and radius $v_k/|\omega_0|$. Furthermore, all agents' circular motion centers converge locally to the desired formation shape defined by the distances d_{kj}.*

Proof Let us define the following Lyapunov function candidate

$$V = \frac{1}{4}\epsilon^T\epsilon = \frac{1}{4}\sum_{l=1}^{m}\epsilon_{l_{ij}}^2 = \frac{1}{4}\sum_{(k,j)\in\mathscr{E}}\left(\|c_k - c_j\|^2 - d_{kj}^2\right)^2. \tag{2.13}$$

The quadratic function V in (2.13) satisfies (i) $V \geq 0$ for all $c \in \mathbb{C}^n$, (ii) $V = 0$ if and only if $\epsilon = 0$, and (iii) $V \to \infty$ for $\epsilon \to \infty$. Hence, V defined in (2.13) is a suitable Lyapunov function to assess the stability and convergence of the formation control system with the proposed control law in (2.12). Note that ϵ is a vector function of (r, θ) and we may also rewrite $V(\epsilon)$ as $V(r, \theta)$. Its derivative can be calculated as

$$\dot{V}(r,\theta) = \frac{1}{2}\sum_{(k,j)\in\mathscr{E}}\left(\|c_k - c_j\|^2 - d_{kj}^2\right)\langle c_k - c_j, \dot{c}_k - \dot{c}_j\rangle. \tag{2.14}$$

By the definition of c_k and the control (2.12), one has

$$\dot{c}_k = -\gamma v_k e^{i\theta_k}\left\langle R_k^T\epsilon, v_k e^{i\theta_k}\right\rangle \tag{2.15}$$

Thus, the derivative of V in (2.13) along the solution of the formation system (2.1) with the controller (2.12) can be further written as

$$\dot{V}(r,\theta) = -\gamma\sum_{k=1}^{n}\left\langle R_k^T\epsilon, v_k e^{i\theta_k}\right\rangle^2 \leq 0 \tag{2.16}$$

The set on which $\dot{V} = 0$ is characterized by

$$\mathscr{O}(r,\theta) = \{(r,\theta) : u_k = \omega_0, \left\langle R_k^T\epsilon, v_k e^{i\theta_k}\right\rangle = 0\}. \tag{2.17}$$

By LaSalle's Invariance Principle and similar arguments as in Theorem 2.1, all trajectories converge to the largest invariant set contained in $\bar{\mathscr{O}}$ described as

$$\bar{\mathscr{O}}(\epsilon,\theta) = \{(\epsilon,\theta) : u_k = \omega_0, R^T\epsilon = 0\}. \tag{2.18}$$

Thus in the limit, the trajectory of each agent (agent k) converges to a stable circular motion with angular velocity ω_0 and radius $v_k/|\omega_0|$. Furthermore, the minimal and infinitesimal rigidity of the target formation implies that R is of full row rank for a formation close to the target formation, and therefore the null space of R^T is the zero vector. If the initial distances between the center positions (i.e., $\|c_k - c_j\|^2$) are close to the target distances d_{kj}, the limit set for which $R^T\epsilon = 0$ also implies $\epsilon = 0$, i.e., the distance error also converges to zero (see e.g. [21]). The remaining analysis is similar to that in Sect. 2.5. □

2.7 Simulation Examples

In this section, we show some simulation examples to illustrate the performance of the two proposed controllers. Consider a unicycle-like agent group consisting of four agents with constant speeds $v_1 = 1.0, v_2 = 1.1, v_3 = 1.2, v_4 = 1.3$. The parameters in the control laws are set as $\gamma = 0.1$ and $\omega_0 = 1$. First consider the displacement-based control law, which aims to stabilize a formation shape with desired displacement vectors $\hat{c}_{21} = 8 + 4i, \hat{c}_{32} = 5 - 3i, \hat{c}_{43} = -9 - 5i, \hat{c}_{14} = -4 + 4i, \hat{c}_{24} = 4 + 8i$, while the underlying graph is undirected and connected. The initial positions are chosen randomly in the simulation. The simulated trajectories by using the controller (2.4) are shown in Fig. 2.2, in which all four agents with constant speeds achieve their respective stable circular motions and also form the desired formation shape.

We then consider the distance-based formation control discussed in Sect. 2.6. Suppose four agents in a group are tasked to achieve a rigid rectangle shape with the desired distance set 3, 4, 5, 4, 3 under the control (2.12). The initial positions are chosen such that the initial relative center distances are close to the desired distances. The trajectories of each agent and the final shape are depicted in Fig. 2.3, which shows that stable circular motions for each agent and a rigid target formation are well achieved.

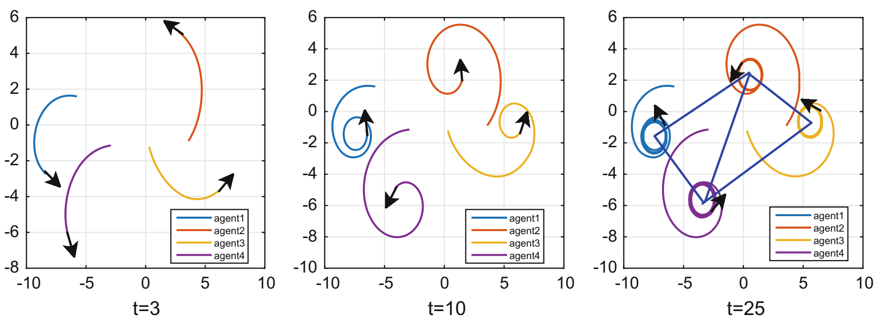

Fig. 2.2 Performance of the displacement-based formation controller (2.4) of four constant-speed unicycle agents with limited interaction

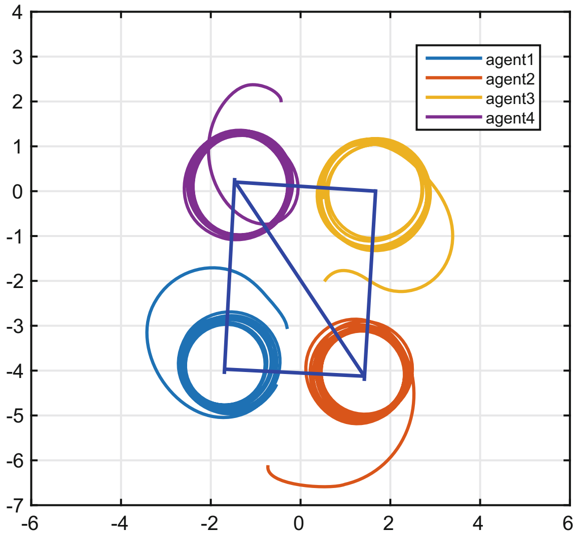

Fig. 2.3 Performance of the distance-based formation controller (2.12) of four constant-speed unicycle agents with limited communication

2.8 Conclusions

In this chapter, we have considered the formation stabilization problem for a group of unicycle-like agents with nonidentical and fixed cruising speeds. By respecting the dynamics constraints caused by constant speeds for each agent, the target formation shape is defined with respect to the rotation center arising from stable circular motions. Two different formation controllers based on different formation specifications and measurement requirements are proposed to coordinate multiple constant-speed agents in achieving a target formation shape and stable circular motions.

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