Wireless Mesh Networking

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Chapter Two

Initially, there is no information available to reach the data sink. Thus, a distributed selection via contention is initiated and next-hop node information is learned. This is stored in the forwarding table. A timer is associated to control the lifetime of the entry. If a subsequent data packet at the sender needs to be delivered to the same data sink, the sender can exploit the next-hop information in the forwarding table to directly unicast the data packet. If the unicast is successful, the timer related to the entry is restarted. If a unicast fails, OGF assumes that the next-hop information is stale and initiates the contention phase to discover a new next-hop node to forward its data packets. When the timer expires and no data packet needs to be delivered to that data sink, the entry in the forwarding table is deleted. The expiration time of a timer is a protocol parameter.

2.5.5.5 On the Critical Connectivity Radii in WM$^2$Snets$^5$

Related Background on Modeling WM$^2$Snets using Graph Theory A graph theoretical model of WM$^2$Snets is first introduced. Although most of the issues and theories discussed throughout this work can be extended to 3D ($\mathbb{R}^3$), we focus on 2D ($\mathbb{R}^2$) problems to simplify the illustration of the basic concepts.

A graphical model of a WM$^2$Snet can be built by using a vertex in the graph to represent every node and an edge in the graph to represent every node pair for which the two nodes can directly communicate with each other. The resulting graph $G = (V, E)$, where $V$ is the vertex set and $E$ is the edge set, is called the underlying graph of the network. Let $|X|$ denote the number of elements in a set $X$; then $|V| = |E|$ represents the number of vertices (edges) in the graph $G = (V, E)$.

A widely used graphical model of a WM$^2$Snet is the unit disk graph. Suppose in the WM$^2$Snet any two nodes can directly communicate with each other if and only if their Euclidean distance is smaller than a given threshold $r$, known as the connectivity radius of the WM$^2$Snet. Then the associated graphical model is known as a unit disk graph.

Another important concept in modeling WM$^2$Snets by graphs rests on the fact that in many WM$^2$Snets, nodes are effectively randomly located. Suppose that nodes are independently and randomly distributed in a $d$-dimensional space ($d$ equals 2 unless otherwise specified) following either a uniform distribution, parameterized by the number of nodes in a defined region, typically the unit square, or a Poisson distribution, parameterized by a node density per unit area. Any two nodes $i$ and $j$ can communicate directly with each other if and only if their Euclidean distance is smaller than a given threshold $r$. The underlying graph of that WM$^2$Snet is called a random geometric graph (Penrose, 2003), which is denoted by $G_n(r)$, or $G_n(r(n))$ when we need to emphasize the dependence of the connectivity radius $r$ on $n$, as will occur later. The parameter $n$ denotes either the total number of nodes in the case of a uniform distribution or the node density per unit area in the case of a Poisson distribution.

In the following section we provide answers to the above questions using properties of graph theory.

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$^5$Excerpt from the invited article "On the critical connectivity radii in wireless mesh sensor networks,"

$^1$Guoqiang Mao, $^2$Brian D. O. Anderson and Bars Fidan, $^3$Jia Fang and A. Stephen Morse ("The work of G. Mao, B. D. O. Anderson and B. Fidan was supported by National ICT Australia. The work of J. Fang and A. S. Morse was supported in part, by grants from the U.S. Army Research Office and the U.S. National Science Foundation and the Xerox Corporation; $^1$University of Sydney and National ICT Australia, Sydney, Australia; $^2$Australian National University and National ICT Australia, Canberra, Australia; $^3$Yale University)."
Using Graph Theory to Model WM²Snets  Whether or not one is working with random networks, a number of WM²Snet problems can be studied in the framework of graph theory. For example, in power control, a critical problem is the minimum transmitter power required to achieve a connected network (Gupta and Kumar, 1998) in which each node in the network has a path to all nodes in the network. If the transmitter power is too large, it may result in excessive interference and power consumption. On the other hand, a small transmitter power may result in the network degenerating into isolated components. Because of the relation between the transmitter power and the connectivity radius, the problem can be cast into graph theory as establishing the minimum $r$ required for a set of nodes for which the underlying unit disk graph is a connected graph. When the graph is random, the question becomes one of determining the minimum $r$ required to guarantee with a prescribed probability, usually $1 - \varepsilon$ for some prescribed small $\varepsilon$, that $G_n(r)$ is connected; alternatively the value $r(n)$ is sought for which, as $n \to \infty$, $G_n(r(n))$ is connected, while $G_n(r(n) - \varepsilon)$ is not connected for any $\varepsilon > 0$. In routing, a problem of concern is that there exist $k$ independent paths between any two nodes for some positive integer $k$. For a random graph, the problem can be formulated as given the connectivity radius $r$, determining the probability that $G_n(r)$ is a $k$-connected graph, or what is virtually equivalent, given a probability that $G_n(r)$ is $k$-connected, determining the value of $r$ that could ensure this.

Another important WM²Snet problem that can be studied in the framework of graph theory is localization. In particular, there is a large class of distance-based localization algorithms that estimate the Euclidean positions of all nodes in a network given the knowledge of the Euclidean positions of some nodes (i.e., anchors) as well as internode distances between certain pairs of nodes (Mao et al., 2006).

The distance-based localization problem can be formulated using graph theory as follows. Consider a WM²Snet with a set $V$ of nodes, a set $D$ of known distances $d_{ij}$ between certain pairs of nodes $v_i, v_j \in V (i, j \in \{1, \ldots, |V|\}, i \neq j)$, and a set $V_a$ of anchors $v_{ai}, (a_i \in \{1, \ldots, |V_a|\}, V_a \subseteq V)$ whose Euclidean coordinates $p(a_i)$ are known. Note that the distances between anchors are given implicitly and hence are known values. The localization problem is one of finding a mapping $p : V \to \mathbb{R}^d$ which assigns coordinates $p(i) \in \mathbb{R}^d$ to each node $v_i$ such that $\|p(i) - p(j)\| = d_{ij}$ holds for all pairs $i, j$ for which $d_{ij}$ is given, and the assignment is consistent with the known anchor positions. The greatest interest is in the case when there is only a single assignment $p(i)$ possible for every $i$, that is, the case when the network is uniquely localizable. This is illustrated in Fig. 2.17.

The WM²Snet localization problem can be split into two fundamental problems: an analytic existence/solvability problem and an algorithmic problem. The analytic existence/solvability problem is concerned with the properties of the network required to uniquely localize all nodes. The algorithmic problem is concerned with the design of efficient algorithms to solve the localization problem, the computational complexity of the localization algorithms, and the properties of the WM²Snet that can be exploited to simplify the computational complexity of the localization algorithms.

In the following section we show that answers to the earlier questions can be found in the framework of graph theory.

Graphical Properties of Uniquely Localizable Networks  A fundamental problem in localization is whether every node in a WM²Snet with a given set of nodes and internode distances is uniquely localizable. Unique localizability is a fundamental property of the WM²Snet, which does not depend on specific localization algorithms. The unique
localizability problem has been studied in the framework of graph theory (Goldenberg et al., 2005; Eren et al., 2004; Aspnes et al., 2006; Anderson et al., 2006). In this section, we shall investigate the graphical properties of a uniquely localizable WM$^2$Snet.

A graphical model $G(V, E)$ of a WM$^2$Snet can be built using the procedure described in the last section. A $d$-dimensional framework $G(V, p)$ is a graph $G(V, E)$ together with a mapping $p : V \rightarrow \mathbb{R}^d$. A framework is called a realization if the associated mapping $p$ satisfies $\|p(i) - p(j)\| = d_{ij}$ for all pairs of $i, j \in V$ where there is an edge between $i$ and $j$. Two frameworks $G(V, p)$ and $G(V, q)$ are equivalent if $\|p(i) - p(j)\| = \|q(i) - q(j)\|$ holds for every pair $i, j \in V$ connected by an edge. Two frameworks $G(V, E)$ and $G(V, q)$ are congruent if $\|p(i) - p(j)\| = \|q(i) - q(j)\|$ holds for every pair $i, j \in V$ no matter whether there is an edge between them. For two congruent frameworks, one can be obtained from the other by applying one or more of a translation, rotation and reflection.

A framework $G(V, p)$ is called generic if the set containing the coordinates of all its points is algebraically independent over the rationales. A framework $G(V, p)$ is called rigid if there exists a sufficiently small positive constant $\varepsilon$ such that if $G(V, p)$ is equivalent to $G(V, q)$ and $\|p(i) - q(i)\| < \varepsilon$ for all $i \in V$, then $G(V, q)$ is congruent to $G(V, p)$. The closeness qualification produced by $\varepsilon$ is critical in the definition, which is to be contrasted with the definition below of global rigidity. A graph $G(V, E)$ is called rigid if there is an associated framework $G(V, p)$ that is generic and rigid. Intuitively, if the underlying graph of a WM$^2$Snet (with at least three anchors in generic positions in $\mathbb{R}^2$) is rigid, there can only be a finite number of solutions to the localization problem and there is no continuous deformation that can move a node (nodes) to a different position (positions) while satisfying the distance constraints. If the underlying graph is nonrigid, there are an infinite number of solutions to the localization problem.

A framework $G(V, p)$ is globally rigid if every framework equivalent to $G(V, p)$ is also congruent to $G(V, p)$. A graph $G(V, E)$ is called globally rigid if there is an associated framework $G(V, p)$ that is generic and globally rigid. If the underlying graph of a WM$^2$Snet (with at least three anchors in generic positions in $\mathbb{R}^2$) is globally rigid and the associated framework is generic, there can only be one solution to the localization
problem, that is, every node in the network is uniquely localizable. Note that rigidity is a generic property in ℝ². In other words, provided the mapping p is generic, the graph G alone determines the rigidity of the framework. One can speak of a rigid graph: this would be a graph such that a generic framework corresponding to the graph is rigid. Global rigidity is also known to be a generic property in ℝ².

For a WM²Snet containing three or more anchors in generic positions in ℝ² and whose underlying graph is globally rigid, any generic framework associated with the graph is also globally rigid, that is, the WM²Snet is unique localizable; and many nongeneric frameworks associated with the graph may also be globally rigid.

For a WM²Snet containing three or more anchors in generic positions in ℝ², and whose underlying graph is a unit disk graph, global rigidity of the graph is only a sufficient condition for unique localization and rigidity of the graph is a necessary condition for unique localization. Only in generic situations where a priori information is not helpful, global rigidity is both a sufficient condition and a necessary condition. An example of a nongeneric situation can arise in a WM²Snet whose underlying graph is a unit disk graph. For such a WM²Snet, even when its underlying graph is a rigid graph but not a globally rigid graph, it may still be uniquely localizable because the ambiguities associated with the nonglobally rigid nature of the underlying graph may sometimes be eliminated using the unit disk graph properties. This is illustrated in Fig. 2.18.

The following theorem summarizes the aforementioned discussions:

**Theorem 3.1** (Eren et al., 2004; Aspnes et al., 2006): Consider a 2-dimensional WM²Snet with at least three anchors generically positioned in ℝ². In generic situations where a priori information is not helpful, the necessary and sufficient condition for unique localization of the WM²Snet is that its underlying graph is globally rigid. Otherwise,
if the underlying graph has the property of being a unit disk graph, then a sufficient condition for unique localization is that the underlying graph is globally rigid and a necessary condition is that the underlying graph is rigid.

In practice, given that a wireless node has a limited transmission range, a WM²Snet can be regarded as a unit disk graph. Therefore, global rigidity is only a sufficient condition for unique localization of a WM²Snet. Determining the necessary conditions for unique localization remains a challenging research issue.

Test for Uniquely Localizable Networks In the previous paragraphs, we have established the necessary and sufficient conditions for a uniquely localizable WM²Snet. A question of practical significance is: How do we test the underlying graph of a given WM²Snet (with the associated internode distances) for the property of rigidity or of global rigidity?

Generally, rigidity can be tested by examining the rank of a matrix whose entries are formed from the coordinates of the vertices (Eren et al., 2004). In $\mathbb{R}^2$, an alternative well-known combinatorial necessary and sufficient condition for rigidity is given in Laman's theorem (Laman, 1970):

**Theorem 3.2 (Laman's Theorem):** A graph $G(V, E)$ in $\mathbb{R}^2$, where $|V| > 1$, is rigid if and only if there exists a subset $E' \subseteq E$ satisfying the following two conditions:

1. $|E'| = 2|V| - 3$;
2. For any nonempty subset $E'' \subseteq E'$, $|E''| \leq 2|V(E'')| - 3$, where $V(E'')$ denotes the set of all end vertices of the edges in $E''$.

Currently, no such combinatorial necessary and sufficient condition is available in $\mathbb{R}^3$.

In $\mathbb{R}^2$, an elegant necessary and sufficient condition for the global rigidity of a graph is given in the following:

**Theorem 3.3 (Hendrickson, 1992; Jackson and Jordan, 2005):** A graph $G(V, E)$ in $\mathbb{R}^2$ with $|V| \geq 4$ vertices is globally rigid if and only if it is **redundantly rigid** and 3-connected (see below).

A graph is **redundantly rigid** if it remains rigid with the removal of any edge in the graph. In $\mathbb{R}^2$, there is a variant of Laman’s theorem called the pebble game (Jacobs and Hendrickson, 1997) for simply testing rigidity and redundant rigidity (Goldenberg, 2006). A graph is k-connected if there exist at least k paths which have no edge or vertices in common (apart from the end vertices) between any two vertices, or equivalently, it is not possible to find $k - 1$ vertices whose removal would render the graph disconnected. Testing of 2-connectivity and 3-connectivity is discussed below.

In addition, for random graphs, there is a probabilistic but asymptotic result linking k-connectedness and the minimum node degree, as given in the following theorem.

**Theorem 3.4.** (Penrose, 2003, 1999): Consider a WM²Snet whose underlying graph is a random geometric graph, represented by $G_n(r)$, with the nodes of the WM²Snet uniformly randomly distributed within a unit square in $\mathbb{R}^2$. Let $r_n$ denotes the minimum $r$ that makes $G_n(r)$ k-connected and let $\sigma_n$ denotes the minimum $r$ for which vertices in $G_n(r)$ have a minimum node degree $k$. Then $\Pr(\sigma_n = r_n) \to 1$ as $n \to \infty$. 
An intuitive explanation of Theorem 3.4 is at a large value of \( n \), with a very high probability the same connectivity radius which makes the graph \( \mathcal{G}_n(r) \) have a minimum node degree of \( k \) also makes the graph \( k \)-connected. We are of course interested in finite \( n \) behavior, and this is taken up in Theorem 4.3 below. When the number of nodes \( n \) is large, the difference between a network whose nodes are uniformly randomly distributed in a defined region \( D \) and one whose nodes are Poisson distributed in \( D \) and the mean number of vertices is \( n \) is small (Gupta and Kumar, 1998), and likewise for the associated graphs. Therefore, Theorem 3.4 may be extended to a graph whose vertices are Poisson distributed. Similarly, Theorem 4.2 in the next section may also be extended to a graph whose vertices are Poisson distributed.

**Generation of Uniquely Localizable Networks**  Given a WM\(^2\)Snet that has been determined as not uniquely localizable, an interesting question is how we transform this WM\(^2\)Snet into one that is uniquely localizable by adding additional information about the network, that is, adding extra internode distances. Before we delve into technical discussions, some notations need to be introduced. Let \( G = (V, E) \) be a graph. Then the graph \( G^2 \) is defined as \( (V, E \cup E^2) \) where for any \( v_i \neq v_j \in V, (v_i, v_j) \in E^2 \) if and only if there exists a \( v_k \in V \) such that \( (v_i, v_k) \in E \) and \( (v_k, v_j) \in E \). Thus \( G^2 \) is obtained from \( G \) by adding edges between the vertex pairs of \( G \) which are separated by exactly one intermediate vertex. Analogously, one can also obtain a graph \( G^3 = (V, E \cup E^2 \cup E^3) \) where for any \( v_i \neq v_j \in V, (v_i, v_j) \in E^3 \) if and only if there exist two vertices \( v_k, v_m \in V \) such that \( (v_i, v_k), (v_k, v_m), (v_m, v_j) \in E \). A main result of this section is the following theorem:

*Theorem 3.5 (Anderson et al., 2006; Cheung and Whiteley, 2005):* Let \( G = (V, E) \) be an edge 2-connected graph in \( \mathbb{R}^2 \). Then \( G^2 = (V, E \cup E^2) \) is globally rigid.

A graph is edge \( k \)-connected if there exists \( k \) paths between any two vertices that have no edge in common between any two vertices. It is obvious that a \( k \)-connected graph is also an edge \( k \)-connected graph but the converse is not necessarily true. There is a simple algorithm, known as the "ear decomposition" (Ramachandran, 1992), that can be used to test edge 2-connectivity, 2-connectivity, and 3-connectivity.

Based on Theorem 3.5, consider a WM\(^2\)Snet whose underlying graph is a unit disk graph with connectivity radius \( r \); if the underlying graph is not globally rigid but edge 2-connected, by doubling the connectivity radius of the WM\(^2\)Snet, the graph becomes globally rigid and the associated mesh network becomes uniquely localizable. Doubling the connectivity radius can be achieved by an increase in the transmitter power. Note that this upward adjustment in connectivity radius needs to be performed only once, or at least occasionally. During the rest of the time, a lower transmitter power suffices to maintain a topology with a simple connectivity-based property, or with other desirable properties. Other alternatives to increasing the connectivity radius include increasing the density of nodes or employing additional measurements, for example, bearing measurements (Anderson et al., 2006).

The edge-2 connectedness condition in Theorem 3.5 can be mildly relaxed (Cheung and Whiteley, 2005). Specifically, if \( G = (V, E) \) is connected and if the removal of any edge in \( E \) which disconnects \( G \) results in one of the two components being a single vertex, then \( G^2 \) is globally rigid.

**Graphical Properties of Connected Networks and Easily Localizable Networks**  In the last section, we discussed the analytic existence/solvability problem in WM\(^2\)Snet localization.
Chapter Two

An equally important problem is the computational complexity of WM²Snet localization algorithms. The computational complexity of WM²Snet localization algorithms has been studied in the literature. Generally, the computational complexity of localization algorithms is NP-hard and probably exponential in the number of vertices (Saxe, 1979). In this section, we show that for a mesh network whose underlying graph has certain properties, for example, bilateration graph and trilateration graph, in addition to global rigidity, the localization algorithm can be greatly simplified.

A graph \( G = (V, E) \) is called a bilateration graph with seeds \( v_1, v_2, \) and \( v_3 \) if its vertices can be ordered as \( v_1, v_2, \ldots, v_{|V|} \) such that \((v_1, v_2) \in E\) and each vertex \( v_i, i = 3, 4, \ldots, |V| \) is adjacent to at least two of the vertices in \( v_1, v_2, \ldots, v_{i-1} \). The ordering of the vertices \( v_1, v_2, \ldots, v_{|V|} \) is called a bilaterative ordering. A network is called a bilateration network if its underlying graph is a bilateration graph. Similarly, a graph \( G = (V, E) \) is called a trilateration graph if there exists an ordering of the vertices \( v_1, v_2, \ldots, v_{|V|} \) such that the vertices \( v_1, v_2, v_3 \) induce a complete graph and each vertex \( v_i, i = 4, 5, \ldots, |V| \) is adjacent to at least three of the vertices in \( v_1, v_2, \ldots, v_{i-1} \). The ordering of the vertices \( v_1, v_2, \ldots, v_{|V|} \) is called a trilaterative ordering. A network is called a trilateration network if its underlying graph is a trilateration graph. Given a graph known to be a bilateration (trilateration) graph, there can be more than one bilaterative (trilaterative) ordering that is consistent with the bilateration (trilateration) property.

A bilateration graph can be obtained from a connected graph by doubling the connectivity radius of a network with a connected underlying graph:

**Theorem 4.1:** (Anderson et al., 2006): Let \( G \) be a connected graph in \( \mathbb{R}^2 \). Then \( G^2 = (V, E \cup E^2) \) is a bilateration graph.

An important result about connected graphs is given in the following. It is relevant for bilateration graphs as well, since node radius doubling in a network with a connected graph results in a bilateration graph.

**Theorem 4.2:** (Gupta and Kumar, 1998): Let \( G_n(r(n)) \) be a random geometric graph derived from a network whose nodes are uniformly randomly placed a unit disk area in \( \mathbb{R}^2 \). If the connectivity radius \( r(n) \) of the network satisfies \( r(n) = \sqrt{\frac{\log n + c(n)}{hn^2}} \), then the graph \( G_n(r(n)) \) is connected with probability one as \( n \to \infty \) if and only if \( c(n) \to +\infty \).

When \( \lim_{n \to \infty} \sup c(n) < +\infty \), the graph is disconnected with a positive probability (Gupta and Kumar, 1998).

Connectivity is a **monotone property** of a graph. A graphical property is called an increasing property, if the property is preserved when edges are added to the graph. A graph property \( A \) is monotone if either \( A \) or \( A^c \) is increasing. In random geometric graphs, a monotone property is known to have a sharp threshold (Goel et al., 2004). The following Theorem in conjunction with Theorem 4.2 sheds more light on the connectivity of random geometric graphs with a finite (but large) number of vertices.

**Theorem 4.3:** (Goel et al., 2004): If \( A \) is a monotone property, then for \( 0 < \varepsilon < \frac{1}{2} \), let \( r(n, \varepsilon) = \inf \{ r > 0 : \Pr[G_n(r) \in A \geq \varepsilon] \} \). Define further \( \delta(n, \varepsilon) = r(n, 1 - \varepsilon) - r(n, \varepsilon) \). Then, for every monotone property in \( \mathbb{R}^2 \) it is \( \delta(n, \varepsilon) = O \left( \frac{\log^{3/4} n}{\sqrt{n}} \right) \).
We examine now the style of algorithms that can be used to localize nodes where the underlying graph is a bilateration graph. Note first though that a bilateration graph is a rigid graph but is not necessarily globally rigid nor is a globally rigid graph necessarily a bilateration graph. This is illustrated in Fig. 2.19.

If the underlying graph of a network is both globally rigid and a bilateration graph, a sequential localization algorithm called "sweep" can be developed (Fang et al., 2006; Fang et al., 2006a) and (Goldenberg et al., 2006a) to efficiently localize all nodes in the network. The principle of the sweep algorithm is illustrated through the special case in Fig. 2.20. Consider a graph $G$ that is both globally rigid and a bilateration graph with a bilaterative ordering $v_1, v_2, \ldots, v_k$ $(k = 8 \text{ in Fig. 2.20})$. With some abuse of notation, let us also regard $v_1, v_2, \ldots, v_k$ as nodes in the WM$^2$Snet (and thus, there is associated with $v_i$ a position).
Starting with three seed nodes $v_1, v_2$ and $v_3$ with known coordinates, without loss of generality the coordinates of the three nodes are assumed to be $v_1 = (0, 0), v_2 = (a, 0)$ and $v_3 = (b, c)$ with $a, c > 0$. Then knowledge of the internode distances $v_1v_4$ and $v_3v_4$ gives the position of $v_4$ with a binary ambiguity, that is, $v_4$ has two possible positions. Consider $v_5$ with known internode distances $v_1v_5$ and $v_3v_5$ to $v_1$ and $v_3$ respectively, for each possible position of $v_4$, $v_5$ has two possible positions that makes the total number of possible positions for $v_5$ four. Successively, we obtain the positions of $v_6, v_7, v_8$ with $2^3, 2^4$ and $2^5$ ambiguities. However, $v_9$ is also connected to $v_2$ (in the example shown in Fig. 2.20, the edge $v_2v_8$ is necessary to make a globally rigid graph), knowledge of distance to $v_2$ resolves the ambiguities in the position of $v_9$ and also reduces the ambiguities in the positions of other vertices. Finally, knowledge of internode distance $v_1v_7$ and position of $v_8$ sequentially removes all ambiguities in $v_7, v_6, \ldots, v_4$ and the unique localization of the network is achieved.

There are $\frac{1}{6}|V|(|V| - 1)(|V| - 2)$ different choices of three nodes from $V$. Therefore, the computational complexity involved in searching for the seed nodes is $O(|V|^3)$, that is, at most polynomial in the number of nodes. If one knows the seed but does not know the ordering, the computational complexity in choosing the ordering is $O(|V| + |E|)$. Another potential problem with the sweep algorithm is that the number of possible positions grows exponentially with the number of nodes. This exponential growth in the number of possible positions can be mitigated by considering all known distances of the newly swept nodes to the already swept nodes, instead of two distances only. When a node with more than two known distances to the already swept nodes is added to the set of swept nodes, the additional distance information will not only eliminate some of its own possible positions, but it also removes some of the possible positions of already swept nodes (Goldenberg et al., 2006a). The number of possible positions also reduces dramatically when the sweep process meets an anchor. Simulation using 250 nodes randomly uniformly distributed in a square region showed that when the average node degree equals six, the number of possible positions reaches its maximum value of 250 (Goldenberg et al., 2006a). Generally, the running time of the sweep algorithm grows linearly in the number of nodes (Goldenberg et al., 2006a). Details on the design of the algorithm can be found in the work by Goldenberg et al. (2006a) where extensions of the basic sweep algorithm to handle noisy distance measurements as well as distance and angle measurements are reported.

In the previous paragraphs, we have shown that if the underlying graph of a network is a globally rigid bilateration graph, an efficient localization algorithm can be designed whose computational complexity is at most polynomial in the number of nodes provided that the ambiguities are bounded. An even more efficient algorithm can be designed if the underlying graph has the property of a trilateration graph. A trilateration graph can be obtained from a connected graph by tripling the connectivity radius.

**Theorem 4.4 (Anderson et al., 2006):** Let $G = (V, E)$ be a connected graph with $n$ vertices, and let $v_1, v_2, \ldots, v_n$ be an ordering of the vertices of $G$ such that for all $m > 1$, the subgraph of $G$ induced by the vertex set $V_m = \{v_1, v_2, \ldots, v_m\}$, denoted by $G_m$, is connected. Then $G^3 = (V, E \cup E^2 \cup E^3)$ is a trilateration graph with a trilaterative ordering $v_1, v_2, \ldots, v_n$.

In $\mathbb{R}^2$, a trilateration graph is globally rigid but a globally rigid graph is not necessarily a trilateration graph.
In $\mathbb{R}^2$, if the underlying graph of a mesh network is a trilateration graph and at least three of the meshes are anchors, then the whole network can be easily and uniquely localized. The computational complexity involved in searching for the seed nodes and a trilaterative ordering is the same as those involved in searching for the seed nodes and a bilaterative ordering in a bilateration network. These are $O(|V|^3)$ and $O(|V| + |E|)$ respectively. Now considering one has found the seed nodes and the corresponding trilaterative ordering and assuming temporarily the three seed nodes $v_1$, $v_2$ and $v_3$ have the known coordinates, $v_1 = (0, 0)$, $v_2 = (a, 0)$ and $v_3 = (b, c)$ with $a, c > 0$, the values of $a$, $b$ and $c$ can be derived from the known internode distances between the seed nodes. Then, it is obvious that other nodes can be localized relative to the three seed nodes sequentially, in a single sweep and in time $O(|V|)$. The estimated positions differ from the true positions by a translation, rotation and possible reflection. At this stage, knowledge of the anchor positions has not been used. By aligning the estimated positions of anchors with their true positions, the required translation, rotation and reflection to transform the estimated coordinates into coordinates in which anchors positions are consistent with their true positions can be obtained. The new estimated positions follow for the rest of the nodes through application of the same translation, rotation and reflection.

Though the computational complexity involved in searching for the seed nodes and the vertex ordering is the same as that involved in a bilateration network, the computational complexity involved in estimating a node’s position is much simpler than that in a bilateration network and there is no growth in the number of possible positions. Therefore, localization algorithms in a trilateration network can be more efficient than those in a bilateration network. However, a trilateration network requires knowledge of a higher number of edges (or equivalently a larger connectivity radius) than a bilateration network.

**Simulation Results**  Simulations were conducted to evaluate the critical connectivity radii required for a connected, a 2-connected network, a 3-connected network, a rigid network, a redundantly rigid network, a globally rigid network, a globally rigid bilateration network and a trilateration network respectively. Compared with the simulation results by Anderson et al. (2006), more extensive results, including the critical connectivity radii for a globally rigid bilateration network and a trilateration network, were obtained. We generate 50 instances of test network each with 200 nodes uniformly distributed in a unit square area.

Simulation results are shown in Fig. 2.21. Comparing $r_1$, $r_2$, $r_g$, $r_{tri}$ it can be readily concluded that $r_g \leq 2r_2$ and $r_g \leq r_{tri} \leq 3r_1$. This is an expected result from Theorem 3.5 and Theorem 4.4. Moreover, it is observed that $r_g = r_g^{bi}$ in all simulations. As shown in Fig. 2.19, a bilateration graph is not necessarily globally rigid nor is a globally rigid graph necessarily a bilateration graph. However, simulation results seem to suggest that the scenario shown in Fig. 2.19 rarely occurs and with a very high probability, a globally rigid graph is also a bilateration graph. It is also noted that the difference between $r_g$ and $r_{tri}$ is usually small which indicates that a small increase in the connectivity radius will transform a globally rigid graph into a trilateration graph.

### 2.5.6 Multicasting in Mobile Wireless Networks

**Background** A number of WM$^2$Net scenarios require some degree of one-to-many or many-to-many interactions, such that a node forwards its data to multiple destinations.