

**Control and Modeling
of Complex Systems**
Cybernetics in the 21st Century

Festschrift in Honor of Hidenori Kimura
on the Occasion of his 60th Birthday

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Analysis of Weight Change in H_∞ Control Design

Brian D. O. Anderson and Xavier Bombois

ABSTRACT H_∞ control design is generally performed iteratively. At each iteration, the weights constraining the desired closed loop transfer functions are adapted. The way in which the weights are adapted is generally purely heuristic. It is consequently very interesting to build some insights about the influence of a weight modification on the obtained (central) controller and, more importantly, on the obtained closed-loop transfer functions. In this paper, we analyze this influence in the case of a classical two-block problem under the assumption of "small" modifications in the weights. The concept *small modification* must be understood in the sense of small enough to allow first order approximation.

Keywords: H_∞ control design, J -spectral factorization, Weight selection, Robust control design

1 Dedication

Hidenori Kimura has been associated with H_∞ control since the 1980s, and he is one of a comparatively small number of workers in the area who can be said to have command of all the different approaches to considering H_∞ control, as is amply evidenced by his fine book [7]. His early work focused on use of interpolation theory to solve control problems, see e.g., [5, 6]. In [7], he married many ideas of a classical kind and a modern system theory kind. The classical ideas included the use of scattering representations for networks or more generally interconnected systems, and frequency domain factorization; here, the notion of Wiener factorization which underpins linear-quadratic control and filtering is replaced by a type of generalization, J -spectral factorization. An allied concept especially associated with Kimura Sensei is J -lossless factorization, which relates to J -spectral factorization in a similar manner to the way Darlington synthesis of networks relates to Wiener or spectral factorization.

Hidenori Kimura is a consistent supporter of the idea that each new view of a problem adds to one's understanding of it. A corollary is that if there are certain aspects of a problem one wishes to understand, there may be preferred views of the problem when one is seeking that understanding. That precept motivated our use in this paper of a tool highly familiar to Kimura Sensei, and yet described rather less frequently in the most modern

H_∞ textbooks, that of J -spectral factorization. For we are seeking to isolate some frequency domain properties of H_∞ design, with little concern for tracking state-variable dimensions, let alone state-variable matrices.

It is with admiration for his work and in gratitude for the inspiration it has provided us that we dedicate this paper to Kimura Sensei.

2 Introduction

In recent years, H_∞ control design has become a well-known method to design a model-based controller satisfying a number of constraints expressed by amplitude bounds (weights) on the "to-be-designed" closed-loop transfer functions. This method whose theoretical basis can be found in the works [2, 8] has known numerous applications for control design on real-life systems (see e.g., [1]).

The design of a controller using H_∞ control design generally follows an iterative procedure. In a first step, only the sensitivity function is effectively constrained (i.e., the constraints on the other transfer functions are chosen in such a way that they remain ineffective). A first controller is obtained in this way. However, this controller has generally an unsatisfactory performance with respect to the closed-loop transfer functions for which the constraints were (in this first step) too loose to be effective. Consequently, in a second step, the weights on these closed-loop transfer functions are adapted in order to improve the closed-loop behaviour of the controller and a second controller is computed using these adapted weights. This procedure is pursued until the obtained controller is judged satisfactory enough.

The way with which the weights are adapted at each "iteration" is generally purely heuristic. It is consequently very interesting to build some insights about the influence of a weight modification on the obtained (central) controller and, more importantly, on the obtained closed-loop transfer functions. In this work, we analyze this influence in the case of a classical two-block problem (for a scalar model) under the assumption of "small" modifications in the weights. The concept *small modification* must be understood in the sense of small enough to allow first order approximation.

In order to solve an H_∞ control design problem, two different methods are available. The most attractive one with respect to computational efficiency is the one developed in [2], and relies on a state-space formulation. The second one found for example in [3, 4, 7] is based on the J -spectral factorization of the augmented plant. This second method is less attractive computationally speaking. However, it can of course be used for analysis purposes. In order to develop our analysis, we will use the second method since this is based on frequency domain expressions which are important for our purpose. In particular, we contemplate variation of weight functions

that may change their McMillan degree, and yet in frequency domain terms are small. Such variations almost certainly could not easily be cast as small for state-space descriptions¹, involving as they do degree change.

In this work, the first contribution is to give a frequency domain approximation of the change in the central controller due to a (small) weight modification after a first H_∞ control design step. This approximation is a function of the weight modification frequency response and of the variables involved in the initial control design problem, and is therefore computable before performing the new control design step (i.e., the one with the modified weights). We show also that the modification in the central controller persists outside the frequency band where the weight modification is mainly located. A last contribution is to analyze the influence of this modification of the central controller on the modified closed-loop transfer functions. Note that we restrict attention to scalar systems and to the two-block H_∞ problem. However it is our opinion that the results can be extended to multivariable systems and to the four-block H_∞ problem. In the latter, two J -spectral factorizations are performed instead of one in the two-block case [3].

Overview. In Section 3, we recall the method to solve two-block H_∞ control problems using J -spectral factorization. In Section 4, we introduce the particular two-block problem that we will consider and we state the objectives. In Section 5, we analyze the modification on the matrix defining the central controller, induced by a weight modification. Using the result of Section 6, we give an approximation of the modification of the central controller in Section 5 and an approximation of the modified closed-loop transfer functions in Section 7. We finish by an illustration (Section 8) and some conclusions (Section 9). \square

3 J -spectral factorization and H_∞ control design

In this section, we first recall how we can solve a classical two-block H_∞ control design problem (for a scalar model G_{mod}) using J -spectral factorization (see also [3, 4, 7] for more details). For this purpose let us first introduce the notion of homographic transformation.

Definition 3.1 ([7]). Consider a scalar transfer function $Q(s)$ and a transfer matrix $H(s)$ partitioned as follows and H_{22} are scalars. Define the homographic transformation $HOM(H, Q)$ as

$$HOM(H, Q) \triangleq \frac{H_{11}Q + H_{12}}{H_{21}Q + H_{22}}. \quad (3.1)$$

\square

¹Used in the method of [2].

Having defined the notion of homographic transformation, we may now present the following result about the solution of a two-block H_∞ control design problem.

Proposition 3.2 ([7]). *Consider the two-block H_∞ control problem consisting of finding a stabilizing controller C for the model G_{mod} such that*

$$\left\| \begin{array}{c} W_1(s) T_1(G_{mod}, C) \\ W_2(s) T_2(G_{mod}, C) \end{array} \right\|_\infty < 1 \quad (3.2)$$

where W_1, W_2 are two stable and inversely stable weights and T_1, T_2 are two closed-loop transfer functions of the to-be-designed loop $[C G_{mod}]$. Suppose that this control design problem satisfies the general conditions of solvability. Then the central controller C_c corresponding to this control design problem is given by

$$C_c = HOM(\Theta, 0) \quad (3.3)$$

where Θ is a 2×2 stable and inversely stable transfer matrix whose inverse $\Pi(s) \triangleq \Theta^{-1}$ is obtained via the J -spectral factorization of the augmented plant $H(s)$ i.e.,

$$H^* \overbrace{\begin{pmatrix} I_2 & 0 \\ 0 & -1 \end{pmatrix}}^{\triangleq J_3} H = \Pi^* \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}^{\triangleq J_2} \Pi \quad (3.4)$$

where $X^*(s)$ denotes the adjoint of $X(s)$ i.e., $X^T(-s)$. The augmented plant $H(s)$ is here defined as the transfer matrix such that

$$HOM(H, C) = \begin{pmatrix} W_1(s) T_1(G_{mod}, C) \\ W_2(s) T_2(G_{mod}, C) \end{pmatrix}. \quad (3.5)$$

Important remark. The matrix $\Pi(s)$ delivered by the J -spectral factorization (3.4) of $H(s)$ is always defined to within left multiplication by a constant J -unitary matrix. In other words, there are different possible matrices $\Pi(s)$ solving (3.4). However, for some problems, there is a natural choice for $\Pi(j\infty)$, which thus pins down all of $\Pi(s)$. \square

4 Considered problem

As stated in the introduction, the objective in this paper is to analyze the influence of a (small) modification in the weights on the obtained central controller C_c and, more importantly, on the obtained closed-loop transfer functions $T_1(G_{mod}, C_c)$ and $T_2(G_{mod}, C_c)$. In the sequel, we will restrict

attention to one particular (and classical) two-block problem. This problem is the one for which expression (3.2) is given by

$$\left\| \begin{array}{c} W_1(s) \frac{C}{1+CG_{mod}} \\ W_2(s) \frac{1}{1+CG_{mod}} \end{array} \right\|_\infty < 1. \quad (4.1)$$

In other words, we consider the case where $T_1 = C/(1 + CG_{mod})$ and $T_2 = 1/(1 + CG_{mod})$.

For this particular two-block problem, we will analyze the following (classical) situation. We assume that, in a first step, a central controller C_c has been designed using the criterion (4.1) with an appropriate constraint W_2 on the sensitivity function, but a very loose constraint W_1 on the other closed-loop transfer function. For instance, in this first step, the weight W_1 could have been chosen equal to a very small constant. However, this is of course not a requirement.

In a second step, the weight W_1 is adapted and becomes $W_{1,bis} = W_1 + \Delta W_1$, a second H_∞ design procedure is achieved with the adapted weights $W_{1,bis} = W_1 + \Delta W_1$ and W_2 , and a new central controller $C_{c,bis} = C_c + \Delta C$ is obtained delivering modified closed-loop transfer functions $T_{1,bis}$ and $T_{2,bis}$:

$$T_{1,bis} = \frac{C_c + \Delta C}{1 + (C_c + \Delta C)G_{mod}} (= \hat{T}_1 + \Delta T_1), \quad (4.2)$$

$$T_{2,bis} = \frac{1}{1 + (C_c + \Delta C)G_{mod}} (= T_2 + \Delta T_2). \quad (4.3)$$

The final objective of our research is to fully understand the link existing between a change in the weights and the obtained closed-loop transfer functions. In this work, we will nevertheless restrict attention to the following tasks:

- find a frequency domain approximation ΔC_{appr} for ΔC , as a function of ΔW_1 and of the variables involved in the initial two-block problem (i.e., the one with W_1 and W_2).
- And then, knowing ΔC_{appr} , to analyze the influence of this modification on the modified closed-loop transfer functions $T_{1,bis}$ and $T_{2,bis}$.

For this purpose, we will need to assume that the change ΔW_1 is small (so we may use first order approximations) and since we are interested in frequency domain expressions, we will use the frequency domain expression (3.4) as basis of our analysis. It is to be noted that all approximations given in this paper will therefore only be relevant in the frequency domain.

In effect, we will construct a mapping $\Delta W_1 \rightarrow \Delta C_{appr} \rightarrow \Delta T_1$ and ΔT_2 . The smallness assumption means that the mapping is linear, but

Since Π and $\Pi_{bis} = \Pi + \Delta\Pi$ are both stable and inversely stable, the right-hand side of (5.10) is the sum of a stable transfer matrix $J_2\Delta\Pi\Theta$ and its complex conjugate $\Theta^*\Delta\Pi^*J_2$. Let us now decompose the left-hand side of (5.10) as in (5.4).³ From (5.4) and (5.10), it is then obvious that

$$J_2\Delta\Pi\Theta \approx \Psi. \quad (5.11)$$

From (5.11), we can write successively the following:

$$\Delta\Pi \approx J_2\Psi\Pi, \quad (5.12)$$

$$\Pi_{bis} = \Pi + \Delta\Pi \approx (I_2 + J_2\Psi)\Pi, \quad (5.13)$$

$$\Theta_{bis} = \Pi_{bis}^{-1} \approx \Pi^{-1}(I_2 - J_2\Psi) = \Theta - \Theta J_2\Psi. \quad (5.14)$$

Note that, in order to invert Π_{bis} , we have made use of the fact that the changes are small. \square

6 Modification in the central controller

Proposition 5.1 gives us a (frequency domain) approximation of the modified transfer matrix Θ_{bis} . This approximation is a function of ΔW_1 and the variables involved in the initial two-block problem (i.e., the one with W_1 and W_2).

6.1 Approximation ΔC_{appr} of ΔC

This result will now allow us to deduce an approximate expression for the modified central controller $C_{c,bis} = C_c + \Delta C$ using the relation between Θ_{bis} and the central controller. This expression is given in the following proposition.

Proposition 6.1. *Consider the matrix Θ involved in the two-block H_∞ problem (4.1) with weights W_1 and W_2 . Consider also the stable matrix Ψ defined in (5.4) and (5.5). Then the modified controller $C_{c,bis}$ delivered by the two-block H_∞ problem (4.1) with weights $W_{1,bis} = W_1 + \Delta W_1$ and W_2 can be approximated as*

$$C_{c,bis} = C_c + \Delta C \approx C_c + \overbrace{\Delta C_{appr}}^{\frac{\det(\Theta)}{\theta_{22}^2}} \psi_{12}, \quad (6.1)$$

³The matrix Ψ resulting from (5.4) is defined up to an additive constant skew matrix. This constant matrix is in our problem chosen to be zero and $\Theta_{bis}(j\infty)$ is again diagonal, with $C_{c,bis}$ again strictly proper.

where $\det(A)$ denotes the determinant of the matrix A and ψ_{ij} are the entries of Ψ .

Proof. From Expression (3.3), we can write

$$C_{c,bis} = HOM(\Theta_{bis}, 0) = \frac{\theta_{12,bis}}{\theta_{22,bis}}, \quad (6.2)$$

where $\theta_{ij,bis}$ are the entries of Θ_{bis} . Using (5.3), it is easy to find (approximate) expressions for $\theta_{12,bis}$ and $\theta_{22,bis}$ i.e.,

$$\theta_{12,bis} \approx \theta_{12} - \overbrace{(\theta_{11} \ \theta_{12})}^{=\alpha^T} \overbrace{\begin{pmatrix} \psi_{12} \\ -\psi_{22} \end{pmatrix}}^{=\beta}, \quad (6.3)$$

$$\theta_{22,bis} \approx \theta_{22} - \overbrace{(\theta_{21} \ \theta_{22})}^{=\gamma^T} \begin{pmatrix} \psi_{12} \\ -\psi_{22} \end{pmatrix}. \quad (6.4)$$

Using (6.3) and (6.4), the expression of the modified controller (i.e., (6.2)) can be approximated as

$$C_{c,bis} \approx \frac{\theta_{12} - \alpha^T \beta}{\theta_{22} - \gamma^T \beta}, \quad (6.5)$$

and using the first order Taylor expansion around $\beta = 0$ of the ratio between $\theta_{12} - \alpha^T \beta$ and $\theta_{22} - \gamma^T \beta$, we finally obtain

$$C_{c,bis} \approx \frac{\theta_{12} - \alpha^T \beta}{\theta_{22} - \gamma^T \beta} \approx \frac{\theta_{12}}{\theta_{22}} + \frac{\theta_{12} \gamma^T - \theta_{22} \alpha^T}{\theta_{22}^2} \beta = C_c - \frac{\det(\Theta)}{\theta_{22}^2} \psi_{12}. \quad (6.6)$$

□

Remark. From the approximation of the controller $C_{c,bis}$ given in (6.1), we can deduce that the transfer function ψ_{12} must be strictly proper i.e.,

$$\psi_{12}(j\infty) = 0 \quad (6.7)$$

in order that $\Delta C_{appr}(j\infty) = 0$ which is in accordance with the fact that $C_c(j\infty) = C_{c,bis}(j\infty) = 0$. This property is also in accordance with the definition and properties of the matrix Ψ given in (5.4)–(5.5). Since (5.4) only defines Ψ to within a skew matrix, $\psi_{12}(j\infty)$ can be adjusted. □

6.2 Link between ψ_{12} and ΔW_1

Proposition 6.1 delivers an approximation ΔC_{appr} of the modification in the central controller due to the deviation ΔW_1 . This approximation is

a function of the matrix Θ of the initial two-block problem and of the second entry ψ_{12} of the matrix Ψ defined implicitly in (5.4). In order to be able to compute ΔC_{appr} , we show in this subsection how to compute ψ_{12} (explicitly) and moreover we analyze the relation between this transfer function ψ_{12} and the deviation ΔW_1 .

Let us deduce the following from (5.4):

$$\psi_{12} + \psi_{21}^* = \overbrace{(W_1^* \Delta W_1 + W_1 \Delta W_1^*) \theta_{11}^* \theta_{12}}^{f(s)} \quad (6.8)$$

where $f(s)$ is a known transfer function, since it is a function of some entries of the matrix Θ , the weight W_1 and the change of weight ΔW_1 . Notice also that f is equal to 0 when $\Delta W_1 = 0$. An expression for ψ_{12} can now be computed by partial fraction decomposition of $f(s)$ into its unstable part ψ_{21}^* and its stable part ψ_{12} .

Important comments. The deviation ΔW_1 will generally be a stable filter having a form similar to

$$\Delta W_1 = \frac{\pm K s}{(s + \omega_1)(s + \omega_2)}.$$

The modification of the weight due to ΔW_1 is therefore mainly restricted to the frequency band $B_{\Delta W_1} = [\omega_1 \ \omega_2]$. Indeed, outside this band $B_{\Delta W_1}$, the amplitude of ΔW_1 converges to zero (i.e., $|\Delta W_1(j\omega)| \rightarrow 0$ when $\omega \rightarrow 0$ and when $\omega \rightarrow \infty$). The representation of such a filter ΔW_1 can be found in Figure 8.4 (in Section 8). \square

Due to the decomposition into partial fractions, the transfer function ψ_{12} will generally have a different behaviour than ΔW_1 . The amplitude of ψ_{12} will indeed only converge to 0 in high frequencies (see (6.7)). In low frequencies, this amplitude will generally converge to a non-zero constant and, in the band $B_{\Delta W_1} = [\omega_1 \ \omega_2]$, $\psi_{12}(j\omega)$ will have a non-constant dynamic behaviour. An example of such a transfer function ψ_{12} can be found in Figure 8.4. Consequently, the region where $\psi_{12}(j\omega)$ will have a significant influence on ΔC_{appr} (i.e., $B_{\psi_{12}} = [0, \omega_2]$) is much more important than the region $B_{\Delta W_1}$ for ΔW_1 . This is in fact a well-known property of the decomposition into partial fractions.

6.3 Comments about ΔC_{appr}

The approximation ΔC_{appr} (see (6.1)) can be computed *a-priori*, that is without having first to solve the second two-block H_∞ problem. Indeed, ΔC_{appr} is only a function of the transfer matrix Θ (involved in the initial two-block H_∞ problem) and of the transfer function ψ_{12} which has been proved computable in the previous subsection.

Due to (6.1) and the properties of ψ_{12} with respect to ΔW_1 presented in the previous subsection, we can also state that the modification of the central controller due to ΔW_1 will not only be restricted to the band $B_{\Delta W_1}$, but will also persist in low frequencies. In high frequencies, the amplitude ΔC_{appr} converges to 0 because of (6.7).

7 Consequences for the closed-loop transfer functions

The results presented in the previous section allow one to compute very easily an approximation of the modified central controller $C_{c,bis} = C_c + \Delta C$ due to the modification ΔW_1 , and this without having to perform the H_∞ control design problem with the weights $W_1 + \Delta W_1$ and W_2 which would have given $C_{c,bis}$ as solution. According to Proposition 6.1, this approximation is given by

$$C_{c,bis} \approx C_c + \frac{\Delta C_{appr}}{\theta_{22}^2} \psi_{12}. \quad (7.1)$$

From this approximation, it is then easy to compute the effects of the change in the weights on the obtained closed-loop transfer functions $T_{1,bis}$ and $T_{2,bis}$ (see (4.2) and (4.3)):

$$T_{1,bis} \approx \frac{C_c + \Delta C_{appr}}{1 + (C_c + \Delta C_{appr})G_{mod}}, \quad (7.2)$$

$$T_{2,bis} \approx \frac{1}{1 + (C_c + \Delta C_{appr})G_{mod}}. \quad (7.3)$$

Let us once again notice that the approximations given in (7.2) and (7.3) are both computable without having to perform the H_∞ control design problem with the weights $W_1 + \Delta W_1$ and W_2 which would deliver $T_{1,bis}$ and $T_{2,bis}$ as closed-loop transfer functions. Consequently, the influence of the change in the weights can be evaluated *a-priori*.

Although the influence of a change of the weights will vary depending on the considered model, the (modified) weights⁴ and so on, we can nevertheless deduce some general comments which will be based on the first order Taylor expansion around $\Delta C_{appr} = 0$ of the approximations (7.2) and (7.3):

⁴Indeed, ΔC_{appr} is a function of all these factors through Θ and ψ_{12} .

$$T_{1,bis} \approx \frac{\overbrace{C_c}^{T_1}}{1 + C_c G_{mod}} \left(1 + \frac{1}{1 + C_c G_{mod}} \frac{\Delta C_{appr}}{C_c} \right) = \frac{C_c + \Delta C_{appr}}{1 + C_c G_{mod}}, \quad (7.4)$$

$$T_{2,bis} \approx \frac{\overbrace{1}^{T_2}}{1 + C_c G_{mod}} \left(1 - \frac{G_{mod}}{1 + C_c G_{mod}} \Delta C_{appr} \right); \quad (7.5)$$

and on the following classical behaviours of these closed-loop transfer functions in low frequencies (LF) and in high frequencies (HF):

$$LF: T_1(j\omega) \approx T_{1,bis}(j\omega) \approx \frac{1}{G_{mod}(j\omega)}, \quad (7.6)$$

$$LF: T_2(j\omega) \approx \frac{1}{G_{mod}(j\omega)C_c(j\omega)}, \quad T_{2,bis}(j\omega) \approx \frac{1}{G_{mod}(j\omega)C_{c,bis}(j\omega)}, \quad (7.7)$$

$$HF: T_1(j\omega) \approx C_c(j\omega), \quad T_{1,bis}(j\omega) \approx C_{c,bis}(j\omega), \quad (7.8)$$

$$HF: T_2(j\omega) \approx T_{2,bis}(j\omega) \approx 1. \quad (7.9)$$

Comments on $T_{1,bis}$

- At low frequencies, according to (7.6), the difference between T_1 and $T_{1,bis}$ will be negligible and this even though $\Delta C_{appr}(j\omega) \neq 0$ at those frequencies.
- At high frequencies, according to (7.8), $|T_{1,bis}(j\omega)| < |T_1(j\omega)|$ if $|C_{c,bis}(j\omega)| < |C_c(j\omega)|$, and vice versa. Moreover, since both controllers are strictly proper, $T_1(j\omega)$ and $T_{1,bis}(j\omega)$ generally converge to zero when $\omega \rightarrow \infty$.
- According to (7.4), we will have the inequality $|T_{1,bis}(j\omega)| < |T_1(j\omega)|$ if $|C_{c,bis}(j\omega)| < |C_c(j\omega)|$ (and vice versa) around the peak of T_1 . \square

Comments on $T_{2,bis}$

- Since $\Delta C_{appr}(j\omega) \neq 0$ at low frequencies, we will have a difference between T_2 and $T_{2,bis}$ at those frequencies. According to (7.7) $|T_{2,bis}(j\omega)| < |T_2(j\omega)|$ if $|C_{c,bis}(j\omega)| > |C_c(j\omega)|$, and vice versa.
- At high frequencies, according to (7.9), $T_{2,bis}(j\omega) \approx T_2(j\omega) \approx 1$.

- A general comment about the difference between $T_{2,bis}$ and T_2 around the resonance peak is not obvious. \square

Further research to be achieved. From the previous comments, we see that a next step in our research should be to understand the link that exists between ΔW_1 and the fact that $|C_{c,bis}(j\omega)| < |C_c(j\omega)|$ (or conversely) at certain frequencies. For this purpose, a further investigation of the expression (6.1) will be necessary. \square

8 Illustration

In this section, we will illustrate the results presented in this paper. We will consider the following system G_{mod} :

$$G_{mod} = \frac{10}{(s-1)(0.2s+1)} \quad (8.1)$$

In the first H_∞ control design problem (see (4.1)), we will as usual only constrain the sensitivity function and choose the constraint W_1 on the other transfer function as a small constant. The chosen weights are: $W_1(s) = 1/6$ and $W_2(s) = (0.1s+1)/(0.003(100s+1))$. The H_∞ problem is solved with these elements and we obtain the following central controller:

$$C_c = \frac{50.2564(s+0.69)(s+5)}{(s+0.01)(s^2+30.85s+423.3)} \quad (8.2)$$

Figures 8.2 and 8.3 represent the amplitude of the closed-loop transfer functions T_1 and T_2 achieved by this controller C_c with the system G_{mod} . Now in a second step, we want to decrease the resonance peak of $T_1(G_{mod}, C_c)$. So, we choose the following new weight $W_{1,bis}$:

$$W_{1,bis} = W_1 + \Delta W_1 = W_1 + \frac{1.8967s}{(s+17.78)(s+5.623)} \quad (8.3)$$

that can be considered as a small deviation of W_1 . The H_∞ problem is solved with this new weight and we obtain the following central controller:

$$C_{c,bis} = \frac{58.9691(s+17.78)(s+5.623)(s+5)(s+0.6782)}{(s+27.12)(s+5.628)(s+0.01)(s^2+32.49s+339.1)} \quad (8.4)$$

The controller $C_{c,bis}$ is represented in Figure 8.1 and the new closed-loop transfer functions $T_{1,bis}$ and $T_{2,bis}$ in Figure 8.2 and 8.3, respectively. In these three last figures, the modified transfer functions are compared to the corresponding transfer function in the initial two-block problem.

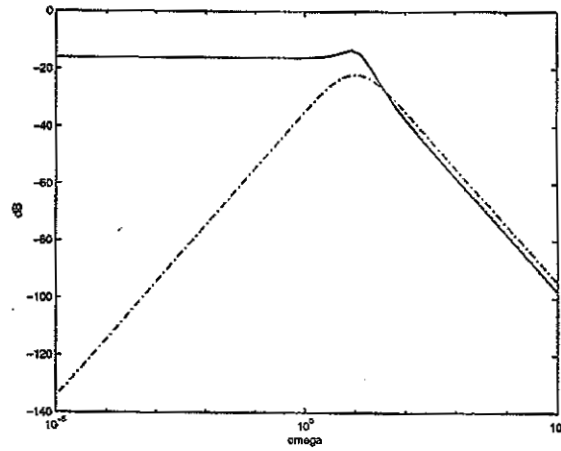


FIGURE 8.4. $|\psi_{12}(j\omega)|$ (solid) and $|\Delta W_1(j\omega)|$ (dashdot) at each frequency

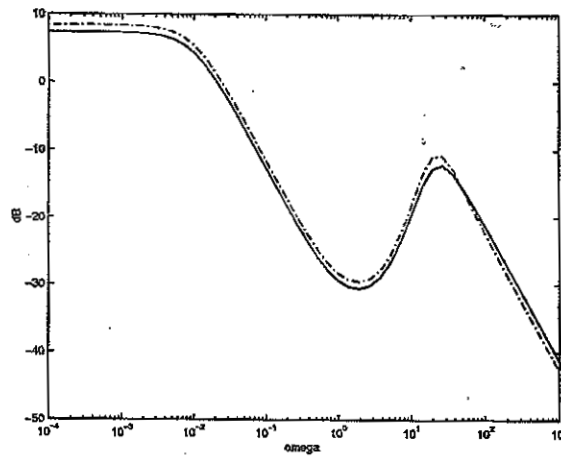


FIGURE 8.5. $|C_{c,bis}(j\omega) - C_c(j\omega)|$ (solid) and $|\Delta C_{appr}(j\omega)|$ (dashdot) at each frequency

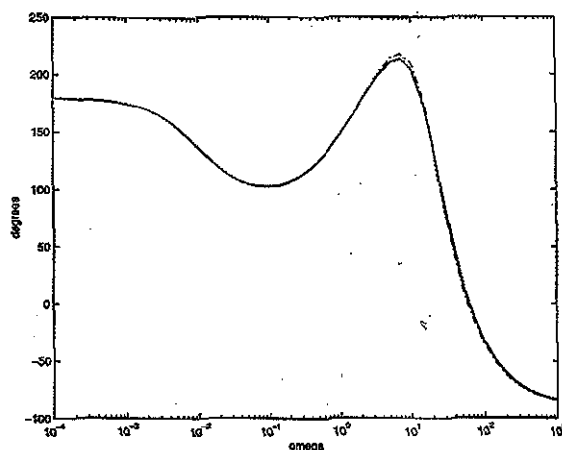


FIGURE 8.6. $\arg(C_{c,bis}(j\omega) - C_c(j\omega))$ (solid) and $\arg(\Delta C_{appr}(j\omega))$ (dashdot) at each frequency

9 Conclusions

In this work, we have analyzed the influence of a small weight modification in a classical two-block H_∞ control design problem on the obtained central controller and on the obtained closed-loop transfer functions. The first contribution has been to give a frequency domain approximation of the change in the central controller induced by the (small) weight modification after a first H_∞ control design step. This approximation is a function of the weight modification frequency response and of the variables involved in the initial control design problem, and is therefore computable before performing the new control design step (i.e., the one with the modified weights). We show also that the modification in the central controller persists outside the frequency band where the weight modification is mainly located. A last contribution is to analyze the influence of this modification of the central controller on the modified closed-loop transfer functions.

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