Due date: November 23, 2011
Marking scheme: Full marks for a formulation that correctly answers the question and clearly shows the steps to obtain the solution.
Solutions to be submitted electronically by Email to Peter.Baumgartner@nicta.com.au. Scans of handwritten solutions are of course acceptable.

Question 1 (2 + 2 pts). True or false? Prove or find a counterexample:

1. For all propositional logic formulas $F$ and $G$ and suitable assignments $A$, if $(F \rightarrow G)$ is valid and $A \not|= G$ then $\neg F$ is satisfiable.

2. For all propositional logic formulas $F$ and $G$, if $(F \rightarrow G)$ is satisfiable and $F$ is satisfiable then $G$ is satisfiable.

Question 2 (6 pts). Is the following propositional clause set $M$ satisfiable? Justify your answer by a proof. (Hint: inductive proof, compactness.)

$$M = \{ A_1 \lor A_2, \neg A_2 \lor \neg A_3, A_3 \lor A_4, \neg A_4 \lor \neg A_5, \ldots \} .$$

Question 3 (10 pts). In class, the proof of Theorem 30 (completeness of propositional Resolution) was sketched by means of an example. Carry out the proof in its generality.

Question 4 (4 pts). In a criminal case the following facts have been shown to hold true:

1. At least one of the persons $X,Y,Z$ is guilty.
2. If $X$ is guilty and $Y$ is not guilty, then $Z$ is guilty.
3. If $Y$ is guilty then $Z$ is guilty.

Use propositional resolution to prove that one of $X, Y, Z$ is guilty (who?). What can be said about the others?

Question 5 (3 + 3 pts). Convert these formulas to clause normal form:

1. $(((A \rightarrow B) \lor C) \rightarrow ((A \leftrightarrow B) \land C))$
2. $\forall x \ ((\forall x \ P(x)) \rightarrow \exists y \ Q(x, y))$

Question 6 (3 + 3 pts). Apply the unification algorithm presented in class to these sets of equations and read off the result, i.e., either FAIL or the unifier ($a$ is a constant, $x$ and $y$ are variables):

1. $U = \{ x = y, f(f(x)) = f(y) \}$
2. \( U = \{ a = x, f(x, z) = y, f(z, x) = y \} \)

**Question 7** (4 pts). Give a resolution refutation of the clause set

\[ M = \{ P(x) \lor P(y), Q(x, f(x)) \lor \neg P(x), \neg Q(g(y), z) \} \]

**Question 8** (5 + 5 pts). Given the following facts.

(i) Every barber shaves all persons who do not shave themselves.

(ii) No barber shaves a person who shaves himself.

(iii) There is no barber.

1. Formalize (i), (ii) and (iii) in first-order logic. Use \( B(x) \) for “\( x \) is a barber” and \( S(x, y) \) for “\( x \) shaves \( y \)”.

2. Use the automated theorem prover “Otter” to prove that (iii) follows from (i) and (ii).

Otter is available from [http://www.cs.unm.edu/~mccune/otter/](http://www.cs.unm.edu/~mccune/otter/). Otter is usually very easy to install and comes with a good manual. Looking at the example problem [examples/auto/steam.in](http://www.cs.unm.edu/~mccune/otter/examples/auto/steam.in) should give you enough clues to get started. Remember that otter reads its input from standard input. This means otter needs to be invoked as “otter < myproblem.in”, not “otter myproblem.in”.