

A Tuned Eigenspace Technique for Articulated Motion Recognition

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Abstract

In this paper, we introduce a tuned eigenspace technique so as to classify human motion. The method presented here overcomes those problems related to articulated motion and dress texture effects by learning various human motions in terms of their sequential postures in an eigenspace. In order to cope with the variability inherent to articulated motion, we propose a method to tune the set of sequential eigenspaces. Once the learnt tuned eigenspaces are at hand, the recognition task then becomes a nearest-neighbor search over the eigenspaces. We show how our tuned eigenspace method can be used for purposes of real-world and synthetic pose recognition. We also discuss and overcome the problem related to clothing texture that occurs in real-world data, and propose a background subtraction method to employ the method in out-door environment. We provide results on synthetic imagery for a number of human poses and illustrate the utility of the method for the purposes of human motion recognition.

1 Introduction

In computer vision and pattern recognition, there is a considerable body of work aimed at understanding and developing appearance-based methods. Appearance-based methods can cope with illumination, reflectance and pose effects based upon the appearance of the scene in the image. The bulk of this work focuses on using PCA to build a subspace representation of the scene which is then used for purposes of appearance-based object and pose recognition. Turk and Pentland [1] have shown how this PCA-based representation, called the eigenspace, can be used to perform face recognition. In a related development, Murase and Nayar [2] have performed object and pose recognition by projecting the views under study onto a basis formed by the eigenspace components. Kopp-Borotschnig *et al.* [3] have developed a method to recognise objects from ambiguous viewpoints using an active vision approach. Hall, Marshall and Martin [4] have shown how appearance models can be updated based upon addition and subtraction

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of eigenspaces. Recently Schechtman and Irani [5] have introduced a behaviour-based similarity measure which is computed from intensity information.

One of the main arguments levelled against these methods is that they are not robust to occlusion, shadows or background texture. Ohba and Ikeuchi [6] have proposed a method to cope with partially occluded objects by storing partial appearances of on an “eigenwindow”. A mean eigenwindow method has also proposed by Rahman and Ishikawa[7] for reducing partial occlusion. Leonardis and Bischof [8] have shown how the coefficients of the eigenimages can be computed so as to cope with occlusion and segmentation. Black *et al.* [9] have used robust estimators to model structured noise and corruption. Yilmaz and Gokmen [10] have overcome problems related to illumination changes by applying the eigenspace representation to the edge images rather than the intensity values.

Despite effective, the methods above are prone to error due to texturing and articulated object variation such as the one present in human body motion. Thus, in this paper, we introduce a novel development of the appearance-based technique to recognise human motion. Here, we propose a tuned eigenspace so as to represent and recognise human posture and/or motion that has which considers dress-changes, pose variation, imaging noise and background clutter. We depart from the eigenspace technique of Murase and Nayar [2]. As mentioned earlier, this method makes use an eigenspace which is prone to variations in pose, dress-texture and clothing variation. Therefore, we generalise the eigenspace projection approach so that we can overcome these problems. In addition, we make use of a blurred edge image so as to solve to make the eigenspace projection robust to dress-texture variations. Further, in order to learn the eigenspace for a variety of human motions, we propose a mean posture matrix created from similar pose-windows. This is done by collecting similar poses from a particular subject and recovering the mean posture matrix. This mean posture matrix is then used to learn the eigenspace for the human motion under study. The eigenspace recovered from the mean posture matrix is what we called a tuned eigenspace. With these ingredients, the recognition of unobserved motions can be posed as a nearest neighbour search over the learnt tuned eigenspace. The study conducts a number of experiments for investigating the human dress-texture effect in the eigenspace and how the proposed method recovers it. Furthermore, We propose a background subtraction method in order to introduce this method in out-door application. We also compare our results with the conventional method.

2 Generating the Eigenspace

In order to develop a tuned eigenspace which can handle dress-texture and articulated human motion, we consider $P = \{p_1, p_2, \dots, p_{|P|}\}$ successive views. Each of these views is, in practice, an image comprised by $M_{rows} \times N_{cols}$ pixels, where M_{rows} is the height and N_{cols} is the width of the image p_i . These pixels can be rearranged in a raster scan manner into a column vector of the form $\mathbf{x}_p = [x_{1p}, x_{2p}, \dots, x_{Np}]^T$, where $N \cong M_{rows} \times N_{cols}$. In the sake of simplicity, we assume that this vector is already normalised to unity, i.e., $\|\mathbf{x}_p\| = 1$.

For a set \mathcal{M} of different human motions of order M , we denote the vector \mathbf{x} cor-

responding to the m^{th} motion as \mathbf{x}_p^m . For each motion, its image stream is sampled P times. These $P \times M$ images are collected into a single matrix X of the form $\mathbf{X} = [\mathbf{x}_1^1 - \hat{\mathbf{x}} \mid \mathbf{x}_2^1 - \hat{\mathbf{x}} \mid \dots \mid \mathbf{x}_p^1 - \hat{\mathbf{x}} \mid \mathbf{x}_1^2 - \hat{\mathbf{x}} \mid \mathbf{x}_2^2 - \hat{\mathbf{x}} \mid \dots \mid \mathbf{x}_p^2 - \hat{\mathbf{x}} \mid \dots \mid \mathbf{x}_1^M - \hat{\mathbf{x}} \mid \mathbf{x}_2^M - \hat{\mathbf{x}} \mid \dots \mid \mathbf{x}_p^M - \hat{\mathbf{x}}]$, where $\hat{\mathbf{x}}$ is the mean for the set of all vectors \mathbf{x}_i^j , i.e.

$$\hat{\mathbf{x}} = \frac{1}{P \times M} \sum_{i=1}^P \sum_{j=1}^M \mathbf{x}_i^j \quad (1)$$

The matrix X contains $P \times M$ columns and N rows. For the matrix X , the covariance matrix \mathbf{C} is defined by $\mathbf{C} = \mathbf{X}\mathbf{X}^T$.

We can use PCA [11], we can construct a subspace representation for the covariance matrix \mathbf{C} as follows. Let $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_N$ be the N eigenvalues of the covariance matrix \mathbf{C} arranged in decreasing order of rank. We can then select the first k eigenpairs, i.e. the eigenvectors \mathbf{e}_i and eigenvalues λ_i such that $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k$ so as to build a k -dimensional space which we denote the eigenspace of X . The image \mathbf{x}_p^m is then projected into a point \mathbf{g}_p^m in the eigenspace by the following equation

$$\mathbf{g}_p^m = [\mathbf{e}_1 \mid \mathbf{e}_2 \mid \dots \mid \mathbf{e}_k]^T \mathbf{x}_p^m \quad (2)$$

For each motion, $|P|$ points, which correspond to each of the p_i successive observations in P , describe a trace in the eigenspace. Since a motion is smooth, these points conform a smooth curved line. This is called a motion line. If a motion starts and ends with the same pose, the motion line composes a closed loop, which is referred to as a motion trajectory hereafter. A global eigenspace is that which contains M motion loops so as to capture multiple motions.

3 Developing a Tuned Eigenspace

As mentioned in the previous section, a human posture is represented by a point in the eigenspace, projected making use of Equation 2. A motion is described by a set of successive points that can provide a motion line. For H subjects, the motion lines in the eigenspace, corresponding to a particular motion, should ideally coincide with one other. In practice, this is not the case. Therefore we compute a mean expression of the postures for every of the motions under study. In this way, we take into account a general pattern which is comprised by the mean over all the motion lines for the motion under study. The proposed eigenspace containing the mean expression is called a tuned eigenspace. Consider a set H of human motion subjects. Let $\mathbf{x}_p^{m,h}$ denote the image stream corresponding to the p^{th} view of the motion indexed m , for the subject h . For the subject h , the matrix \mathbf{X} becomes

$$\mathbf{X}_h = [\mathbf{x}_{1,h}^1 \mid \mathbf{x}_{2,h}^1 \mid \dots \mid \mathbf{x}_p^{1,h} \mid \mathbf{x}_1^{2,h} \mid \mathbf{x}_2^{2,h} \mid \dots \mid \mathbf{x}_p^{2,h} \mid \dots \mid \mathbf{x}_1^{M,h} \mid \mathbf{x}_2^{M,h} \mid \dots \mid \mathbf{x}_p^{M,h}] \quad (3)$$

With the matrix \mathbf{X}_h at hand, we define the matrix $\tilde{\mathbf{X}} = [\mathbf{X}_1 \mid \mathbf{X}_2 \mid \dots \mid \dots \mid \mathbf{X}_{|H|}]$, which can be regarded as a higher-order analogous of X . For every of the $|H|$ subjects, we can project the image stream $\mathbf{x}_p^{m,h}$ for the subject h into the point $\mathbf{g}_p^{m,h}$ of

the tuned eigenspace making use of the expression $\mathbf{g}_p^{m,h} = [\tilde{\mathbf{e}}_1 \mid \tilde{\mathbf{e}}_2 \mid \dots \mid \tilde{\mathbf{e}}_K]^T \mathbf{x}_p^{m,h}$, where $\tilde{\mathbf{e}}_i$ is the i^{th} eigenvector of the covariance matrix $\tilde{C} = \tilde{X}\tilde{X}^T$. For the set H of subjects, we have $|H|$ such points, i.e., $\mathbf{g}_p^{m,h}$; $h = \{1, 2, \dots, H\}$. Thus, the points in the tuned eigenspace are given by the average point $\bar{\mathbf{g}}_p^m = \frac{1}{H} \sum_{h=1}^H \mathbf{g}_p^{m,h}$, which captures the p^{th} postures of a particular motion m learnt from a set H of subjects. The set of $|P|$ points $\bar{\mathbf{g}}_p^m | p = (1, 2, \dots, P)$ defines a mean line for the motion m . Hence, in the paper, we call the mean motion line for the M motions a *global tuned eigenspace*.

4 Dress Texture

In order to employ the global tuned eigenspace for purposes of human motion recognition, motion representation should be generalised so as to be robust to dress-texture and clothe variations. The standard eigenspace technique, however, is prone to error due to the changes in appearance introduced by variations in clothes and dress-texture. Therefore, here we follow Yilmaz and Gokmen [10] and employ, to recover the eigenspace, edge images as an alternative to the gray-scale views. In contrast with their approach, we have used a blurred edge image so as to introduce a Gaussian kernel over the edge-image for our set of views. Thus, every of our views is comprised by a blurred edge image $E(x, y)$ computed from the original image $I(x, y)$, which is given by $E(x, y) = G_{\sigma_2}(x, y) * D(G_{\sigma_1}(x, y) * I(x, y))$.

Here $G_{\sigma_1}(x, y)$ is a Gaussian kernel with a standard deviation σ_1 . The Gaussian kernel $G_{\sigma_1}(x, y)$ is convolved with the Image $I(x, y)$ in order to reduce random jitter and image noise. The resultant image is differentiated making use of differential operator D , which in our experiments is given by the Sobel operator. The differentiated image is, again, convolved with a Gaussian kernel whose standard deviation is σ_2 .

5 Recognition Strategy

Our aim in this paper is to perform human motion recognition based upon the tuned eigenspace introduced in the previous sections. Consider an image containing a data view of an unknown human motion. We want to decide if that view belongs to any of the learnt motions and in the case it does belong to one of the learnt motion classes, relate it to the views that characterise the motion to which it belongs. Let p' denote the data view under consideration. The view p' is then projected onto a discrete point $\mathbf{g}_{p'}^{m'}$ in the learnt global tuned eigenspace. To perform recognition, we make use of the minimum Euclidean distance $d_{p^*}^{m^*}$ in the learnt tuned eigenspace given by $d_{p^*}^{m^*} = \min_{p \in P; m \in \mathcal{M}} \|\mathbf{g}_{p'}^{m'} - \bar{\mathbf{g}}_p^{m'}\|$.

Thus, $d_{p^*}^{m^*}$ is such that the nearest learned point in the eigenspace to our data point $\mathbf{g}_{p'}^{m'}$ is related to both, a particular motion $m \in \mathcal{M}$ and an observation $p \in P$. Therefore, our strategy of motion recognition does not rely only on the recognition of a particular view but on the mean for the learnt set of views. Furthermore, since we employ the Euclidean distance between the data point in the tuned eigenspace and the

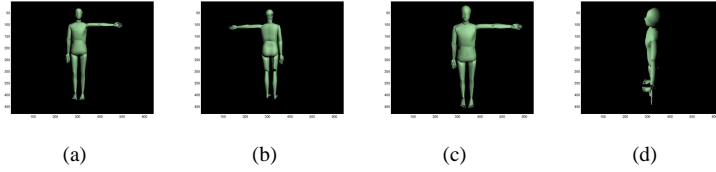


Figure 1: Sample poses (out of a total of 120) obtained from the 3 different camera rotations about the subject under study

mean motion line, our recognition strategy can be viewed as the search over the mass-centres for the points in the eigenspace corresponding to the observations for every of the learnt motions.

6 Experimental Results

In this section, we conduct a number of experiments in order to verify the effectiveness of our method for purposes of human motion recognition. This section is divided into three parts. In the first of these, we perform recognition using a set of synthetic motion views rendered using camera rotations. We then provide results on real-world data for 6 cricket umpiring motions obtained from 5 persons. We conclude the section by conducting an extensive sensitivity study on dress-texture and its impact on our tuned eigenspace technique. Along these lines, we propose a background subtraction method to overcome background noise and jitter and perform experiments so as to evaluate the proposed method under various noise levels.

6.1 Synthetic motion representation and recognition

We commence by providing results on synthetic imagery. Here, we have modelled synthetic motion by rotating the viewpoint. Since the positions of the subject under study and the camera are relative, this camera rotation procedure is equivalent to the appearance changes induced by subject position variation. We have used 3D Studio

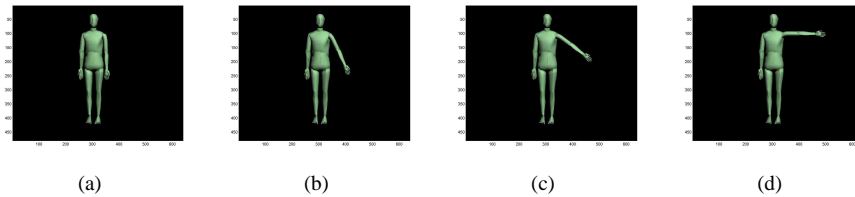


Figure 2: (a),(b) and (c): Poses used to learn the tuned eigenspace; (d): Pose used to render our data views

Max to create a set of four articulated motions in which the camera rotates about the vertical, sagittal and temporal axis of the subject under study. For each motion, we have used a subject with a different pose and rendered 120 frames rotating the camera in 4.5° degree intervals. In Figure 1, we show example views for our 3 different camera rotations. In Figure 2, we show the four poses used in our experiments. The pose in the right-most panel of Figure 2 constitutes our data pose. The other three poses are used for purposes of learning the tuned eigenspace. It is worth noting that the position of the arm and hand of the subject vary in an articulated fashion. To learn this articulated variation of the subject’s limb position, we have used 360 views, i.e. 120×3 . We have then used a fourth sequence of 120 views of the same subject in a different pose as our data set. For our recognition task, we consider a view to have been classified correctly if it corresponds to the point in the tuned eigenspace for the set of view in our learning set whose camera position is the same as that of the data view. This is, the rotation of the camera for the views in the learning set and that of the data view are the same. We have done this since the camera rotations along with hand movement give us various appearance-change. Therefore, for our synthetic data, the camera rotation and the pose determine the appearance. In our experiments, the recognition rate was of 99.3%. In other words, 118 views out of the 120 data views were classified correctly. An eigenspace obtained from 120 sample views is shown in Figure 3(a) and a tuned eigenspace generated from the three subject’s poses is also shown in the Figure 3(b).

6.2 Human motion representation and recognition

For our real-world experiments, we have employed 6 prominent actions ($M = 6$) of an umpire arbitrating a cricket match, i.e. “wide”, “no”, “boundary”, “over-boundary”, “leg bye”, and “out”. Sample views for each of these are shown in Figure 4. The motions were captured using a digital video camera. For each motion, we have used 10 views, i.e. ($P = 10$). For purposes of recognition, we have used the blurred edge images computed making use of the procedure introduced earlier in the paper. For our gaussian blurring, we have chosen $\sigma_1 = 0.30$ and $\sigma_2 = 2.0$. As a result,

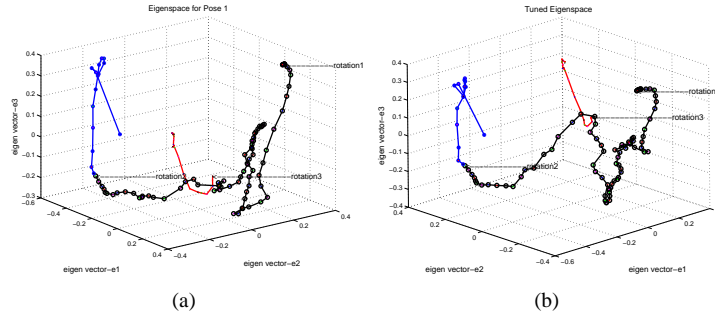


Figure 3: Eigenspaces obtained from the articulated motions : (a) Eigenspaces of a single pose, and (b) tuned eigenspace obtained from 3 poses



Figure 4: Real world motions used in the experiment.

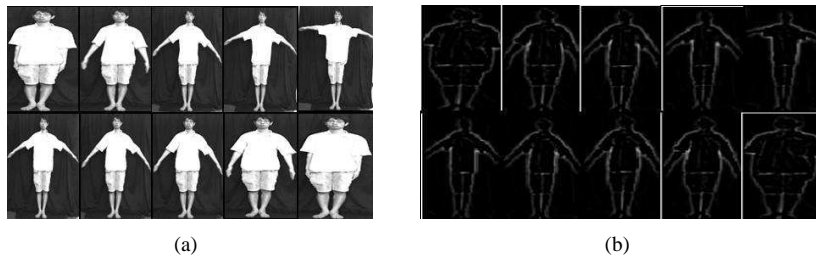


Figure 5: (a) Sequential images of "wide motion" (a) Sobel-edge images

$P \times M = 60$ edge-images were used to learn our global eigenspace. In the left-hand panel of Figure 5(a) we show 10 successive images of the "wide" motion. Their blurred edge images are shown in the right-hand panel of the figure 5(b). A graphical representation of a global eigenspace is shown in Figure 7. In the figure, individual motion trajectories are indicated by different colors/markers in the graph. Since all the motions start and end with an identical pose, i.e., a natural standing posture, every motion makes a closed loop. As a result, the global eigenspace in Figure 7 contains 6 motion loops originating from a common point. In order to illustrate how the tuned eigenspace reflects the eigenspaces for each of the 6 motions, in Figure 8 we have plotted the motion trajectories in the eigenspace for individual motions. In the top row of Figure 8, we show the trajectories of the "wide" and "no" motions, respectively, for five subjects. These have been obtained using our method. It is worth noting that, despite the models all wear different clothes, this do not the recovered eigenspace. As a result, each motion trajectory is very similar to one another. We have also compared our



Figure 6: Persons involved in performing the experiments. Models where background subtraction method is: (a) not employed, and (b) employed.

Table 1: Experimental results. MPM denotes mean posture matrix

Experiment	Training Set/MPM (Postures)	Testing Set (Postures)	Eigen Dimension	Recog. Rate (Average)
Human Motion	4 (240)	1(60)	6	87.5%
Dress	9(324)	1(36)	6	88.88%
Background	16(576)	51(36)	6	86.9%

results with those obtained using the method of Murase and Nayar [2]. In the bottom row of Figure 8, we show the results for the method in [2]. The motion trajectories are less congruent and show more variation than those recovered using our method.

6.2.1 Motion recognition using tuned eigenspaces

Since our method employed primarily 5 motions for recognizing human motions via posture recognition, a leave-one-out scheme is applied for selecting the image set. It means that we always choose 4 data sets for generating a tuned eigenspace and leave one data set for testing. A tuned eigenspace obtained from 4 data sets is shown in Figure 11(a). The obtained recognition results are shown in Table 1. We have obtained an average of 86.5% recognition rates where background issue were not considered. It is worth noting that the obtained motion recognition is 100%.

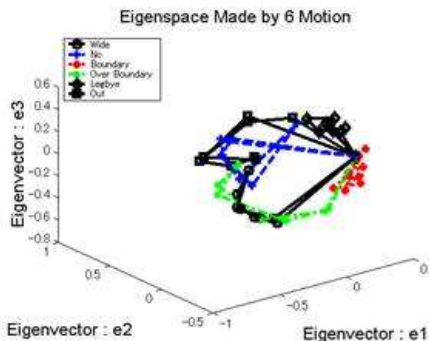


Figure 7: A global eigenspace of 6 motions. Only 3 prominent dimensions are displayed.

6.3 Special experiment considering Clothing Problem

We have further performed another experiment where the attention was focused in the clothing problem with a number of typical dressing schemes. In the experimental setup, we have used a camera for taking a video image of a turning motion (therefore $m = 1$) of a particular subject wearing 10 typical clothes. The dresses are shown in Figure 9. From the 10 different clothes, we have obtained $P = 360$ sampled views. For the comparison, the study employed a conventional method [2] where an original gray image was employed for generating an eigenspace. Figure 10 shows the closed motion trajectories generated from various clothes. Dress texture has made an undesirable effect by the conventional method, as shown in Figure 10(a) producing dissimilar motion trajectories, despite having identical models and motions. On the other hand, the motion loops are mutually quite similar using the proposed method as shown in

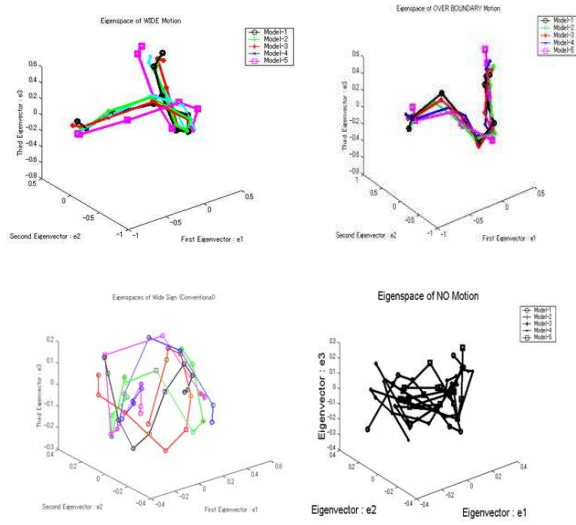


Figure 8: Top row: comparison of motion trajectories obtained from 5 persons: Similar motion trajectories obtained from the proposed approach; Bottom row: Motion trajectories affected by the model's variations in the conventional method.

Figure 10(b). For obtaining the recognition performance, we have employed the earlier mentioned leave-one-out scheme for selecting the tuned eigenspace. Therefore, 9 data set are used for training and one data set is always left for the testing. An average of 87% recognition rate is achieved for this particular data set as shown in Table 1.

6.4 Background subtraction method

A background subtraction method is applied in order to prove the effectiveness of the method. We have conducted an experiment employing 17 human models as shown in Figure 6(b). The motion categories and segmentation process were same as described in section 6.3. However, respective backgrounds have been subtracted automatically from the sampled images and silhouette images are obtained. Figure 12 shows the result of this subtraction method. Figures (from left to right) show an original image, background subtracted image, a segmented image containing human portion and the sobel-edge image. Once again, we have employed leave-one-out method for generating tuned eigenspace and obtaining the recognition results. The recognition results are listed in the Table 1.

6.5 Comparison results

We have compared our results with the conventional method [2] where original images are used for generating the eigenspace. It is also mentioned that conventional method employed only one data sample obtained from the best search scheme for creating the



Figure 9: Models used for investigating the clothing problem.

eigenspace. Once again, the proposed method has employed earlier described image pre-processing techniques for overcoming the clothing and noise effect, and a posture matrix for creating a tuned eigenspace. Since we have employed a leave-one-out method for selecting the data sets for creating the tuned eigenspace, it confirms use of every image data either for training and/or testing. The comparisons are two manifold: representation of eigenspaces in the presence of clothing effects, model variations and appearance-change. The proposed method has always generated eigenspaces with similar pattern with respect to the motions. Therefore, an eigenspace of a particular motion can be used for testing the other models. The requirement of eigen dimensions were also reasonable in the proposed method as shown in Figure 14. In contradictory, eigenspace obtained form the conventional way have always been affected by the preceding problems. Therefore, conventional method is not suitable for flexible object recognition. Consequently, poor recognition rates (i.e., 44.4% of using the data used in the experiment 6.2 and 42.1% from the data used in the experiment 6.3) have been achieved from the conventional method.

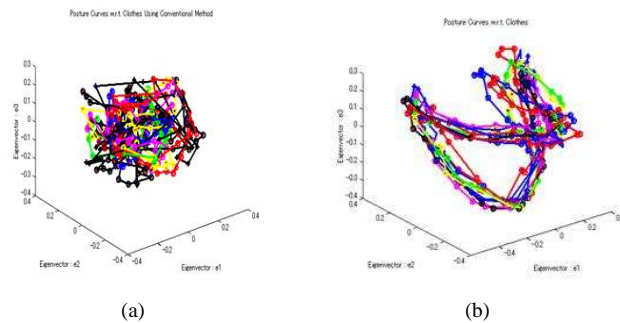


Figure 10: Motion's trajectories with all of 10 dresses: (a) the conventional method, and (b) the proposed method.

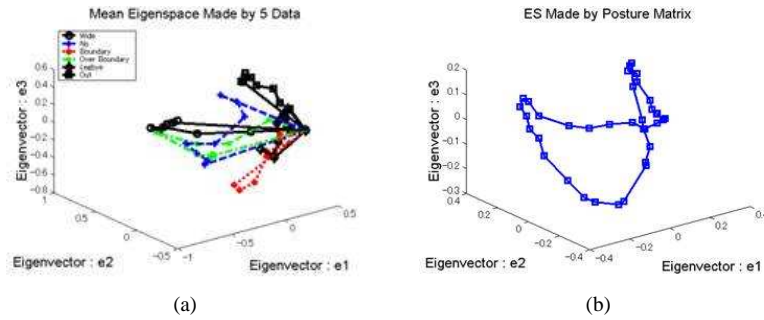


Figure 11: Tuned eigenspace for the 5 data sets in Figure 6(a), and (b) 5 data sets in Figure 9

6.6 Noise reduction

As stated earlier, double gaussian kernel are used mainly for reducing random noise and clothing texture effects. Therefore, our method is also effective under noisy image environments. We have made a comparison how the proposed method works under various noise levels. Figure 13 shows the noise level used in the experiment. We have used 20% salt and pepper noise to the images shown in the Figure 6(a) and they have used for creating eigenspaces and for the recognition. If we do not use the gaussian blurring, the posture recognition rate is shown always less than 70% even using the proposed method. Therefore, the pre-image processing techniques has provided us the noise reduction capability in a significant level.

7 Discussion and Conclusions

In this paper, we have introduced a novel appearance-based method for articulated motion recognition and illustrated its utility in recognition tasks. We have validated the proposed method in a number of ways using synthetic and real-world data. The proposed tuned eigenspace has the robustness to work under both, real human and articulated motions. Furthermore, the method also has the robustness to work under random imaging noise and background variations.



Figure 12: Background subtraction method: (from left to right) original image, background subtracted image, segmented image and Sobel edge image.

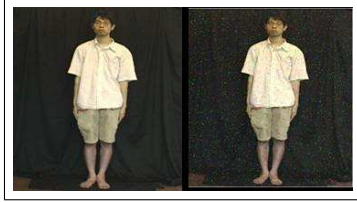


Figure 13: Imaging noise used in the experiment. (left) original image and (right) Image with 20% salt and pepper

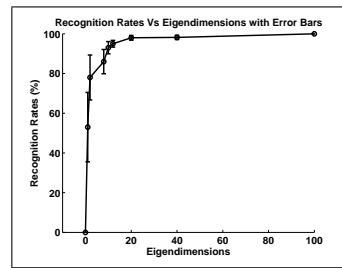


Figure 14: Requirement of eigen dimensions. The error bars correspond to the standard error for the recognition rate.

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