Linking losses for density ratio and class-probability estimation

Aditya Krishna Menon    Cheng Soon Ong

NICTA and The Australian National University
Linking losses for density ratio and class-probability estimation

Aditya Krishna Menon    Cheng Soon Ong

Data61 and The Australian National University
Class-probability estimation (CPE)

From labelled instances
Class-probability estimation (CPE)
From labelled instances, estimate probability of instance being +’ve
• e.g. using logistic regression

0.6

0.6

0.6
Density ratio estimation (DRE)

Given samples from densities $p, q$
Density ratio estimation (DRE)

Given samples from densities $p, q$, estimate density ratio $r = p/q$
Application: covariate shift adaptation

Marginal training distribution

Can overcome by reweighting training instances using ratio between test and training densities.
Application: covariate shift adaptation

Marginal training distribution $\neq$ marginal test distribution

Can overcome by reweighting training instances
use ratio between test and training densities
train e.g. weighted class-probability estimator
Application: covariate shift adaptation

Marginal training distribution $\neq$ marginal test distribution

Can overcome by reweighting training instances

- use ratio between test and test densities
- train e.g. weighted class-probability estimator
This paper
Formal link between CPE and DRE

- CPE
- Proper losses
  - Logistic
  - Exponential
  - Square Hinge
- DRE
- Ranking
- KLIEP
- LSIF
- Bregman
This paper
Formal link between CPE and DRE

- existing DRE approaches $\rightarrow$ implicitly performing CPE
This paper
Formal link between CPE and DRE

- existing DRE approaches $\rightarrow$ implicitly performing CPE
- CPE $\rightarrow$ Bregman minimisation for DRE
This paper

Formal link between CPE and DRE

- existing DRE approaches $\rightarrow$ implicitly performing CPE
- CPE $\rightarrow$ Bregman minimisation for DRE
- new application of DRE losses to “top ranking”

Proper losses

- Logistic
- Exponential
- Square Hinge

Bregman

CPE

DRE

Ranking
DRE and CPE: formally
Distributions for learning with binary labels

Fix an instance space $\mathcal{X}$ (e.g. $\mathbb{R}^n$)

Let $\mathcal{D}$ be a distribution over $\mathcal{X} \times \{\pm 1\}$, with $\mathbb{P}(Y = 1) = \frac{1}{2}$ and

$$(P(x), Q(x)) = (\mathbb{P}(X = x|Y = 1), \mathbb{P}(X = x|Y = -1))$$
Distributions for learning with binary labels

Fix an instance space $\mathcal{X}$ (e.g. $\mathbb{R}^n$)

Let $\mathcal{D}$ be a distribution over $\mathcal{X} \times \{\pm 1\}$, with $\mathbb{P}(Y = 1) = \frac{1}{2}$ and

$$(P(x), Q(x)) = (\mathbb{P}(X = x|Y = 1), \mathbb{P}(X = x|Y = -1))$$

$$(M(x), \eta(x)) = (\mathbb{P}(X = x), \mathbb{P}(Y = 1|X = x))$$
Scorers, losses, risks

A scorer is any $s : \mathcal{X} \rightarrow \mathbb{R}$

- e.g. linear scorer $s : x \mapsto \langle w, x \rangle$
Scorers, losses, risks

A **scorer** is any $s : X \rightarrow \mathbb{R}$

- e.g. linear scorer $s : x \mapsto \langle w, x \rangle$

A **loss** is any $\ell : \{\pm 1\} \times \mathbb{R} \rightarrow \mathbb{R}_+$

- e.g. logistic loss $\ell : (y, v) \mapsto \log(1 + e^{-yv})$
Scorers, losses, risks

A scorer is any \( s : \mathcal{X} \to \mathbb{R} \)

- e.g. linear scorer \( s : x \mapsto \langle w, x \rangle \)

A loss is any \( \ell : \{\pm 1\} \times \mathbb{R} \to \mathbb{R}_+ \)

- e.g. logistic loss \( \ell : (y, v) \mapsto \log(1 + e^{-yv}) \)

The risk of scorer \( s \) wrt loss \( \ell \) and distribution \( \mathcal{D} \) is

\[
\mathbb{L}(s; \mathcal{D}, \ell) = \mathbb{E}_{(X,Y) \sim \mathcal{D}}[\ell(Y, s(X))] 
\]

- average loss on a random sample
CPE versus DRE

Given samples $S \sim \mathcal{D}^N$, with $\mathcal{D} = (P, Q) = (M, \eta)$:
CPE versus DRE

Given samples $S \sim \mathcal{D}^N$, with $\mathcal{D} = (P, Q) = (M, \eta)$:

Class-probability estimation (CPE)
Estimate $\eta$

- class-probability function
CPE versus DRE

Given samples $S \sim \mathcal{D}^N$, with $\mathcal{D} = (P, Q) = (M, \eta)$:

Class-probability estimation (CPE)
Estimate $\eta$

- class-probability function

Density ratio estimation (DRE)
Estimate $r = \frac{p}{q}$

- class-conditional density ratio
CPE approaches: proper composite losses

For suitable $S \subseteq \mathbb{R}^X$, find

$$\arg\min_{s \in S} L(s; D, \ell)$$

where $\ell$ is such that, for some invertible $\Psi : [0, 1] \to \mathbb{R}$,

$$\arg\min_{s \in \mathbb{R}^X} L(s; D, \ell) = \Psi \circ \eta$$

estimate $\hat{\eta} = \Psi^{-1} \circ s$
CPE approaches: proper composite losses

For suitable \( S \subseteq \mathbb{R}^{\mathcal{X}} \), find

\[
\arg\min_{s \in S} \mathbb{L}(s; \mathcal{D}, \ell)
\]

where \( \ell \) is such that, for some invertible \( \Psi : [0, 1] \rightarrow \mathbb{R} \),

\[
\arg\min_{s \in \mathbb{R}^{\mathcal{X}}} \mathbb{L}(s; \mathcal{D}, \ell) = \Psi \circ \eta
\]

- estimate \( \hat{\eta} = \Psi^{-1} \circ s \)

Such an \( \ell \) is called strictly proper composite with link \( \Psi \)
Examples of proper composite losses

Logistic loss
\[ \Psi^{-1} : v \mapsto \sigma(v) \]

Exponential loss
\[ \Psi^{-1} : v \mapsto \sigma(2v) \]

Square hinge loss
\[ \Psi^{-1} : v \mapsto \min(\max(0, (v + 1)/2), 1) \]
DRE approaches: divergence minimisation

For suitable $S \subseteq \mathbb{R}^X$, find

**KLIEP:** (Sugiyama et al., 2008)

$$\arg\min_{s \in S} \text{KL}(p \parallel q \odot s)$$

- constrained KL minimisation

**LSIF:** (Kanamori et al., 2009)

$$\arg\min_{s \in S} \mathbb{E}_{X \sim Q} \left[ (r(X) - s(X))^2 \right]$$

- direct least squares minimisation
Story so far

We begin by showing existing DRE losses implicitly perform CPE.

Proper losses:
- DRE
- CPE
- Logistic
- Exponential
- Square Hinge
- KLIEP
- LSIF

CPE → Logistic
CPE → Exponential
CPE → Square Hinge
CPE → KLIEP
CPE → LSIF

DRE
Roadmap

We begin by showing existing DRE losses implicitly perform CPE
Existing DRE losses are proper composite
Existing DRE approaches

Suppose $\mathcal{D} = (P, Q)$

KLIEP: (Sugiyama et al., 2008)

$$\arg\min_{s \in S} KL(p \parallel q \circ s)$$

LSIF: (Kanamori et al., 2009)

$$\arg\min_{s \in S} \mathbb{E}_{X \sim Q} \left[ (r(X) - s(X))^2 \right]$$
Existing DRE approaches as loss minimisation

Suppose \( \mathcal{D} = (P, Q) \)

**KLIEP:** (Sugiyama et al., 2008)

\[
\arg\min_{s \in S} \mathbb{E}_{(X,Y) \sim \mathcal{D}} [\ell(Y, s(X))]
\]

\[\ell(-1, v) = a \cdot v \text{ and } \ell(1, v) = -\log v\]

for suitable \( a > 0 \)

**LSIF:** (Kanamori et al., 2009)

\[
\arg\min_{s \in S} \mathbb{E}_{(X,Y) \sim \mathcal{D}} [\ell(Y, s(X))]
\]

\[\ell(-1, v) = \frac{1}{2} \cdot v^2 \text{ and } \ell(1, v) = -v\]
Existing DRE approaches as loss minimisation

Suppose $\mathcal{D} = (P, Q)$

**KLIEP:** (Sugiyama et al., 2008)

$$\arg\min_{s \in S} \mathbb{E}_{(X, Y) \sim \mathcal{D}} [\ell(Y, s(X))]$$

$$\ell(-1, v) = a \cdot v \text{ and } \ell(1, v) = -\log v$$

for suitable $a > 0$

**LSIF:** (Kanamori et al., 2009)

$$\arg\min_{s \in S} \mathbb{E}_{(X, Y) \sim \mathcal{D}} [\ell(Y, s(X))]$$

$$\ell(-1, v) = \frac{1}{2} \cdot v^2 \text{ and } \ell(1, v) = -v$$

These are no ordinary losses
For \( u \in [0, 1] \), let

\[
\Psi_{dr}: u \mapsto \frac{u}{1 - u}.
\]

**Lemma**

The LSIF loss is strictly proper composite with link \( \Psi_{dr} \). The KLIEP loss with \( a > 0 \) is strictly proper composite with link \( a^{-1} \cdot \Psi_{dr} \).
Existing DRE approaches as CPE

For $u \in [0, 1]$, let

$$\Psi_{dr}: u \mapsto \frac{u}{1-u}.$$ 

**Lemma**

The LSIF loss is strictly proper composite with link $\Psi_{dr}$. The KLIEP loss with $a > 0$ is strictly proper composite with link $a^{-1} \cdot \Psi_{dr}$.

KLIEP and LSIF perform CPE in disguise!
Proof

For LSIF and KLIEP (with $a = 1$),

$$\frac{\ell'(1, v)}{\ell'(-1, v)} = -\frac{1}{v},$$

so that
Proof

For LSIF and KLIEP (with $a = 1$),

$$\frac{\ell'(1, v)}{\ell'(-1, v)} = -\frac{1}{v'},$$

so that

$$f(v) = \frac{1}{1 - \frac{\ell'(1, v)}{\ell'(-1, v)}} = \frac{v}{1 + v}$$

Proper compositeness follows from (Reid and Williamson, 2010).
Proof

For LSIF and KLIEP (with $a = 1$),

$$\frac{\ell'(1, \nu)}{\ell'(-1, \nu)} = -\frac{1}{\nu},$$

so that

$$f(\nu) = \frac{1}{1 - \frac{\ell'(1, \nu)}{\ell'(-1, \nu)}} = \frac{\nu}{1 + \nu} = \Psi^{-1}_{\text{dr}}(\nu).$$
Proof

For LSIF and KLIEP (with $a = 1$),

$$\frac{\ell'(1, v)}{\ell'(-1, v)} = -\frac{1}{v'},$$

so that

$$f(v) = \frac{1}{1 - \frac{\ell'(1, v)}{\ell'(-1, v)}} = \frac{v}{1 + v} = \Psi^{-1}_{dr}(v).$$

Proper compositeness follows from (Reid and Williamson, 2010).

The link $\Psi_{dr}$ is especially suitable for DRE...
Another view of $\Psi_{dr}$

Bayes’ rule shows targets of DRE and CPE are linked:

$$(\forall x \in X) \ r(x) \cdot \frac{p(x)}{q(x)} = \Psi_{dr}(\eta(x))$$

as is well known (Bickel et al, 2009) KLIEP and LSIF apposite for DRE

Optimal scorer is exactly $\Psi_{dr} \circ \eta = r$
Another view of $\Psi_{dr}$

Bayes’ rule shows targets of DRE and CPE are linked:

$$\left(\forall x \in \mathcal{X}\right) r(x) = \frac{p(x)}{q(x)} = \frac{\eta(x)}{1 - \eta(x)}$$
Another view of $\Psi_{dr}$

Bayes’ rule shows targets of DRE and CPE are linked:

$$(\forall x \in \mathcal{X}) r(x) = \frac{p(x)}{q(x)}$$

$$= \frac{\eta(x)}{1 - \eta(x)}$$

$$= \Psi_{dr}(\eta(x))$$
Another view of $\Psi_{dr}$

Bayes’ rule shows targets of DRE and CPE are linked:

$$(\forall x \in X) r(x) = \frac{p(x)}{q(x)}$$

$$= \frac{\eta(x)}{1 - \eta(x)}$$

$$= \Psi_{dr}(\eta(x))$$

as is well known (Bickel et al, 2009)
Another view of $\Psi_{\text{dr}}$

Bayes’ rule shows targets of DRE and CPE are linked:

$$(\forall x \in X) \ r(x) \propto \frac{p(x)}{q(x)} = \frac{\eta(x)}{1 - \eta(x)} = \Psi_{\text{dr}}(\eta(x))$$

as is well known (Bickel et al, 2009)

KLIEP and LSIF apposite for DRE

- Optimal scorer is exactly $\Psi_{\text{dr}} \circ \eta = r$
Story so far

Existing DRE losses are specific examples of CPE losses
Roadmap

Now consider using arbitrary CPE losses for DRE
CPE as Bregman minimisation
General CPE approach to DRE?

Suppose $\ell$ proper composite with link $\Psi$

Class-probability estimate $\hat{\eta} = \Psi^{-1} \circ s$

- for logistic loss, $\hat{\eta}(x) = 1/(1 + e^{-s(x)})$
General CPE approach to DRE?

Suppose $\ell$ proper composite with link $\Psi$

Class-probability estimate $\hat{\eta} = \Psi^{-1} \circ s$

- for logistic loss, $\hat{\eta}(x) = 1/(1 + e^{-s(x)})$

Density ratio estimate is naturally:

$$\hat{r}(x) = \Psi_{dr}(\hat{\eta}(x)) = \frac{\hat{\eta}(x)}{1 - \hat{\eta}(x)}.$$  

- e.g. for logistic loss, $\hat{r}(x) = e^{s(x)}$
General CPE approach to DRE?

Suppose $\ell$ proper composite with link $\Psi$

Class-probability estimate $\hat{\eta} = \Psi^{-1} \circ s$

- for logistic loss, $\hat{\eta}(x) = 1/(1 + e^{-s(x)})$

Density ratio estimate is naturally:

$$
\hat{r}(x) \doteq \Psi_{dr}(\hat{\eta}(x)) = \frac{\hat{\eta}(x)}{1 - \hat{\eta}(x)}.
$$

- e.g. for logistic loss, $\hat{r}(x) = e^{s(x)}$

Intuitive, but what can we guarantee about this?

- preceding analysis only asymptotic
A Bregman minimisation view of CPE

For proper composite $\ell$, the regret or excess risk of a scorer is

$$\text{reg}(s; \mathcal{D}, \ell) = \mathbb{L}(s; \mathcal{D}, \ell) - \min_{s^* \in \mathbb{R}^X} \mathbb{L}(s^*; \mathcal{D}, \ell)$$
A Bregman minimisation view of CPE

For proper composite $\ell$, the regret or excess risk of a scorer is

$$\text{reg}(s; D, \ell) = \mathbb{L}(s; D, \ell) - \min_{s^* \in \mathbb{R}^X} \mathbb{L}(s^*; D, \ell)$$

$$= \mathbb{E}_{X \sim M} \left[ B_f(\eta(X), \hat{\eta}(X)) \right]$$

for Bregman divergence $B_f$ and loss-specific $f$
A Bregman minimisation view of CPE

For proper composite $\ell$, the regret or excess risk of a scorer is

$$\text{reg}(s; \mathcal{D}, \ell) = \mathbb{L}(s; \mathcal{D}, \ell) - \min_{s^* \in \mathbb{R}^X} \mathbb{L}(s^*; \mathcal{D}, \ell)$$

$$= \mathbb{E}_{X \sim \mathcal{M}} \left[ B_f(\eta(X), \hat{\eta}(X)) \right]$$

for Bregman divergence $B_f$ and loss-specific $f$

- e.g. for logistic loss, regret is a KL projection

$$\text{reg}(s; \mathcal{D}, \ell) = \mathbb{E}_{X \sim \mathcal{M}} \left[ \text{KL}(\eta(X) \| \hat{\eta}(X)) \right]$$
A Bregman minimisation view of CPE

For proper composite $\ell$, the regret or excess risk of a scorer is

$$reg(s; D, \ell) = \mathbb{L}(s; D, \ell) - \min_{s^* \in \mathbb{R}^X} \mathbb{L}(s^*; D, \ell)$$

$$= \mathbb{E}_{X \sim M} [B_f(\eta(X), \hat{\eta}(X))]$$

for Bregman divergence $B_f$ and loss-specific $f$

- e.g. for logistic loss, regret is a KL projection

$$reg(s; D, \ell) = \mathbb{E}_{X \sim M} [\text{KL}(\eta(X) \| \hat{\eta}(X))]$$

Does this imply a Bregman projection onto $r$?
A Bregman identity
The following lemma lets us make progress.

Lemma

Pick any convex and twice differentiable $f : [0, 1] \to \mathbb{R}$. Then,

$$(\forall x, y \in [0, \infty)) \ B_f \left( \frac{x}{1+x}, \frac{y}{1+y} \right)$$

where $f^{\otimes} : z \mapsto (1 + z) \cdot f \left( \frac{z}{1+z} \right)$. 
A Bregman identity
The following lemma lets us make progress.

Lemma

Pick any convex and twice differentiable $f : [0, 1] \rightarrow \mathbb{R}$. Then,

$$(\forall x, y \in [0, \infty)) B_f \left( \frac{x}{1+x}, \frac{y}{1+y} \right) = \frac{1}{1+x} \cdot B_{f^\otimes} (x, y),$$

where $f^\otimes : z \mapsto (1 + z) \cdot f \left( \frac{z}{1+z} \right)$. 
A Bregman identity
The following lemma lets us make progress.

**Lemma**

*Pick any convex and twice differentiable* $f : [0, 1] \rightarrow \mathbb{R}$. *Then,*

$$(\forall x, y \in [0, \infty)) B_f \left( \frac{x}{1 + x} , \frac{y}{1 + y} \right) = \frac{1}{1 + x} \cdot B_{f \boxtimes} (x, y),$$

*where* $f \boxtimes : z \mapsto (1 + z) \cdot f \left( \frac{z}{1 + z} \right)$.

$f \boxtimes$ *is closely related to the perspective transform*
A Bregman identity
The following lemma lets us make progress.

**Lemma**

*Pick any convex and twice differentiable* $f : [0, 1] \to \mathbb{R}$. Then,

$$\left( \forall x, y \in [0, \infty) \right) B_f \left( \frac{x}{1 + x}, \frac{y}{1 + y} \right) = \frac{1}{1 + x} \cdot B_{f^\otimes}(x, y),$$

*where* $f^\otimes : z \mapsto (1 + z) \cdot f \left( \frac{z}{1 + z} \right)$.

$f^\otimes$ is closely related to the perspective transform

Unlike standard dual symmetry,

$$B_f(x, y) = B_{f^\star}(f'(y), f'(x)),$$

order of $x$ and $y$ retained, and only $x$ appears in extra scaling factor.
Proof - I

By (Reid and Williamson 2009, Equation 12),

$$B_f(x, y) = \int_y^x (x - z) \cdot f''(z) \, dz.$$  

Applying this to the LHS,

$$B_f \left( \frac{x}{1 + x}, \frac{y}{1 + y} \right) = \int_{\frac{y}{1+y}}^{\frac{x}{1+x}} \left( \frac{x}{1 + x - z} \right) \cdot f''(z) \, dz.$$
Proof - II

Employing the substitution \( z = \frac{u}{1+u} \), with \( dz = \frac{du}{(1+u)^2} \),

\[
\text{LHS} = \int_y^x \left( \frac{x}{1+x} - \frac{u}{1+u} \right) \cdot f'' \left( \frac{u}{1+u} \right) \cdot \frac{1}{(1+u)^2} \, du
\]
Proof - II

Employing the substitution $z = \frac{u}{1+u}$, with $dz = \frac{du}{(1+u)^2}$,

$$\text{LHS} = \int_y^x \left( \frac{x}{1+x} - \frac{u}{1+u} \right) \cdot f'' \left( \frac{u}{1+u} \right) \cdot \frac{1}{(1+u)^2} \, du$$

$$= \frac{1}{1+x} \cdot \int_y^x (x-u) \cdot f'' \left( \frac{u}{1+u} \right) \cdot \frac{1}{(1+u)^3} \, du$$
Proof - II

Employing the substitution \( z = \frac{u}{1+u} \), with \( dz = \frac{du}{(1+u)^2} \),

\[
\text{LHS} = \int_y^x \left( \frac{x}{1+x} - \frac{u}{1+u} \right) \cdot f'' \left( \frac{u}{1+u} \right) \cdot \frac{1}{(1+u)^2} \, du
\]

\[
= \frac{1}{1+x} \cdot \int_y^x (x-u) \cdot f'' \left( \frac{u}{1+u} \right) \cdot \frac{1}{(1+u)^3} \, du
\]

\[
= \frac{1}{1+x} \cdot B_{f \otimes}(x,y),
\]

since by definition of \( f \otimes \),

\[
(f \otimes)''(z) = f'' \left( \frac{z}{1+z} \right) \cdot \frac{1}{(1+z)^3}.
\]
Proof - II

Employing the substitution \( z = \frac{u}{1+u} \), with \( dz = \frac{du}{(1+u)^2} \),

\[
\text{LHS} = \int_y^x \left( \frac{x}{1+x} - \frac{u}{1+u} \right) \cdot f'' \left( \frac{u}{1+u} \right) \cdot \frac{1}{(1+u)^2} \, du
\]

\[
= \frac{1}{1+x} \cdot \int_y^x (x-u) \cdot f'' \left( \frac{u}{1+u} \right) \cdot \frac{1}{(1+u)^3} \, du
\]

\[
= \frac{1}{1+x} \cdot B_f \otimes (x, y),
\]

since by definition of \( f \otimes \),

\[
(f \otimes )''(z) = f'' \left( \frac{z}{1+z} \right) \cdot \frac{1}{(1+z)^3}.
\]

Not obviously generalisable with another substitution

- RHS does not remain a Bregman divergence
Implication for DRE via CPE

Identity is equivalently

\[ B_f \left( \Psi_{dr}^{-1}(x), \Psi_{dr}^{-1}(y) \right) = \frac{1}{1 + x} \cdot B_{f\otimes}(x, y). \]
Implication for DRE via CPE

Identity is equivalently

\[ B_f \left( \Psi_{dr}^{-1}(x), \Psi_{dr}^{-1}(y) \right) = \frac{1}{1 + x} \cdot B_{f \otimes}(x, y). \]

Apply to \( x = r \), so that \( \Psi_{dr}^{-1}(x) = \eta \).
Implication for DRE via CPE

Identity is equivalently

\[ B_f \left( \Psi^{-1}_{\text{dr}}(x), \Psi^{-1}_{\text{dr}}(y) \right) = \frac{1}{1 + x} \cdot B_{f^\otimes}(x, y). \]

Apply to \( x = r \), so that \( \Psi^{-1}_{\text{dr}}(x) = \eta \)

Lemma

Pick any strictly proper composite \( \ell \) with \( f \) twice differentiable. Then, for any distribution \( D = (P, Q) \) and scorer \( s : X \to \mathbb{R} \),

\[ \text{reg}(s; D, \ell) = \frac{1}{2} \cdot \mathbb{E}_{X \sim Q} \left[ B_{f^\otimes}(r(X), \hat{r}(X)) \right], \]

for \( \hat{r} = \Psi_{\text{dr}} \circ \hat{\eta} = \Psi_{\text{dr}} \circ \Psi^{-1} \circ s \).

Justifies using CPE for DRE

- concrete sense in which \( \hat{r} \) is a good estimate
Story so far

Shown how to perform DRE with range of CPE losses
Roadmap

Final link is to use **DRE losses for CPE problems**

- CPE
- Logistic
- Exponential
- Square Hinge
- KLIEP
- LSIF
- Proper losses
- DRE
- Bregman

Diagram:
- **CPE**
- Logistic
- Exponential
- Square Hinge
- KLIEP
- LSIF
- **DRE**
- Bregman
DRE for bipartite top ranking
Bipartite top ranking

Given $S \sim \mathcal{D}^N$ as before, learn scorer $s: \mathcal{X} \rightarrow \mathbb{R}$ with
Bipartite top ranking

Given $S \sim \mathcal{D}^N$ as before, learn scorer $s : \mathcal{X} \rightarrow \mathbb{R}$ with

**Bipartite ranking**: maximal area under ROC curve

- rank average positives above negatives
- CPE is suitable (Kotlowski et al, 2010, Agarwal, 2014)
Bipartite top ranking

Given $S \sim D^N$ as before, learn scorer $s : \mathcal{X} \rightarrow \mathbb{R}$ with

**Bipartite ranking**: maximal area under ROC curve

- rank average positives above negatives
- CPE is suitable (Kotlowski et al, 2010, Agarwal, 2014)

**Top ranking**: maximal partial area under ROC curve

- rank top positives above negatives
- is CPE suitable?
CPE and weight functions

Any proper composite $\ell$ has weight function $w : [0, 1] \rightarrow \mathbb{R}_*$

- large $w(c) \rightarrow$ more focus on $\eta \approx c$

Logistic loss

$$w(c) = \frac{1}{2 \cdot c \cdot (1-c)}$$

Exponential loss

$$w(c) = \frac{1}{4 \cdot c^{3/2} \cdot (1-c)^{3/2}}$$

Square hinge loss

$$w(c) = 2$$
Top ranking via LSIF

Carefully selected $\ell$ suitable for top ranking

- choose $\ell$ with $w$ focussing on large values of $\eta$

Easy to check that for LSIF,

$$\ell(-1,v) = \frac{1}{2} \cdot v^2 \text{ and } \ell(1,v) = -v.$$  

$$w(c) = \frac{1}{(1-c)^3}.$$  

- focusses on $\eta \approx 1$
- appealing due to closed-form solution!

See paper for details
Conclusion
Summary

Formal links between (losses for) CPE and DRE

Proper losses
- Logistic
- Exponential
- Square Hinge
- KLIEP
- LSIF

CPE

DRE

Bregman
Ranking
Future work

Finite sample analysis

- understanding of when importance weighting doesn’t help

Other applications of DRE losses?

- closed form solution for LSIF is appealing

Other applications for Bregman lemma?
Thanks!\textsuperscript{1}

\textsuperscript{1}Drop by the poster for more (Paper ID 152)