Diagnosis (01)
Definitions

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1. Presentation

2. Modeling of a diagnosis problem

3. Formal definition of diagnosis
1. Presentation
   - Diagnosis problem
   - Diagnosis as a logic problem
   - Model-Based Diagnosis

2. Modeling of a diagnosis problem

3. Formal definition of diagnosis
Diagnosis problem

- **Given**
  - a system
  - a set of observations

- **Goal**
  - find if a problem happens, and if yes which one
  - restore a good behavior
Example: car

- System:

- Observations: the car does not start
- Possible diagnoses: the battery does not work, the starter is broken, the car is out of petrol, etc.
- Possible repair: first, test plan to discriminate between the diagnoses (check the battery, etc.)
Example: human body

- System:

- Observations: Fever (40 degrees), headache
- Possible diagnoses: cold, migraine
- Possible repair: take three pills per day
Famous *syllogism* of Aristotle:

- Socrates is a man
- Every man is mortal

**Deduction**

- Socrates is mortal
Abduction

- Every man is mortal
- Socrates is mortal

Abduction

- Socrates is a man
  (eg. Sherlock Holmes)
Abduction

- Every man is mortal
- Socrates is mortal

Abduction
  - Socrates is a man (eg. Sherlock Holmes)

- Every duck is mortal
- Socrates is mortal

Abduction
  - Socrates is a duck
Abduction

- Every man is mortal
- Socrates is mortal

Abduction

- Socrates is a man (eg. Sherlock Holmes)

- Every ET is mortal
- But ETs do not exist

Not an abduction

- Socrates is an ET
Induction

- Socrates is a man
- Socrates is mortal

**Induction**
- Every man is mortal
- Every mortal is a man
- No man but Socrates is mortal
- *etc.*
What is diagnosis?

- Deduction?
- Abduction?
- Induction?
What is diagnosis?

- Deduction
- Abduction
- Induction
Expert Diagnosis vs Model-based Diagnosis

**Expert Diagnosis**
- Need an expertise (human experience, logs from past experience, *etc.*)
- Efficient: direct mapping from the observations to the diagnosis

**Model-based Diagnosis**
- Need a model of the system
- Robust
- Justification
Historical

- Heuristic approaches
  - Expert systems (70)
- Approaches of static systems based on model (80)
- Approaches of dynamic systems based on model (90)
- Approaches of reconfigurable systems based on model (00)
Historical

- Heuristic approaches
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Static system

System whose state does not depend on the previous states

Example: Davis Circuit

\[
\begin{align*}
A &= 2 & B &= 3 & C &= 3 & D &= 2 \\
E &= 2 & F &= 10 & G &= 12
\end{align*}
\]
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Knowledge about “how the world works”

[Russel and Norvig, 2003]
Model

Knowledge about “how the world works”

[Russel and Norvig, 2003]

Mathematical representation of the behavior of the environment that enables to simulate it.

[Grastien, 2005]
A system model is a couple \((SD, COMP)\) where
- \(SD\) is a set of first-order logic sentences describing the behavior of the system
- \(COMP\) is a set of constants, a constant = one component

An observed system is a tuple \((SD, COMP, OBS)\) where
- \((SD, COMP)\) is a system model
- \(OBS\) is the set of observations
$COMP = \{ a_1, a_2, m_1, m_2, m_3 \}$
Adder \((SD)\):

- \(Add(x) \land \neg Ab(x) \land In1(x, u) \land In2(x, v) \land Sum(u, v, w) \Rightarrow Out(x, w)\)
- \(Add(x) \land \neg Ab(x) \land In1(x, u) \land Out(x, w) \land Sum(u, v, w) \Rightarrow In2(x, v)\)
- \(Add(x) \land \neg Ab(x) \land In2(x, v) \land Out(x, w) \land Sum(u, v, w) \Rightarrow In1(x, u)\)

Multiplier \((SD)\):

- \(Mult(x) \land \neg Ab(x) \land In1(x, u) \land In2(x, v) \land Prod(u, v, w) \Rightarrow Out(x, w)\)
- \(Mult(x) \land \neg Ab(x) \land In1(x, u) \land Out(x, w) \land Prod(u, v, w) \Rightarrow In2(x, v)\)
- \(Mult(x) \land \neg Ab(x) \land In2(x, v) \land Out(x, w) \land Prod(u, v, w) \Rightarrow In1(x, u)\)
Component types (*SD*)

- $Add(a_1)$, $Add(a_2)$, $Mult(m_1)$, $Mult(m_2)$, $Mult(m_3)$

Connections (*SD*)

- $Out(m_1, u) \land In1(a_1, v) \Rightarrow u = v$
- $Out(m_2, u) \land In2(a_1, v) \Rightarrow u = v$
- $Out(m_2, u) \land In1(a_2, v) \Rightarrow u = v$
- $Out(m_3, u) \land In2(a_2, v) \Rightarrow u = v$
- $Out(m_1, u) \land In1(m_3, v) \Rightarrow u = v$
**Observations**

- *OBS* is a set of atomic sentences
- each atomic sentence represents an observation

- $ln1(m_1, 3), ln2(m_1, 2)$
- $ln1(m_2, 2), ln2(m_2, 3)$
- $ln1(m_3, 2), ln2(m_3, 3)$
- $Out(a_1, 10), Out(a_2, 12)$
1. Presentation

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3. Formal definition of diagnosis
A state of the system \((SD, COMP)\) is the Ab-clause denoted \(\Phi_\Delta\) where \(\Delta \subseteq COMP\) defined by:

\[
\bigwedge_{c \in COMP \setminus \Delta} (\neg Ab(c)) \land \bigwedge_{c \in \Delta} (Ab(c))
\]

The components in \(\Delta\) have an abnormal behavior (they are faulty)
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\[
\Delta = \{a_1, a_2\}
\]

\[
Ab(a_1) \land Ab(a_2) \land \neg Ab(m_1) \land \neg Ab(m_2) \land \neg Ab(m_3)
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\(\Delta = \{a_1, a_2, m_1, m_2, m_3\}\)

- \(Ab(a_1) \land Ab(a_2) \land Ab(m_1) \land Ab(m_2) \land Ab(m_3)\)
A diagnosis of the observed system \((COMP, SD, OBS)\) is a state \(\Phi_\Delta\) such that

\[
SD \land OBS \land \Phi_\Delta
\]

is satisfiable (consistent)
Definition of diagnosis

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\[
SD \land OBS \land \Phi_{\Delta}
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is satisfiable (consistent)

- The state is possible according to \((SD, COMP, OBS)\)
Definition of diagnosis

- A diagnosis of the observed system \((COMP, SD, OBS)\) is a state \(\Phi_\Delta\) such that
  \[
  SD \land OBS \land \Phi_\Delta
  \]
  is satisfiable (consistent)

- The state is possible according to \((SD, COMP, OBS)\)

- A diagnosis exists if
  \[
  SD \land OBS
  \]
  is satisfiable. If not, the model is either not well-designed or incomplete
The observations are abnormal if

\[ SD \land OBS \land \Phi_\emptyset \]

is not satisfiable
Example

How many diagnoses can you find in this example?

![Diagram with nodes labeled A, B, C, D, E, X, Y, Z, mult-1, mult-2, mult-3, add-1, add-2, F, G.]

Observations

\[ In1(m_1, 3), In2(m_1, 2), ln1(m_2, 2), ln2(m_2, 3), \]
\[ ln1(m_3, 2), ln2(m_3, 3), Out(a_1, 10), Out(a_2, 12) \]