LETTER

Efficient Multiple Multicast in WDM Networks

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SUMMARY This paper addresses the problem of multiple multicast in WDM networks. It presents three efficient algorithms to construct an optimal/sub-optimal multicast tree for each multicast and minimise the network congestion on wavelengths. The first two algorithm achieve an optimal network congestion for a specific class of networks whose all wavelengths are globally accessible and convertible at a unit cost. The third algorithm produces an approximation solution for the general case of WDM networks.

key words: *algorithm, communication, multicast, WDM networks*

1. Introduction

Wavelength-Division Multiplexing (WDM) network is an optical network which supports the propagation of multiple laser beams through a single fiber-optic link provided that each laser beam uses a distinct optical wavelength. The major applications of the network are video conferencing, scientific visualisation, real-time medical imaging, supercomputing, and distributed computing [1], [9], [12]. A comprehensive overview of its physical theory and applications of this technology can be found in the books by Green [4] and McAulay [7].

In order to solve various application problems on an WDM network, mechanisms must be developed to handle not only point-to-point communication but also group communication involving transporting information from a group of sites (nodes) to another group of sites in the network. A typical group communication is *multicast* that transports information from one source node to a set of destination nodes. A more general version of group communication is *multiple multicast* that contains multiple groups of multicast, each with its own source node and destination set [11]. Multiple multicast covers all existing types of communications.

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A WDM network can be represented by graph $G = (V, E, \Gamma)$ with $|V| = n$ and $|E| = m$, where $\Gamma = \{\Gamma_0, \Gamma_1, \ldots, \Gamma_{m-1}\}$, Γ_e is the set of wavelengths available at edge $e \in E$ with $w(e, \gamma)$ associated with wavelength γ as the cost required to access γ. At each node v, there is a set of converters (switches) associated, each converting a particular wavelength (γ_i) from all incoming edges to another wavelength (γ_j) on outgoing edges at a fixed cost $c(\gamma_i, \gamma_j)$. Figure 1 illustrates wavelength conversion at a node.

In general and most practical cases, each edge in G is bidirectional so that messages can be transmitted in either direction. When both directions of an edge share the same set of wavelengths, this edge can be regarded as unidirectional for simplicity in representation.

There are r groups of multicast $\mathcal{M}_i = \{s_i, \langle d_i^1, \ldots, d_i^{k_i}\rangle\}$, where s_i is a source and $d_i^1, \ldots, d_i^{k_i}$ are the destinations of s_i, $0 \le i \le r-1$, $1 \le k_i < n$. Let $\mathcal{M}_i$ alone (without considering the existence of other groups) can be realized by a multicast tree MT_i, and $MF = \cup MT_i$. Clearly, in the general case many edges of MF will fall into the same edge (e) of G and use the same wavelength (γ), thus causing *congestion* on wavelength γ at edge e when broadcasting these r sources simultaneously. Figure 2 shows an example of congestion caused by 3 multicast trees.

The degree of congestion corresponds to the length of message buffering queue in an ordinary network and indicates the waiting time required to deliver all the messages on the queue. Thus an important objective in designing multiple multicast algorithms is to minimize the congestion with respect to certain measure (total vs. maximum for instance). In this paper we take the measure of maximum congestion. We say that

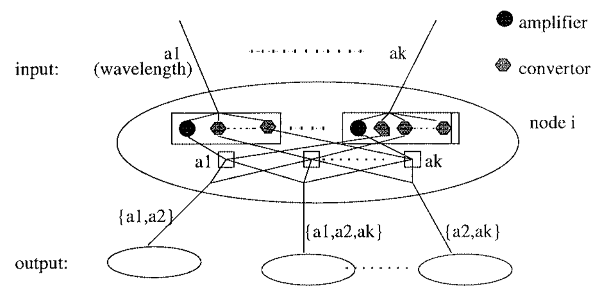

Fig. 1 Wavelength conversion at a node.

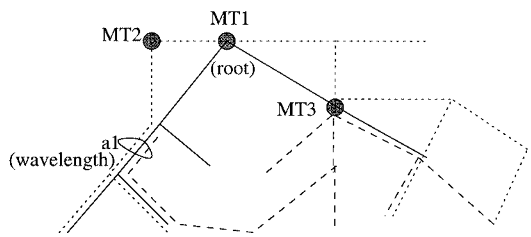

Fig. 2 Congestion caused by 3 muitlcast trees using the same wavelength on the same edge.

the (maximum) congestion of a network is *optimal* if it reaches the minimum.

Define the congestion on an edge e, denoted by l_e, to the maximum congestion on all wavelengths at e, and the congestion on G to be the maximum congestion on all edges in G. Our task is to construct all MT_i's to achieve a minimal network congestion while keeping the cost of MF as small as possible.

While research on point-to-point communication in WDM networks has been quite active [3], [5], [13], not much has been done on group communication due to the degree of its difficulty [8]. In [6] we have shown that the optimal multicast problem in WDM networks is NP-complete and proposed an approximate algorithm for it. Some other work has been done recently by other researchers on different network models [8], [10]. To our knowledge, there is no work reported on the problem of multiple multicast in WDM networks with our definition stated above.

This paper addresses the problem of multiple multicast in WDM networks. In Sect. 2 we describe two simple and efficient algorithms that solve this problem optimally — minimizes the total cost of all multicast trees as well as the network congestion for the simplified case when all wavelengths are globally accessible at any edge in the network, and both access to a wavelength and conversion between a pair of wavelengths require a unit-cost. In Sect. 3 we present an approximation solution for the general case of multiple multicast in WDM networks that uses a greedy approach and employs our previous result on single multicast [6]. Section 4 concludes the paper by proposing some topics for future research.

2. Globally Accessible Unit-Cost Wavelength

We consider a simplified WDM network G based on the following assumptions:

1. All wavelengths of G are globally accessible at any edge in G, that is, $\Gamma_0 = \Gamma_1 = \cdots = \Gamma_{m-1} = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$.
2. Both access to a wavelength and conversion between a pair of wavelengths require a unit-cost, that is, $w(\gamma_i) = c(\gamma_i, \gamma_j) = c$ for $0 \le i \ne j \le m-1$, where c is constant.

Under the above assumptions, it is clear that a multicast $\mathcal{M}$ requires a minimum cost if it follows an optimal multicast tree that is a shortest path tree constructed in G connecting the source to all destinations in $\mathcal{M}$. The problem is now concluded to how to allocate the wavelengths to the edges of r shortest-path trees realizing the given r groups of multicast.

We know from fundamental optics that the magnitude of a wavelength decays in proportional to the distance it travels for message transmission. In order to avoid information loss on any wavelength in the network, we need to maintain the same magnitude of the wavelength at any step of transmission. This requires to set an amplifier at each node for each valid wavelength arrival at the node that amplifies the magnitude of the wavelength on it arrival so that it can be reused with the same reliability in the next step of transmission. Without this on-line amplification, a wavelength would need to undergo the process of off-line energy recharge to restore its original magnitude. We assume this process takes the same length as its transmission of a message over a single edge. Hence the full switching support at each node in a WDM network is a set of amplifiers for magnitude amplification and a set of converters (modulators, filters, etc) for wavelength conversion. We call networks equipped with this support *normal* networks, and networks with only converter support *weak* networks.

These different supports affect the algorithms for multiple multicast in the corresponding networks in the sense of whether the same wavelength is allowed to be used by two successive edges in the network. In the normal network with amplifier support this is allowed, whereas in the weak network without amplifier support it is not allowed. This difference requires to use different approaches and apply different techniques for the design of the algorithm, under the same criteria of minimizing multicast cost and network congestion when the number of wavelengths is given.

In the following we give a simple and efficient algorithm for each type of networks respectively.

For weak networks, each wavelength after use on any edge cannot be reused immediately — it needs to wait for a period of one-edge transmission for off-line energy recharge. To minimize the the network congestion, we make all wavelengths to be used by each group multicast as equally as possible by alternating the wavelengths for different routing steps. We present the following algorithm.

Algorithm WUC-MMulticast($\mathcal{M}$, G, Γ)
{*Compute a communication scheme for r groups of multicast $\mathcal{M} = \{\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M}_{r-1}\}$ in a weak WDM network G with k globally accessible unit-cost wavelengths Γ.*}

1. Construct the multicast tree MT_i on $\mathcal{M}_i$ in G, $0 \le i \le r-1$, by finding the shortest-path tree

connecting the source to all destinations in $\mathcal{M}_{r-1}$}.

2. For $i = 0$ to $r-1$ starting from the root (source at level 0) breadth-first search MT_i and assign wavelength γ_{α_j} to all edges at level j, $1 \le j \le \ell_i$, where the value of α_i is decided as follows:
   If $k \ge 2r$ then
   - $\alpha_j = 2i$ if j is even;
   - $\alpha_j = 2i+1$ if j is odd;

   else
   - $\alpha_j = i \bmod k$ if j is even;
   - $\alpha_j = i \bmod k + 1$ if j is odd.

The degree of congestion can be analyzed as follows. If $k \ge 2r$, since each multicast tree MT_i uses one distinct wavelength ($2i$) if the maximum level number $\ell_i = 1$ and two wavelengths exclusively ($2i$ and $2i+1$) if $\ell_i > 1$, obviously no congestion exists (degree of congestion is 1) in both of these cases. If $k < 2r$, wavelength $i \bmod k + 1$ is shared by MT_i at odd numbered levels (for all ℓ_i) and by MT_{i+1} at even numbered levels (if $\ell_i > 1$). When $\ell_i = 1$, there are r/k multicast trees sharing each wavelength (at level 1), so the degree of maximum congestion is $\lceil r/k \rceil$. In the general case when $\ell_i > 1$, for any wavelength γ_i, $1 \le i \le k$, since there are r/k multicast trees using it at odd numbered levels and another r/k multicast trees using it at even numbered levels, the degree of maximum congestion is therefore $\lceil 2r/k \rceil$. This is the degree of possible maximum congestion for all cases.

Lemma 1: For r groups of multicast in a weak WDM network with k wavelengths, if all wavelengths are accessible and convertible on each edge (node) at a unit cost, the minimum congestion on wavelength in the worst case is $\lceil 2r/k \rceil$.

Proof Since the underlying WDM network is weak, each group must be assigned with at least 2 different wavelengths in the general case when the multicast tree contains more than 2 levels. Therefore, given a total of k wavelengths and r groups, there are at least $\lceil 2r/k \rceil$ groups sharing the same wavelength.

The worst case is that all the r groups have to travel through the same edge in the network at the same time. In this case, all the groups sharing the same wavelength compete for this wavelength. That is, the congestion on the wavelength is at least $\lceil 2r/k \rceil$. □

The above lemma shows that algorithm WUC-MMulticast computes multicast trees with the minimum network congestion. This combined with the fact that each multicast tree is a shortest path tree and therefore has a minimum cost under our unit-cost assumption concludes that all multicast trees constructed by the algorithm are optimal.

Let t_i be the time required for constructing MT_i. The time complexity of the algorithm is $\sum_i^r (t_i + O(|MT_i|)) = O(rn^2 + r^2 n)$, since $t_i = O(n^2)$ for finding a shortest-path tree in G and $|MT_i| \le n-1$. We have thus the following theorem.

Theorem 1: If all k wavelengths in a weak WDM network of n nodes are accessible and convertible on each edge (node) at a unit cost, optimal multicast trees for r groups of multicast can be computed in $O(rn^2 + r^2 n)$ time with an optimal network congestion of 1 if $k \le 2r$ and $\lceil 2r/k \rceil$ otherwise.

We now consider the case of normal WDM networks. In this case since each wavelength can be reused immediately owning to on-line amplification, we should let each tree MT_i use the same wavelength and minimize the total number of wavelengths used by all trees by letting the disjoint part of MF use the same wavelength. Assume that $r > k$, since otherwise the solution is trivial.

Algorithm NUC-MMulticast($\mathcal{M}$, G, Γ)
{*Compute a communication scheme for r groups of multicast $\mathcal{M} = \{\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M}_{r-1}\}$ in a normal WDM network G with k globally accessible unit-cost wavelengths Γ.*}

1. Construct the multicast tree MT_i on $\mathcal{M}_i$ in G, $0 \le i \le r-1$.
2. For $i = 0$ to $r-1$, assign wavelength $i \bmod k$ to all edges in MT_i;
3. Compute $MF = \cup_i MT_i$ and record overlap edges in D and their wavelengths assigned in W;
   {*$W[i]$ holds all wavelengths assigned to edge i with overlap number $D[i]$, $0 \le i \le m-1$.*}
4. **for** $i = 0$ **to** $m-1$ **do**
   if $D[i] > 1$ **then**
   if $W[i] < D[i]$ **then**
   assign $\min\{D[i], k\}$ wavelengths to edge i and each wavelength to $\lceil D[i]/k \rceil$ trees travelling through edge i.

Since $D[i] \le r$ for any r, the following theorem depicting the performance of algorithm NUC-MMulticast can be easily established.

Theorem 2: If all k wavelengths in a normal WDM network of n nodes are accessible and convertible on each edge (node) at a unit cost, an optimal forest consisting of r multicast trees can be computed in $O(rn^2 + r^2 n)$ time with an optimal network congestion of at most $\lceil r/k \rceil$.

3. Partially-Accessible Non-unit Cost Wavelengths

We now consider the general case that all wavelengths are not globally accessible at each edge $e \in E$ — Γ_e can be any subset of $\cup_{e \in E} \Gamma_e$, and different wavelengths and their conversions may carry different costs. Obviously our previous approach given in Sect. 2 is not applicable to this case.

For each individual group of multicast $\mathcal{M}_i$ it is shown NP-hard to find an optimal multicast tree [6]. However, there is a polynomial algorithm for approximation solution as given in [6] by reducing this problem to that of finding a Steiner tree on an auxiliary graph of enlarged size that accommodates all different routing costs, due to the existence of approximation solution for the directed Steiner tree problem [2]. An edge in this auxiliary graph ($G' = (V', E')$) can be denoted by (e, γ) that corresponds to the cost of using wavelength γ on edge e.

For multiple multicast in the general case of WDM networks, we take a greedy approach to find an approximate optimal multicast tree for a multicast and employ our previous algorithm to find an approximate optimal multicast tree for multicast one by one, with the heuristic of increasing the edge weight (routing cost by taking the wavelength corresponding to the edge) by an appropriate amount when an edge is taken by a multicast tree so as to deprioritize that edge (wavelength) to be chosen by the remaining multicast trees and hence minimize the length of the congestion queue on that wavelength.

Let $c_e(\gamma_u, \gamma_v)$ be the conversion cost from wavelength γ_u to γ_v at edge e, $w_e(\gamma)$ the cost of accessing wavelength γ at edge e. Denote by $cost(e, \gamma)$ be the total cost of edge (e, γ). Associate $\Delta_{e,\gamma}$ as the weight increment to edge (e, γ) in G'. $\Delta_{e,\gamma}$ is initialized to zero for all edges of G'. Our algorithm is described as follows:

Algorithm MMulticast ($\mathcal{M}$, G, Γ)
{*Compute a communication scheme for r groups of multicast $\mathcal{M} = \{\mathcal{M}_0, \mathcal{M}_1 \ldots, \mathcal{M}_{r-1}\}$ in a WDM network G with k wavelengths Γ.*}

1. For each (e, γ) in E', compute $cost(e, \gamma) = \sum_{\gamma' \in \Gamma_e, c_e(\gamma, \gamma') \neq \infty} c_e(\gamma, \gamma') + w_e(\gamma)$;
2. Compute $\sum cost(e, \gamma) = \sum_{(e,\gamma) \in E'} cost(e, \gamma)$;
   For $i = 1$ to r do
   (a) find an approximate optimal multicast tree MT_i in the enlarged auxiliary graph G' using algorithm of [6];
   (b) for each edge $(e^*, \gamma^*) \in OMT_i$ (uses wavelength γ^*) assign $\sum cost(e, \gamma)$ to Δ_{e^*,γ^*} and add it into $cost(e^*, \gamma^*)$.

We shall show that once an edge (e^*, γ^*) is chosen by an MT_i, this edge will be deprioritize to the end of the edge queues to be chosen by MT_j for $i + 1 \leq j \leq r$. This can be seen from the simple fact that adding increment $\Delta_(e^*, \gamma^*)$, which is $\sum_{(e,\gamma) \in E'} cost(e, \gamma)$, into $cost(e^*, \gamma^*)$ will make $cost(e^*, \gamma^*)$ greater than the total cost of G', and hence (e^*, γ^*) will be taken by any MT_j only if there is no other choices (alternative routes in G' not through edge (e^*, γ^*) that connect the set of nodes of MT_j. Yet (e^*, γ^*) is still an alive edge because its cost is not infinity so can be chosen if needed.

Let t_i be the time required for computing MT_i. Algorithm MMulticast takes a total time $O(k|E'| + \sum_i^r t_i)$. From [6], we know that $|E'| \leq k(kn + m)$ and $t_i = O((kn)^{1/\epsilon} n)$ for a solution of $O(n^\epsilon)$-OPT on cost based on the current result on directed Steiner tree approximation [2], $0 < \epsilon \leq 1$, so the time complexity of the above algorithm is $O(k^2 n + km + (kn)^{1/\epsilon} rn)$.

Hence we have

Theorem 3: Given a general WDM network of n nodes and m edges and k wavelengths, an approximate solution of $O(n^\epsilon)$-OPT on cost for r groups of multicast in the network can be obtained in $O(k^2 n + km + (kn)^{1/\epsilon} rn)$ time with a small network congestion.

4. Concluding Remarks

We have presented three efficient algorithms for multiple multicast in WDM networks. Our first two algorithms achieve an optimal network congestion when all wavelengths of the network are globally accessible and convertible at any edge. Our third algorithm gives an approximation solution with the heuristic of deprioritizing each wavelength on an edge when it is used by a multicast (tree) so as to avoid using the same wavelength on the same edge by multiple groups of communication.

An interesting open problem for future research is to find an efficient algorithm that achieves optimal network congestion for multiple multicast in the general WDM networks. Another problem of interest is to find efficient algorithms that achieve an approximation solution with a guaranteed good (constant or logarithmic) performance ratio.

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