

# Minimizing the Service Cost of Mobile Chargers while Maintaining the Perpetual Operations of WRSNs

Wenzheng Xu and Weifa Liang

**Abstract** The wireless energy transfer technology based on magnetic resonant coupling has been emerging as a promising technology for lifetime prolongation of wireless sensor networks, by providing controllable yet perpetual energy to sensors. As a result, we can employ mobile chargers (i.e., charging vehicles) to charge sensors with wireless energy transfer when the mobile charger approach lifetime-critical sensors. It is however very costly to dispatch mobile chargers to travel too long to charge sensors since their mechanical movements are energy-consuming too. To minimize the operational cost of wireless sensor networks, in this chapter we study the deployment of multiple mobile chargers to charge sensors in a large-scale wireless sensor network so that none of the sensors will run out of energy and aim to minimize the service cost of the mobile chargers. Specifically, we study the problem of minimizing the total traveling distance of mobile chargers for a given monitoring period, and the problem of deploying the minimum number of mobile chargers to replenish a set of lifetime-critical sensors while ensuring that none of the sensors will run out of energy, respectively. For the former, we propose a novel approximation algorithm with a guaranteed approximation ratio, assuming that the energy consumption rate of each sensor does not change for the given monitoring period. Otherwise, we devise a heuristic algorithm through modifications to the approximation algorithm. Simulation results show that the proposed algorithms are very promising. For the latter, we develop an approximation algorithm with a provable performance guarantee, and experimental results demonstrate that the solution delivered by the proposed approximation algorithm is fractional of the optimal one.

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## 1 Introduction

The limited battery capacities of sensors obstruct the large-scale deployment of wireless sensor networks (WSNs). Although there are many energy-aware approaches developed in the past decade to reduce sensor energy consumptions or balance energy expenditures among sensors [15, 16, 17, 22, 23, 24, 26, 38], the lifetime of WSNs remains a main performance bottleneck in their real deployments, since wireless data transmission consumes substantial sensor energy. The wireless energy transfer technology based on magnetic resonant coupling has been emerging as a promising technology for wireless sensor networks, by providing controllable yet perpetual energy to sensors [32]. In this chapter, we employ multiple mobile chargers (i.e., charging vehicles) to replenish sensor energy in a large-scale WSN for a given monitoring period  $T$  so that none of the sensors will run out of energy, where each sensor can be charged by a mobile charger in its vicinity with the wireless power transfer technique. Since each sensor consumes energy on data sensing, data transmission, data reception, etc., the sensor may need to be charged multiple times during the monitoring period of  $T$  to avoid its energy depletion. It is however very costly to dispatch mobile chargers to travel too long to charge sensors since their mechanical movements are very energy-consuming, or deploy too many mobile chargers to replenish sensors. To minimize the network operational cost, in this chapter we study the deployment of multiple mobile chargers to charge sensors in a large-scale wireless sensor network so that none of the sensors will run out of energy, and aim to minimize the service cost of mobile chargers. Specifically, we investigate two charging optimization problems: the problem of minimizing the traveling distance of mobile chargers for a given monitoring period [35, 36]; and the problem of deploying the minimum number of mobile chargers to replenish a set of lifetime-critical sensors while ensuring that none of the sensors will run out of energy, respectively [18, 19].

Most existing studies on sensor charging scheduling employ mobile chargers to charge all sensors periodically [27, 30, 31], or charge only the sensors that will run out of energy very soon [9, 18, 19, 25, 28, 33, 37, 41]. One major disadvantage of these studies is that the total travelling distance of the mobile chargers in the entire monitoring period can be very long, which may not be necessary, as the energy consumption rates of different sensors usually are significantly different. For example, the sensors near to the base station have to relay data for other remote sensors, their energy consumption rates thus are much higher than that of the others [14]. Therefore, the naive strategy of charging all sensors per charging tour will significantly increase the total travelling distance of the mobile chargers. Similarly, the charging strategy that schedules the mobile chargers to charge only the life-critical sensors also suffers from the same problem as these life-critical sensors may be far away from each other in the monitoring area.

The long total travelling distance of mobile chargers can result in prohibitively high energy consumptions of mobile chargers on their mechanical movements. It is reported that the most fuel-efficient vehicle has an energy consumption of 600 *kJ per km* (i.e., 27 *kWh* per 100 miles) [1] while the energy capacity of a regular

sensor battery is  $10.8 \text{ kJ}$  [27]. This implies that the amount of energy consumed by the vehicle travelling for one kilometer is equivalent to the amount of energy used for charging as many as  $55 (\approx \frac{600 \text{ kJ}}{10.8 \text{ kJ}})$  sensors. Since WSNs usually are deployed for long-term environmental sensing, target tracking, and structural health monitoring [4, 12, 20, 39], the monitoring area of a WSN can be very large (e.g., several square kilometers) [2, 12], the mobile chargers by the existing studies consume a large proportion of their energy on travelling, rather than on sensor charging, thereby leading to a very high cost of network operations.

Unlike existing studies that ignore the energy consumption of mobile chargers on travelling for charging sensors, in this chapter we develop efficient charging scheduling algorithms to dispatch multiple mobile chargers for sensor charging in a large-scale WSN for a long-term monitoring period  $T$ , so that not only none of the sensors runs out of energy but also the total travelling distance of all the mobile chargers for the monitoring period of  $T$  is minimized. As energy consumption rates of different sensors may significantly vary, different sensors have different charging frequencies during the period  $T$ , this poses great challenges for scheduling the mobile chargers, which include

- (1) when should we activate a charging round to dispatch the mobile chargers to replenish sensor energy?
- (2) which sensors should be included in each charging round?
- (3) given a set of to-be-charged sensors, which sensors should be charged by which mobile charger?
- (4) what is the charging order of the sensors assigned to each mobile charger?

In this chapter we will tackle these challenges by first formulating a novel optimization problem, and then devising an efficient approximation algorithm with a performance guarantee and a heuristic algorithm for the problem, depending on whether the energy consumption rate of each sensor is fixed or not for the given monitoring period. On the other hand, most existing studies assumed that one mobile charging vehicle will have enough energy to charge all sensors in a WSN, and the proposed algorithms for vehicle charging scheduling thus are only applicable to small-scale WSNs [27, 29, 30, 41, 34, 36, 35]. However, in a large-scale sensor network, the amount of energy carried by a single mobile charging vehicle may not be enough to charge all nearly-expired sensors, as there are a large proportion of life-critical sensors to be charged. Thus, multiple mobile charging vehicles instead of a single one are needed to be employed. In this chapter we will study the use of minimum numbers of mobile charging vehicles to replenish energy to sensors for a large-scale wireless sensor network such that none of the sensors will run out of energy. We will adopt a flexible on-demand sensor charging paradigm that decouples sensor energy charging scheduling from the design of sensing data routing protocols, and dispatch multiple mobile charging vehicles to charge life-critical sensors in an on-demand way. Specifically, we assume that each mobile charging vehicle can carry only a limited, rather than infinite, amount of energy. We will study a fundamental sensor charging problem. That is, given a set of life-critical sensors to be charged and the energy capacity constraint on each mobile charging vehicle, what is the minimum

number of mobile charging vehicles needed to fully charge these sensors in order to reduce the operational cost of the WSN, while ensuring that none of the sensors runs out of energy? To address this problem, not only should the number of charging vehicles be determined but also the charging tour of each mobile charging vehicle needs to be found so that all life-critical sensors can be charged prior to their expirations, where each vehicle consumes energy on charging sensors in its tour and its mechanical movement along the tour.

There are two closely related studies on minimizing the number of deployed charging vehicles [8, 21]. Specifically, Nagarajan and Ravi studied the *distance constrained vehicle routing problem* (DVRP), in which given a set of nodes in a metric graph, a depot, and an integral distance bound  $D$ , the problem is to find the minimum number of tours rooted at the depot to cover all nodes such that the length of each tour is no more than  $D$  [21]. For the DVRP problem, they presented a  $(O(\log \frac{1}{\varepsilon}), 1 + \varepsilon)$ -bicriteria approximation algorithm for any constant  $\varepsilon$  with  $0 < \varepsilon < 1$ , i.e., the algorithm finds a set of tours that the length of each tour is no more than  $(1 + \varepsilon)D$ , while the number of deployed vehicles is no more than  $O(\log \frac{1}{\varepsilon})$  times the minimum number of vehicles. On the other hand, Dai *et al.* investigated the problem of deploying the minimum number of charging vehicles to fully charge the sensors, by making use of the approximation algorithm in [21], assuming that all sensors have identical energy consumption rates [8]. There are however two essential differences between these two mentioned studies and the work in this chapter. First, the cost of each found tour by the algorithms in [21, 8] may violate the travel distance constraint on the mobile vehicles. In contrast, in this chapter the total energy consumption of each mobile charging vehicle per tour cannot exceed its energy capacity  $IE$ . Otherwise, the vehicle cannot return to the depot for recharging itself. Also, a constant approximation algorithm for the minimum number of mobile chargers deployment problem is devised. Second, the study in [8] assumed that all sensors have identical energy consumption rates. Contrarily, this chapter does not require that all sensors have identical energy consumption rates and the energy consumption rates of different sensors may be significantly different. Therefore, the proposed algorithms in the two mentioned studies cannot be applicable to the problem in this chapter. New approximation algorithms need to be devised, and new algorithm analysis techniques for analyzing the approximation ratio need to be developed, too.

The main contributions of this chapter can be summarized as follows. We first formulate a novel service cost minimization problem of finding a series of charging scheduling of multiple mobile chargers such that the total travelling distance of the mobile chargers for sensor charging is minimized. We also formulate a minimum number of mobile chargers deployment problem while maintaining the perpetual operations of sensors for a given monitoring period, subject to the energy capacity constraint on each mobile charger. We thirdly propose an approximation algorithm for the service cost minimization problem with a provable approximation ratio if energy consumption rates of sensors are fixed during the monitoring period. Otherwise, we devise a heuristic solution through modifications to the approximate solution. Furthermore, we develop an approximation algorithm with a

provable performance guarantee for the minimum number of mobile chargers deployment problem. We finally conduct extensive experiments by simulations to evaluate the performance of the proposed algorithms. Experimental results demonstrate that the proposed algorithms are very promising. To the best of our knowledge, they are the first approximation algorithms for scheduling multiple mobile chargers to charge sensors within a given monitoring period for the service cost minimization problem and the minimum number of mobile chargers deployment problem.

The rest of this chapter is organized as follows. Section 2 introduces preliminaries. Sections 3 and 4 propose efficient approximation and heuristic algorithms for the service cost minimization problem and the minimum number of mobile chargers deployment problem, respectively. Sections 5 and 6 evaluate the performance of the proposed algorithms. Section 7 concludes this chapter.

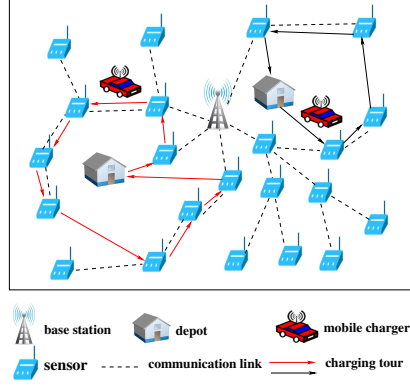
## 2 Preliminaries

In this section, we first introduce the network and energy consumption models, then introduce notations and notions, and finally define the problems precisely.

### 2.1 Network model

We consider a wireless sensor network consisting of  $n$  sensors, which are randomly deployed in a two-dimensional space. Let  $V$  be the set of sensors. Each sensor  $v_i \in V$  generates sensing data with a rate of  $b_i(t)$  (in *bps*) at time  $t$ , and it is powered by a rechargeable battery with energy capacity  $B_i$ . There is one stationary base station in the network. We assume that there is a routing protocol for sensing data collection that relays sensing data from sensors to the base station through multihop relays. For example, each sensor uploads its sensing data to the base station via the path with the minimum energy consumption. Fig. 1 illustrates such a wireless sensor network. Assume that the entire monitoring period is  $T$  ( $T$  typically is quite long, e.g., several months, even years). Since each sensor consumes energy on data sensing, processing, transmission and reception, it requires to be charged multiple times for the period of  $T$  to avoid its energy depletion.

We employ  $q$  wireless mobile chargers to replenish energy to sensors in the network, where mobile charger  $l$  is located at depot  $r_l$ ,  $1 \leq l \leq q$ . Without loss of generality, let  $R = \{r_1, r_2, \dots, r_q\}$  be the set of depot locations of the  $q$  mobile chargers. To determine charging trajectories of the  $q$  mobile chargers, we define a weighted, undirected graph  $G = (V \cup R, E; w)$ , where for any two distinct nodes (sensors or depots)  $u$  and  $v$  in  $V \cup R$ , there is an edge  $e = (u, v) \in E$  between them with their Euclidean distance being the weight  $w(e)$  of edge  $e$ . Assume that each mobile charger has a full energy capacity  $IE$  and a charging rate  $\mu$  for charging a sensor, and the charger travels at a constant speed  $s$ . We further assume that the mechanical movement of the charger is derived from its energy as well. Let  $\eta$  be the energy consumption rate of each charger on travelling per unit-length. Each time mobile charger  $l$



**Fig. 1** A rechargeable wireless sensor network

is dispatched to charge some sensors, it always starts from and ends at its depot  $r_l$  for recharging itself or refuelling its petrol. In other words, each charging tour of a mobile charger  $l$  in  $G$  is a *closed tour* including depot  $r_l$ . For any closed tour  $C$  in  $G$ , denote by  $w(C)$  the weighted sum of the edges in  $C$ , i.e.,  $w(C) = \sum_{e \in E(C)} w(e)$ . We consider a point-to-point charging, i.e., to efficiently charge a sensor by a mobile charger, the mobile charger must be in the vicinity of the sensor [13] and the sensor will be charged to its fully capacity.

We assume that the duration of the  $q$  mobile chargers per charging round that includes the time for charging sensors and their travelling time is several orders of magnitude less than the lifetime of a fully-charged sensor. The rationale behind the assumption is as follows. Once a sensor is fully charged, its lifetime can last from several weeks to months until its next charging, since the sensor energy can be well managed through various existing energy conservation techniques, e.g., duty cycling [5]. On the other hand, the  $q$  mobile chargers can collaboratively finish a charging round within a few hours, since sensor batteries can be made with ultra-fast charging battery materials [11]. For example, in 2009 scientists from MIT implemented an ultra-fast charging, in which a battery can be fully charged within a few seconds [11]. We thus envision that ultra-fast charging batteries will be commercialized in the near future and will be widely used for smartphones, sensors, electric vehicles, etc. Therefore, we ignore the time spent by the  $q$  mobile chargers per charging round. Note that [10, 40] and [41] also adopted the similar assumption.

## 2.2 Energy consumption models

Each sensor will consume energy on data sensing, data transmission, and data reception, and the energy consumption models for these three components are shown in Eq. (1), Eq. (2), and Eq. (3), respectively [14].

$$P_{\text{sense}} = \lambda \times b_i, \quad (1)$$

$$P_{Tx} = (\beta_1 + \beta_2 d_{ij}^\alpha) \times b_i^{Tx}, \quad (2)$$

$$P_{Rx} = \gamma \times b_i^{Rx}, \quad (3)$$

where  $b_i$  (in *bps*) is the data sensing rate of sensor  $v_i$ ,  $b_i^{Tx}$  and  $b_i^{Rx}$  are the data transmission rate and the reception rate of sensor  $v_i$ , respectively,  $d_{ij}$  is the Euclidean distance between sensors  $v_i$  and  $v_j$ ,  $\alpha$  is a constant that is equal to 2 or 4, and the values of other parameters are as follows [14].

$$\begin{aligned}\lambda &= 60 \times 10^{-9} \text{ J/b}, \\ \beta_1 &= 45 \times 10^{-9} \text{ J/b}, \\ \beta_2 &= 10 \times 10^{-12} \text{ J/b/m}^2, \text{ when } \alpha = 2, \\ \text{or } \beta_2 &= 1 \times 10^{-15} \text{ J/b/m}^4, \text{ when } \alpha = 4, \\ \gamma &= 135 \times 10^{-9} \text{ J/b}.\end{aligned}$$

The residual lifetime of each sensor  $v_i \in V$  at time  $t$  is defined as  $l_i(t) = \frac{RE_i(t)}{\rho_i(t)}$ , where  $RE_i(t)$  and  $\rho_i(t)$  are the amounts of residual energy and energy consumption rate of  $v_i$  at time  $t$ , respectively. The base station keeps a copy of the energy depletion rate  $\rho_i(t)$  and the residual energy  $RE_i(t)$  of each sensor  $v_i \in V$ .

We assume that each sensor is able to monitor its residual energy  $RE_i(t)$  and estimate its energy consumption rate  $\rho_i(t)$  in the near future through some prediction techniques such as linear regressions. We further assume that the energy consumption rate of each sensor does not change within a charging round, or such minor changes can be neglected as the duration of a charging round usually is short (e.g., a few hours). But the energy consumption rate of each sensor is allowed to change at a different charging round. Thus, each sensor can estimate its residual lifetime  $l_i(t)$  prior to the next charging round. Recall that for each sensor  $v_i \in V$ , there is a record of its energy consumption rate  $\rho_i(t)$  at the base station, and this value is subject to be updated if the energy consumption profile of the sensor in the future will experience significantly changes. To accurately measure the energy consumption rate of each sensor, each sensor adopts a lightweight prediction technique to estimate its energy consumption rate in the near future, e.g., a sensor can make use of a linear regression,  $\hat{\rho}_i(t) = \omega \rho_i(t-1) + (1-\omega)\hat{\rho}_i(t-1)$ , where  $\hat{\rho}_i$  is the estimation and  $\rho_i$  is the actual value at that moment and  $\omega$  is a weight between 0 and 1 [7]. Let  $\theta > 0$  be a small given threshold. For each sensor  $v_i \in V$ , the updating of its energy consumption rate is as follows. If  $|\hat{\rho}_i(t) - \hat{\rho}_i(t-1)| \leq \theta$ , no updating report from sensor  $v_i$  will be forwarded to the base station; otherwise, the updated energy consumption rate and its residual energy of  $v_i$  will be sent to the base station through a charging request is issued by  $v_i$ . The base station then performs the updating accordingly.

### 2.3 Notations and notions

A *charging scheduling* of  $q$  mobile chargers is to dispatch each of the  $q$  mobile chargers from its depot to collaboratively visit a set of to-be-charged sensors in the current round, and each charger will return to its depot after finishing its charging tour. Assume that at time  $t_j$ , let closed tours  $C_{j,1}, C_{j,2}, \dots, C_{j,q}$  be the charging tours of the  $q$  mobile chargers, where tour  $C_{j,l}$  of mobile charger  $l$  contains its depot  $r_l$  and  $1 \leq l \leq q$ . Let  $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \dots, C_{j,q}\}$  be the set of the  $q$  tours at time  $t_j$ . Notice that it is likely that some tours  $C_{j,l}$ s may contain none of the sensors, and

if so,  $V(C_{j,l}) = \{r_l\}$  and  $w(C_{j,l}) = 0$ . For the sake of simplicity, we represent each charging scheduling by a 2-tuple  $(\mathcal{C}_j, t_j)$ , where all sensors in tour  $C_{j,l} \in \mathcal{C}_j$  will be charged to their full energy capacities by mobile charger  $l$ , all the  $q$  mobile chargers are dispatched at time  $t_j$ , and  $0 < t_j < T$ . Denote by  $V(C_{j,l})$  and  $V(\mathcal{C}_j)$  the set of nodes in  $C_{j,l}$  and  $\mathcal{C}_j$ , respectively. Then,  $V(\mathcal{C}_j) = \cup_{l=1}^q V(C_{j,l})$ .

The *charging cycle* of a sensor  $v_i \in V$  is the duration between its two consecutive chargings, and its *maximum charging cycle*  $\tau_i$  is the maximum duration in which it will not run out of its energy. Since different WSNs adopt different sensing and routing protocols, different sensors may have different energy consumption rates and different maximum charging cycles. If the energy consumption rate of each sensor  $v_i \in V$  does not vary for the period of  $T$ , denote by  $\rho_i$  and  $\tau_i$  its energy consumption rate and maximum charging cycle, then  $\tau_i = \frac{B_i}{\rho_i}$ , where  $B_i$  is the energy capacity of sensor  $v_i$  and the energy consumption rate  $\rho_i$  of sensor  $v_i$  usually is determined by the data generation rate of the sensor and the sum of data rates from other sensors that the sensor must forward to the base station [5]. It is obvious that sensors with shorter maximum charging cycles need to be charged more frequently than sensors with longer maximum charging cycles. Since each time the  $q$  mobile chargers are dispatched to charge a set of sensors, they will consume their electricity or petrol, thereby incurring a service cost. We thus define the *service cost* of the  $q$  mobile chargers as the *sum of their travel distances* for charging sensors in the period of  $T$ .

## 2.4 Problem definitions

In this chapter we investigate the problem of minimizing the traveling distance of mobile chargers for a given monitoring period, and the minimum number of mobile chargers deployment problem, which are precisely defined as follows.

### 2.4.1 The Service Cost Minimization Problem

We note that not every sensor must be replenished in each charging round as the energy consumption rates of different sensors may be significantly different. Therefore, a naive strategy of charging all sensors per round will increase the service cost substantially. Also, as some to-be-charged sensors and their nearest depots in a large-scale sensor network can be far away from each other, it is crucial to schedule the  $q$  mobile chargers, by taking into account both the maximum charging cycles and the geographical locations of the sensors. We assume that each of the  $q$  mobile chargers has enough energy to charge the sensors assigned to it in each charging tour [27, 29, 30, 31].

Given a metric complete graph  $G = (V \cup R, E)$  with  $q$  mobile chargers located at  $q$  depots in  $R$ , a distance function  $w : E \mapsto \mathbb{R}^+$ , a monitoring period  $T$ , and a maximum charging cycle function  $\tau : V \mapsto \mathbb{R}^+$ , assume that the location coordi-



nates  $(x_i, y_i) \in (X, Y)$  of each sensor  $v_i \in V$  are given. *The service cost minimization problem with fixed maximum charging cycles* in  $G$  is to find a series of charging schedulings  $(\mathcal{C}_1, t_1), (\mathcal{C}_2, t_2), \dots, (\mathcal{C}_p, t_p)$  of the  $q$  mobile chargers such that the total length of all closed tours (or the service cost) is minimized, where  $p$  is a positive integer to be determined by the algorithm. Specifically, the problem can be mathematically formulated as follows.

$$\text{minimize} \quad \sum_{j=1}^p w(\mathcal{C}_j) = \sum_{j=1}^p \sum_{l=1}^q w(C_{j,l}), \quad (4)$$

subject to the following conditions: That is, for each sensor  $v_i \in V$ , we have

1. the time gap between its any two consecutive charging schedulings  $(\mathcal{C}_{j_1}, t_{j_1})$  and  $(\mathcal{C}_{j_2}, t_{j_2})$  is no more than its maximum charging cycle  $\tau_i$  (assuming that  $t_{j_1} < t_{j_2}$ ), i.e.,  $t_{j_2} - t_{j_1} \leq \tau_i$ , where sensor  $v_i$  is contained in both charging schedulings  $\mathcal{C}_{j_1}$  and  $\mathcal{C}_{j_2}$  and there is no charging scheduling  $(\mathcal{C}_j, t_j)$  such that sensor  $v_i$  is contained in  $\mathcal{C}_j$  and  $t_{j_1} \leq t_j \leq t_{j_2}$ ;
2. the duration from its last charging to the end of period  $T$  is no more than  $\tau_i$ ,

where  $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \dots, C_{j,q}\}$ ,  $C_{j,l}$  is the charging tour of mobile charger  $l$  located at depot  $r_l$ ,  $1 \leq l \leq q$ , and  $0 < t_1 < t_2 < \dots < t_p < T$ .

For this problem, we not only need to determine the number of rounds  $p$  to schedule mobile chargers for sensor charging but also to decide which sensors to be charged in which rounds and by which chargers. Intuitively, during the period of  $T$ , if more rounds are scheduled, then there are less number of sensors to-be-charged in each round. On the other hand, if less number of rounds is scheduled, there are more sensors to-be-charged in each round. Our objective is to minimize the total traveling distance of the  $q$  mobile chargers for the  $p$  charging rounds. The challenge of this optimization problem is to determine both  $p$  and the set of to-be-charged sensors in each round in order to minimize the total traveling distance of  $q$  mobile chargers. The service cost minimization problem is NP-hard, by a reduction from the well-known NP-hard problem - travelling Salesman Problem (TSP), omitted.

So far, we have assumed that the maximum charging cycle of each sensor  $v_i \in V$  in the entire period  $T$  is fixed. However, in reality, it may experience significant changes over time, since the data rates of different sensors usually depend on the specific application of a WSN, some sensors may be required to increase their data rates for better monitoring the area of these sensors at some time while the others may be required to reduced their data rates for saving their energy. For this general setting, we define *the service cost minimization problem with variable maximum charging cycles* as follows. Given a wireless sensor network  $G$ , a period  $T$ ,  $q$  mobile chargers located at  $q$  depots, the maximum charging cycle  $\tau_i(t)$  of each sensor  $v_i$  that varies with time  $t$ , the problem is to find a series of charging schedulings of the  $q$  mobile chargers such that the service cost of them is minimized, subject to that none of the sensors runs out of energy for the period of  $T$ .

We finally define a *q-rooted TSP problem*, which will be used as a subroutine for the problems of concern. Assume that there is a set of to-be-charged-sensors  $V^c \subseteq V$  at some time point. Given a subgraph  $G^c = (V^c \cup R, E^c; w)$  of  $G$  with  $|R| = q \geq 1$

and  $q$  mobile chargers, the problem is to find  $q$  closed tours  $C_1, C_2, \dots, C_q$  in  $G^c$  such that the total length of the  $q$  tours,  $\sum_{l=1}^q w(C_l)$ , is minimized, subject to that these  $q$  tours cover all sensors in  $V^c$ , i.e.,  $V^c \subseteq \bigcup_{l=1}^q V(C_l)$ , and each of the  $q$  tours contains a distinct depot in  $R$ . The  $q$ -rooted TSP problem is NP-hard as the classical TSP problem is a special case of it when  $q = 1$ .

#### 2.4.2 The Minimum Number of Mobile Chargers Deployment Problem

We notice there is no need that every sensor must be charged at each round. Also, sensor charging tours are not necessarily periodic, instead sensors should be charged in an on-demand fashion. The rationale behind this is that in some applications such as event detections, if there are no events happening in a monitoring area, sensors usually perform duty-cycling to save energy, thus they can run much longer than keeping in wake-up statuses. When an event does occur, the sensors within the event region will keep in wake-up statuses to capture the event and report their sensing results to the base station, while for the sensors not in the event region, they continue maintaining their wakeup-and-sleep duty-cycling statuses, thus consuming much less energy. It can be seen from this case that not all sensors in the network need to be charged in each energy charging round, only the sensors in the regions where the event happened are needed to be charged.

Let  $l_{max}$  be the longest duration of a mobile vehicle tour for charging all sensors in the network. Consider that all sensors in the network will be charged by only one mobile charger. Then,  $l_{max}$  should be no more than the sum of the time spent on traveling and the time spent on charging sensors on its tour by a mobile charging vehicle. Thus, the value of  $l_{max}$  is upper bounded as  $l_{max} \leq \frac{L_{TSP}}{s} + \frac{\min\{IE, \sum_{v_i \in V_s} B_i\}}{\mu}$ , where  $L_{TSP}$  is the length of a TSP tour including all sensors and the depot which can be approximately found by applying Christofides' algorithm [6],  $s$  is the travel speed of the charging vehicle,  $IE$  is the battery capacity of the vehicle,  $B_i$  is the battery capacity of sensor  $v_i$ , and  $\mu$  is the charging rate for sensors. In other words, to ensure that none of the sensors fail due to its energy expiration, a sensor should be charged when its residual lifetime is no greater than  $l_{max}$ .

We define the *critical time point* of a sensor as the time point that the sensor can survive for the next  $l_{max}$  time units. We say that a sensor  $v_i$  at time  $t$  is in a *critical lifetime interval* if  $l_{max} \leq l_i(t) \leq \alpha \cdot l_{max}$  with a given constant  $\alpha \geq 1$ , where  $l_i(t)$  is the residual lifetime of sensor  $v_i$  at time  $t$ . Following the definition of the critical lifetime interval, only the sensors within their critical lifetime intervals need to be charged to avoid running out of their energy completely. Without loss of generality, in the rest of this section, we assume that  $V$  is the set of sensors within their critical lifetime intervals, i.e.,  $V_s = \{v_i \mid v_i \in V, l_{max} \leq l_i(t) \leq \alpha \cdot l_{max}\}$ , where  $l_i(t)$  is the residual lifetime of sensor  $v_i$  at time  $t$ . Clearly,  $V_s \subseteq V$ .

We propose a flexible on-demand sensor energy charging paradigm as follows. We assume that there is only one depot  $r$  in the monitoring region, where there are a number of mobile vehicles available to meet sensor charging demands. Each sensor will send an energy-charging request to the base station for its energy re-

plenishment when its the residual lifetime is below the critical lifetime  $l_{max}$ . The energy-charging request contains the identity, the amount of residual energy, and the energy consumption rate of the sensor. Once the base station receives a set of such requests from the sensors, it then performs a scheduling to dispatch a number of mobile charging vehicles to charge the sensors in the set, where a sensor  $v_i$  at time  $t$  is in its critical lifetime interval if  $l_{max} \leq l_i(t) \leq \alpha \cdot l_{max}$ . Hence, the result of each scheduling consists of the number of mobile charging vehicles needed, a closed tour for each of the mobile vehicle, and the charging duration at each to-be-charged sensor node along the tour. Finally, the mobile charging vehicles are dispatched from the depot to perform charging tasks.

Given a rechargeable sensor network  $G = (V, E)$  consisting of sensors, one stationary base station, and a depot with multiple mobile vehicles, following the on-demand sensor energy charging paradigm, assume that at a specific time point, the base station receives charging requests from the sensors within their critical lifetime intervals. The base station then starts a new round scheduling by dispatching a certain number of mobile charging vehicles to charge these sensors so that none of sensors will run out of energy. Let  $V_s$  be the subset of sensors in  $G$  to-be-charged (within their critical lifetimes) in the next round ( $V_s \subseteq V$ ). Assume that for each sensor  $v_i \in V_s$ , its energy consumption rate  $\rho_i$  does not change during each charging round (or such changes are marginal and can be ignored), and its residual energy  $RE_i$  is given (at the base station), *the minimum number of mobile chargers deployment problem* is to find a scheduling of mobile charging vehicles to fully charge the sensors in  $V_s$  by providing a closed tour for each vehicle, such that the number of mobile vehicles deployed is minimized, subject to the energy capacity constraint  $IE$  on each mobile vehicle. The minimum number of mobile chargers deployment problem is NP-hard, through a reduction from the well-known NP-hard Travelling Salesman Problem (TSP).

The rest is to define the  $p$ -optimal closed tour problem, which will serve as a subroutine of the proposed algorithms for the minimum number of mobile chargers deployment problem. Given a node and edge weighted complete metric graph  $G_s = (V_s, E_s; h, w)$ , a root node  $r \in V_s$ , and an integer  $p \geq 1$ , where  $h : V_s \mapsto \mathbb{R}^{\geq 0}$  and  $w : E_s \mapsto \mathbb{R}^{\geq 0}$  (i.e., the node weight  $h(v)$  of each sensor node  $v \in V_s$  is the amount of energy to be charged to sensor  $v$ , and the edge weight  $w(u, v)$  of each edge  $(u, v) \in E_s$  represents the amount of energy consumed by a mobile vehicle travelling along the edge), *the  $p$ -optimal closed tour problem* in  $G_s$  is to find  $p$  node-disjoint closed tours covering all nodes in  $V_s$  except the root  $r$  that appears in each of the tours such that the maximum total cost among the  $p$  closed tours is minimized, where *the total cost* of a closed tour is the weighted sum of nodes and edges in it.

### 3 Algorithms for the Service Cost Minimization Problem

In this section, we devise efficient algorithms for the service cost minimization problem. We first devise an algorithm for a  $q$ -rooted TSP problem in Subsection 3.1, which will be served as a subroutine of the proposed algorithms. We then propose

an approximation algorithm in Subsection 3.2 and a heuristic algorithm in Subsection 3.3 for the problem under fixed and variable sensor energy consumption rates, respectively.

### 3.1 Algorithm for the $q$ -rooted TSP problem

We propose a 2-approximation algorithm for the  $q$ -rooted TSP problem, which will serve as a subroutine of the approximation algorithm for the service cost minimization problem.

The basic idea of the algorithm for the  $q$ -rooted TSP problem is that we first find  $q$ -rooted trees with the minimum total cost, and we then show that the total cost of the  $q$ -rooted trees is a lower bound on the optimal cost of the  $q$ -rooted TSP problem. We finally convert each of the trees into a closed tour with the cost of the tour no more than twice the cost of the tree.

We start with the  *$q$ -rooted minimum spanning forest ( $q$ -rooted MSF) problem*: given a graph  $G^c = (V^c \cup R, E^c; w)$ ,  $q = |R|$ , and  $w : E^c \mapsto \mathbb{R}^+$ , the problem is to find  $q$  trees  $T_1, T_2, \dots, T_q$  spanning all nodes in  $V^c$  with each tree containing a distinct depot in  $R$  such that the total cost of the  $q$  trees,  $\sum_{l=1}^q w(T_l)$ , is minimized.

For the  $q$ -rooted MSF problem, an exact algorithm is given as follows. We start by constructing an auxiliary graph  $G_r = (V^c \cup \{r\}, E_r; w_r)$  from  $G^c = (V^c \cup R, E^c; w)$  by contracting the  $q$  depots in  $R$  into a single root  $r$ : (i) remove the  $q$  depots in  $R$  and introduce a new node  $r$ ; (ii) for each  $r_l \in R$ , introduce an edge  $(v, r) \in E_r$  for each edge  $(v, r_l) \in E^c$ , where  $v \in V^c$ ; (iii)  $w_r(v, r) = \min_l \{w(v, r_l)\}$ . We then find an MST  $T$  of  $G_r$ . We finally break  $T$  into  $q$  disjoint trees  $T_1, T_2, \dots, T_q$  by un-contracting the roots in  $R$ . This un-contraction means that an edge  $(v, r)$  is mapped to an edge  $(v, r_l)$ , where  $w_r(v, r) = w(v, r_l)$ . Note that each tree  $T_l$  roots at depot  $r_l$ . The detailed algorithm is presented in Algorithm 1.

---

#### Algorithm 1: $q$ -rooted MSF

---

**Input:**  $G^c = (V^c \cup R, E^c; w)$ ,  $w : E^c \mapsto \mathbb{R}^+$ , and  $q = |R|$ .

**Output:** a solution for the  $q$ -rooted MSF problem

- 1 Construct a graph  $G_r = (V^c \cup \{r\}, E_r; w_r)$  from  $G^c$  by contracting the  $q$  depots in  $R$  into a single root  $r$ ;
  - 2 Find an MST  $T$  in  $G_r$ ;
  - 3 Decompose the MST  $T$  into  $q$  disjoint rooted trees  $T_1, T_2, \dots, T_q$  by un-contracting depots in  $R$ ;
- 

**Lemma 1.** *There is an algorithm for the  $q$ -rooted MSF problem, which delivers an optimal solution and takes  $O(n^2)$  time, where  $n = |V^c \cup R|$ .*

*Proof.* Assume that trees  $T_1^*, T_2^*, \dots, T_q^*$  form an optimal solution to the  $q$ -rooted MSF problem. We show that the solution consisting of trees  $T_1, T_2, \dots, T_q$ , delivered by Algorithm 1, is optimal. On one hand, since the  $q$  trees  $T_1, T_2, \dots, T_q$  form a feasible solution, then  $\sum_{l=1}^q w(T_l^*) \leq \sum_{l=1}^q w(T_l)$ . On the other hand, as each tree  $T_l^*$  contains a depot  $r_l \in R$ , we can construct a spanning tree  $T'$  in graph  $G_r$  by

contracting the  $q$  depots into a single root  $r$ , and  $w(T') = \sum_{l=1}^q w(T_l^*)$ . As the MST  $T$  is the minimum one, we have  $w(T) \leq w(T')$ . Since  $\sum_{l=1}^q w(T_l) = w(T)$ ,  $\sum_{l=1}^q w(T_l) = w(T) \leq w(T') \leq \sum_{l=1}^q w(T_l^*)$ . Therefore,  $\sum_{l=1}^q w(T_l) = \sum_{l=1}^q w(T_l^*)$ , i.e., the found trees  $T_1, T_2, \dots, T_q$  form an optimal solution to the problem. The time complexity of Algorithm 1 is analyzed as follows. Constructing graph  $G_r$  takes time  $O(E^c) = O(n^2)$ . Finding the MST  $T$  in  $G_r$  takes  $O(n^2)$  time, while un-contracting the MST  $T$  also takes time  $O(E^c) = O(n^2)$ . Algorithm 1 thus runs in  $O(n^2)$  time.  $\square$

With the help of the exact algorithm for the  $q$ -rooted MSF problem, we now devise a 2-approximation algorithm for the  $q$ -rooted TSP problem in Algorithm 2.

---

**Algorithm 2:**  $q$ -rooted TSP

---

**Input:**  $G^c = (V^c \cup R, E^c; w)$ ,  $w : E^c \mapsto \mathbb{R}^+$ , and  $q = |R|$ .

**Output:** A solution  $\mathcal{C}$  for the  $q$ -rooted TSP problem

- 1 Find  $q$  optimal trees  $T_1, T_2, \dots, T_q$  for the  $q$ -rooted MSF problem in  $G^c$  by calling Algorithm 1;
  - 2 For each tree  $T_l$ , double the edges in  $T_l$ , find a Eulerian tour  $C'_l$ , and obtain a less cost closed tour  $C_l$  by short-cutting repeated nodes in  $C'_l$ . Let  $\mathcal{C} = \{C_1, C_2, \dots, C_q\}$ ;
- 

We show that Algorithm 2 delivers a 2-approximate solution.

**Theorem 1.** *There is a 2-approximation algorithm for the  $q$ -rooted TSP problem, which takes time  $O(|V^c \cup R|^2)$ .*

*Proof.* Assume that closed tours  $C_1^*, C_2^*, \dots, C_q^*$  form an optimal solution to the  $q$ -rooted TSP problem in  $G^c$ . For each tour  $C_l^*$ , we can obtain a tree  $T'_l$  by removing any edge in  $C_l^*$ . Then,  $w(T'_l) \leq w(C_l^*)$ ,  $1 \leq l \leq q$ . It is obvious that trees  $T'_1, T'_2, \dots, T'_q$  form a feasible solution to the  $q$ -rooted MSF problem. As trees  $T_1, T_2, \dots, T_q$  form the optimal solution by Lemma 1,  $\sum_{l=1}^q w(T_l) \leq \sum_{l=1}^q w(T'_l) \leq \sum_{l=1}^q w(C_l^*)$ . Also, we can see that the total cost of each found tour  $C_l$  is no more than twice the total cost of tree  $T_l$ , i.e.,  $w(C_l) \leq 2w(T_l)$ . Therefore,  $\sum_{l=1}^q w(C_l) \leq \sum_{l=1}^q 2w(T_l) \leq 2\sum_{l=1}^q w(C_l^*)$ . The time complexity analysis is straightforward, omitted.  $\square$

### 3.2 Approximation algorithm with fixed maximum charging cycles

In this subsection, we devise an approximation algorithm for the service cost minimization problem, assuming that each sensor has a fixed maximum charging cycle. We start with the basic idea behind the algorithm. We then present the approximation algorithm, and we finally analyze the approximation ratio of the proposed approximation algorithm.

#### 3.2.1 Overview of the approximation algorithm

Given a maximum charging cycle function:  $\tau : V \mapsto \mathbb{R}^+$  and a monitoring period  $T$ , if there is a series of mobile charger schedulings for  $T$  such that no sensor depletes its

energy, then we say that these schedulings form a *feasible solution* to the service cost minimization problem, i.e., for each sensor  $v_i \in V$ , the maximum duration between its any two consecutive chargings is no more than  $\tau_i$ . A *series of feasible charging schedulings* of the  $q$  mobile chargers is an *optimal solution* if the service cost of the solution is the minimum one.

The basic idea behind the proposed approximation algorithm is to construct another charging cycle function  $\tau'(\cdot)$  for the sensors based on the maximum charging cycle function  $\tau(\cdot)$ , by exploring the combinatorial property of the problem. We construct a very special charging cycle function  $\tau'(\cdot)$  such that charging cycles of the  $n$  sensors will form a geometric sequence as follows.

Let  $\tau_1, \tau_2, \dots, \tau_n$  be the maximum charging cycles of sensors  $v_1, v_2, \dots, v_n$  in the network. Assume that  $\tau_1 \leq \tau_2 \leq \dots \leq \tau_n$ . Let  $\tau'_1, \tau'_2, \dots, \tau'_n$  be the charging cycles of the sensors and  $\tau'_i \leq \tau'_j$  if  $\tau_i \leq \tau_j$ . We construct  $\tau'(\cdot)$  as follows. We partition the set  $V$  of the sensors into  $K+1$  disjoint subsets  $V_0, V_1, \dots, V_K$ , where  $K = \lfloor \log_2 \frac{\tau_n}{\tau_1} \rfloor$ , and sensor  $v_i \in V$  with its maximum charging cycle  $\tau_i$  is contained in  $V_k$  if  $2^k \tau_1 \leq \tau_i < 2^{k+1} \tau_1$ . Then,  $k = \lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor$ . Let  $\tau'_i = 2^k \tau_1$ . We assign each sensor in  $V_k$  with the identical charging cycle  $2^k \tau_1 = 2^k \tau_1$ . Consequently, the charging cycles of sensors in  $V_0, V_1, \dots, V_K$  are  $\tau_1, 2\tau_1, \dots, 2^K \tau_1$ , respectively. We can see that the assigned charging cycle  $\tau'_i$  of sensor  $v_i$  is no less than the half its maximum charging cycle  $\tau_i$ , since

$$\tau'_i = 2^{\lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor} \tau_1 > 2^{\log_2 \frac{\tau_i}{\tau_1} - 1} \tau_1 = \frac{\tau_i}{2}, \quad \forall v_i \in V. \quad (5)$$

### 3.2.2 Approximation algorithm

Given the charging cycle function  $\tau'(\cdot)$ , we can see that  $\tau'_i$  is divisible by  $\tau'_j$  for any two sensors  $v_i$  and  $v_j$  if  $\tau_i \leq \tau_j$  and  $1 \leq i < j \leq n$ . For simplicity, assume that the monitoring period  $T$  is divisible by the maximum assigned charging cycle  $\tau'_n$ , let  $T = 2m\tau'_n = 2m2^K \tau_1$ , where  $m$  is a positive integer. Furthermore, we assume that each sensor is fully charged at time  $t = 0$ . The solution delivered by the proposed algorithm consists of a series of schedulings of the  $q$  mobile chargers. Specifically, we first find a sequence of schedulings for a period  $\tau'_n$ . Then, we repeat the found schedulings for the next time period of  $\tau'_n$ , and so on. We repeat the scheduling sequence for period  $T$  no more than  $\lfloor T/\tau'_n \rfloor - 1 = 2m - 1$  times.

In the following, we construct a series of schedulings for a period  $\tau'_n = 2^K \tau_1$ . Recall that we have partitioned the sensor set  $V$  into  $K+1$  disjoint subsets  $V_0, V_1, \dots, V_K$ , and the charging cycle of each sensor in  $V_k$  is  $2^k \tau_1$ ,  $0 \leq k \leq K$ . We further partition the period  $\tau'_n$  into  $2^K$  equal time intervals with each interval lasting  $\tau_1$ , and label them from the left to right as the 1st, 2nd, ..., and the  $2^K$ th time interval. Clearly, all sensors in  $V_0$  must be charged at each of these  $2^K$  time intervals; all sensors in  $V_1$  must be charged at every second time interval; and all sensors in  $V_k$  must be charged at every  $2^k$  time interval,  $0 \leq k \leq K$ . That is,

At time  $\tau_1$ , charge the sensors in  $V_0$ .  
 At time  $2\tau_1$ , charge the sensors in  $V_0 \cup V_1$ .  
 At time  $3\tau_1$ , charge the sensors in  $V_0$ .  
 At time  $4\tau_1$ , charge the sensors in  $V_0 \cup V_1 \cup V_2$ .  
 $\vdots$   
 At time  $j\tau_1$ , charge the sensors in  $\cup_{(j \bmod 2^k)=0} V_k$  where  $0 \leq k \leq K'$ ,  $K' = \lfloor \log_2 j \rfloor$ , and  $1 \leq j \leq 2^K$ .  
 $\vdots$   
 At time  $2^K \tau_1$ , charge the sensors in  $\cup_{i=0}^K V_i = V$ .

There are  $2^K$  charging schedulings of the  $q$  mobile chargers and one charging scheduling is dispatched at each time interval. Let  $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \dots, C_{j,q}\}$  be the set of closed tours of the  $q$  mobile chargers at time interval  $j$ , where  $1 \leq j \leq 2^K$ . Furthermore, it can be seen that in the  $2^K$  charging schedulings, there are  $2^{K-1}$  identical charging schedulings with each only containing the sensors in  $V_0$ , there are  $2^{K-2}$  identical charging schedulings with each containing the sensors only in  $V_0 \cup V_1$ . In general, there are  $2^{K-1-k}$  identical charging schedulings with each containing the sensors only in  $V_0 \cup V_1 \dots \cup V_k$ ,  $0 \leq k \leq K-1$ . Finally, there is one charging scheduling containing the sensors in  $V_0 \cup V_1 \dots \cup V_K = V$ . Denote by  $\mathcal{D}_k = \{D_{k,1}, D_{k,2}, \dots, D_{k,q}\}$  the set of  $q$  closed tours for the  $q$ -rooted TSP problem in the induced graph  $G[R \cup V_0 \dots \cup V_k]$ , which is delivered by Algorithm 2.

The series of charging schedulings for a period  $\tau'_n$  thus is  $(\mathcal{C}_1, \tau_1), \dots, (\mathcal{C}_j, j\tau_1), \dots, (\mathcal{C}_{2^K}, 2^K \tau_1)$ , where the 2-tuple  $(\mathcal{C}_j, j\tau_1)$  represents that the  $q$  mobile chargers are dispatched at time  $j\tau_1$  and the set of to-be-charged sensors is  $\cup_{C_{j,i} \in \mathcal{C}_j} V(C_{j,i}) = \cup_{(j \bmod 2^k)=0} V_k$ ,  $0 \leq k \leq K'$ ,  $K' = \lfloor \log_2 j \rfloor$ , and  $1 \leq j \leq 2^K$ . As a result, there are  $p = 2m \cdot 2^K - 1$  charging schedulings found for a period of  $T = 2m\tau'_n$  as follows.

$(\mathcal{C}_1, \tau_1), \dots, (\mathcal{C}_{2^K-1}, (2^K-1)\tau_1), (\mathcal{C}_{2^K}, 2^K \tau_1),$   
 $(\mathcal{C}_1, \tau'_n + \tau_1), \dots, (\mathcal{C}_{2^K-1}, \tau'_n + (2^K-1)\tau_1), (\mathcal{C}_{2^K}, \tau'_n + 2^K \tau_1),$   
 $\vdots$   
 $(\mathcal{C}_1, (2m-1)\tau'_n + \tau_1), \dots, (\mathcal{C}_{2^K-1}, (2m-1)\tau'_n + (2^K-1)\tau_1).$

Note that we do not perform a charging scheduling at time  $T = 2m\tau'_n$  as there is no such need at the end of period  $T$ . The proposed algorithm is described in Algorithm 3.

### 3.2.3 Algorithm analysis

In the following we dedicate ourselves to analyzing the approximation ratio of the proposed approximation algorithm. We start by showing that Algorithm 3 delivers a feasible solution to the service cost minimization problem by Lemma 2. We then provide a lower bound on the minimum cost of the problem by Lemma 3. We finally derive the approximation ratio of Algorithm 3 based on the lower bound in Theorem 2.

**Algorithm 3:** *MinDis*


---

**Input:**  $G = (V \cup R, E; w)$ , maximum charging cycles  $\tau : V \mapsto \mathbb{R}^+$ ,  $q$  chargers, and a monitoring period  $T$ .

**Output:** A series of charging schedulings  $\mathcal{C}$  for period  $T$

- 1 Let  $\tau_1, \tau_2, \dots, \tau_n$  be the sorted maximum charging cycles of sensors  $v_1, v_2, \dots, v_n$  in ascending order;
- 2 For each sensor  $v_i$ , let  $\tau'_i = 2^{\lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor}$ ;
- 3 Partition sensors in  $V$  into  $K + 1$  disjoint subsets  $V_0, V_1, \dots, V_K$ , where sensor  $v_i \in V_k$  if  $2^k \tau_1 = 2^{\lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor} \tau_1$ ,  $0 \leq k \leq K$ , and  $K = \lfloor \log_2 \frac{\tau_n}{\tau_1} \rfloor$ . All sensors in  $V_k$  have the same charging cycle  $2^k \tau'_1$ ;
- 4 **for**  $k \leftarrow 0$  **to**  $K$  **do**
- 5     Find  $q$  charging tours  $\mathcal{D}_k = \{D_{k,1}, D_{k,2}, \dots, C_{j,q}\}$  in the induced subgraph  $G[R \cup V_0 \dots \cup V_k]$  by applying Algorithm 2;
- 6 **end for**
- 7  $\mathcal{C} \leftarrow \emptyset$ ; /\* the solution \*/
- 8 /\* Construct schedulings  $(\mathcal{C}_1, \tau_1), \dots, (\mathcal{C}_{2^K}, 2^K \tau_1)$  \*/
- 9 **for**  $j \leftarrow 1$  **to**  $2^K$  **do**
- 10     /\* Find the charging scheduling  $\mathcal{C}_j$  of the  $q$  mobile chargers at time  $t_j = j\tau_1$  \*/;
- 11     Let  $\mathcal{C}_j = \mathcal{D}_k$ , where  $k$  is the largest integer so that  $j \bmod 2^k = 0$ , where  $0 \leq k \leq K'$  and  $K' = \lfloor \log_2 j \rfloor$ ;
- 12      $\mathcal{C} \leftarrow \mathcal{C} \cup \{(\mathcal{C}_j, t_j)\}$ ;
- 13 **end for**
- 14 **for**  $m' \leftarrow 2$  **to**  $\lfloor T/\tau'_n \rfloor$  **do**
- 15     **for**  $j \leftarrow 1$  **to**  $2^K$  **do**
- 16          $\mathcal{C} = \mathcal{C} \cup \{(\mathcal{C}_j, m' \cdot \tau'_n + t_j)\}$ ;
- 17     **end for**
- 18 **end for**
- 19 **return**  $\mathcal{C}$ .

---

**Lemma 2.** Algorithm 3 delivers a feasible solution to the service cost minimization problem.

*Proof.* It is obvious that the solution delivered by Algorithm 3 is feasible, as the charging cycle  $\tau'_i$  of each sensor  $v_i \in V$  in the solution is no more than its maximum charging cycle  $\tau_i$ , i.e.,  $\tau'_i \leq \tau_i$ . Thus, no sensors will die in the period  $T$ .  $\square$

The following lemma provides a lower bound on the optimal service cost, which bounds the service cost of the solution delivered by Algorithm 3.

**Lemma 3.** Given the sensor set partitioning  $V_0, V_1, \dots, V_K$  based on the maximum charging cycles of sensors, each sensor in  $V_k$  is assigned with the same charging cycle  $2^k \tau_1$ ,  $0 \leq k \leq K$ . Let  $OPT$  be the service cost of an optimal solution to the service cost minimization problem. Denote by  $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \dots, D_{k,q}^*\}$



the optimal  $q$  closed tours for the  $q$ -rooted TSP problem in the induced graph  $G[R \cup V_0 \cup V_1 \cup \dots \cup V_k]$ , then  $OPT \geq m2^{K-k} \cdot w(\mathcal{D}_k^*)$ , assuming that  $T = 2m\tau'_n$ , where  $w(\mathcal{D}_k^*) = \sum_{l=1}^q w(D_{k,l}^*)$ ,  $K = \lfloor \log_2 \frac{\tau_n}{\tau_1} \rfloor$ , and  $0 \leq k \leq K$ .

*Proof.* To show that  $OPT \geq m2^{K-k} \cdot w(\mathcal{D}_k^*)$ , we partition the entire period  $T = 2m\tau'_n = 2m \cdot 2^K \tau_1$  into  $m \cdot 2^{K-k}$  time intervals with each lasting time  $t_k = 2^{k+1} \tau_1$ . Let  $(0, t_k], (t_k, 2t_k], \dots, ((j-1)t_k, jt_k], \dots, ((m2^{K-k}-1)t_k, m2^{K-k}t_k]$  be these  $m \cdot 2^{K-k}$  intervals, where time interval  $j$  is the interval  $((j-1) \cdot t_k, j \cdot t_k]$ ,  $1 \leq j \leq m \cdot 2^{K-k}$ . Note that  $m2^{K-k}t_k = m2^{K-k}2^{k+1}\tau_1 = T$ .

In the following, we first show that there is at least one time interval among the  $m2^{K-k}$  time intervals such that (i) the service cost of charging schedulings within the interval is no more than  $\frac{1}{m2^{K-k}}$  of the service cost  $OPT$  in the optimal solution; (ii) each sensor in  $\bigcup_{i=0}^k V_i$  must be charged at least once in this interval; and (iii) the service cost within this interval in the optimal solution is no less than the cost  $w(\mathcal{C}_k)$  of a feasible solution  $\mathcal{C}_k$  to the  $q$ -rooted TSP problem in graph  $G[R \cup V_0 \cup \dots \cup V_k]$ . Since  $\mathcal{D}_k^*$  is the optimal solution to the  $q$ -rooted TSP problem,  $w(\mathcal{D}_k^*) \leq w(\mathcal{C}_k) \leq \frac{OPT}{m2^{K-k}}$ .

Assume that an optimal solution consists of  $p$  charging schedulings  $(\mathcal{C}_1^*, t_1^*), (\mathcal{C}_2^*, t_2^*), \dots, (\mathcal{C}_p^*, t_p^*)$  with  $0 < t_1^* \leq \dots \leq t_p^* < T$ . Recall that  $OPT$  is the sum of lengths of the  $p$  charging schedulings i.e.,  $OPT = \sum_{s=1}^p w(\mathcal{C}_s^*) = \sum_{s=1}^p \sum_{l=1}^q w(C_{s,l}^*)$ . We partition the  $p$  charging schedulings into  $m2^{K-k}$  disjoint groups according to their dispatching times, the charging scheduling  $\mathcal{C}_s^*$  is in group  $j$  if its dispatching time  $t_s^*$  is within time interval  $j$ , i.e.,  $t_s^* \in ((j-1)t_k, jt_k]$ , where  $1 \leq s \leq p$  and  $1 \leq j \leq m2^{K-k}$ . Denote by  $\mathcal{G}_j$  and  $w(\mathcal{G}_j)$  the set of charging schedulings in group  $j$  and the cost sum of charging schedulings in  $\mathcal{G}_j$ , respectively, i.e.,  $w(\mathcal{G}_j) = \sum_{\mathcal{C}_s^* \in \mathcal{G}_j} w(\mathcal{C}_s^*)$ ,  $1 \leq j \leq m2^{K-k}$ . Then,  $\sum_{j=1}^{m2^{K-k}} w(\mathcal{G}_j) = OPT$ . Among the  $m2^{K-k}$  groups, there must be a group  $\mathcal{G}_j$  whose service cost  $w(\mathcal{G}_j)$  is no more than  $\frac{1}{m2^{K-k}}$  of the optimal cost  $OPT$ , i.e.,

$$w(\mathcal{G}_j) \leq \frac{OPT}{m2^{K-k}}. \quad (6)$$

We then show that each sensor in  $\bigcup_{i=0}^k V_i$  must be charged at least once by the charging schedulings in  $\mathcal{G}_j$  by contradiction. Assume that there is a sensor  $v_i \in \bigcup_{i=0}^k V_i$  which will not be charged by any charging scheduling in  $\mathcal{G}_j$ . Since  $v_i \in \bigcup_{i=0}^k V_i$ , its maximum charging cycle  $\tau_i$  must be strictly less than  $2 \cdot 2^k \tau_1 = 2^{k+1} \tau_1$  by inequality (5), i.e.,  $\tau_i < 2^{k+1} \tau_1$ . On the other hand, as  $v_i$  will not be charged by any charging scheduling in  $\mathcal{G}_j$  while it is still survived, this implies that its maximum charging cycle must be no less than the length  $t_k$  of the time interval, i.e.,  $\tau_i \geq t_k = 2^{k+1} \tau_1$ , this results in a contradiction. Thus,  $v_i$  must be charged by at least one charging scheduling in  $\mathcal{G}_j$ .

We finally construct a feasible solution  $\mathcal{C}_k = \{C_{k,1}, C_{k,2}, \dots, C_{k,q}\}$  to the  $q$ -rooted TSP problem in graph  $G[R \cup V_0 \cup \dots \cup V_k]$  based on the charging schedulings in  $\mathcal{G}_j$  such that the service cost  $w(\mathcal{C}_k)$  is no more than  $w(\mathcal{G}_j)$ . Since each closed tour in  $\mathcal{G}_j$  contains a depot  $r_l \in R$ , we partition the closed tours in  $\mathcal{G}_j$  by the depot that each tour contains. To this end, we partition tours in  $\mathcal{G}_j$  into  $q$  disjoint subgroups

$\mathcal{G}_{j,1}, \mathcal{G}_{j,2}, \dots, \mathcal{G}_{j,q}$ , where subgroup  $\mathcal{G}_{j,l}$  includes all closed tours in  $\mathcal{G}_j$  that contains depot  $r_l$ ,  $1 \leq l \leq q$ . For each subgroup  $\mathcal{G}_{j,l}$ , since each tour contains depot  $r_l$ , the union of all close tours in  $\mathcal{G}_{j,l}$  forms a connected Eulerian graph. Then, we can derive a Eulerian circuit  $C'_{k,l}$  from this Eulerian graph and  $w(C'_{k,l}) = w(\mathcal{G}_{j,l})$ . We further obtain a closed tour  $C_{k,l}$  including only nodes in  $R \cup V_0 \cup \dots \cup V_k$  once from  $C'_{k,l}$ , by the removal of the nodes not in  $R \cup V_0 \cup \dots \cup V_k$  and the nodes with multiple appearances, and performing path short-cutting. Since edge weights satisfy the triangle inequality, we have

$$w(C_{k,l}) \leq w(C'_{k,l}) \leq w(\mathcal{G}_{j,l}), \quad 1 \leq l \leq q. \quad (7)$$

As each sensor in  $\bigcup_{i=0}^k V_i$  will be charged at least once by the charging schedulings in  $\mathcal{G}_j$ , and tour  $C_{k,l}$  contains depot  $r_l$ , we have  $\bigcup_{i=0}^k V_i \subseteq \bigcup_{l=1}^q V(C_{k,l})$ . Then, all tours in  $\mathcal{G}_k$  form a feasible solution to the  $q$ -rooted TSP problem in graph  $G[R \cup V_0 \cup \dots \cup V_k]$ . Let  $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \dots, D_{k,q}^*\}$  be the optimal  $q$  tours. Then,

$$\sum_{l=1}^q w(D_{k,l}^*) \leq \sum_{l=1}^q w(C_{k,l}). \quad (8)$$

By combining inequalities (6), (7), and (8), the lemma then follows.  $\square$

According to Lemmas 2 and 3, we show the approximation ratio of Algorithm 3 by the following theorem.

**Theorem 2.** *There is a  $2(K+2)$ -approximation algorithm for the service cost minimization problem with fixed maximum charging cycles, which takes time  $O(\lfloor \log \frac{\tau_{\max}}{\tau_{\min}} \rfloor n^2 + \frac{T}{\tau_{\min}} n)$ , where  $\tau_{\max} = \max_{i=1}^n \{\tau_i\}$ ,  $\tau_{\min} = \min_{i=1}^n \{\tau_i\}$ , and  $K = \lfloor \log_2 \frac{\tau_n}{\tau_1} \rfloor$ .*

*Proof.* By Lemma 2, Algorithm 3 delivers a feasible solution. The rest is to analyze its approximation ratio. Recall that the charging schedulings delivered by Algorithm 3 for period  $T = 2m\tau'_n$  are:  $(\mathcal{C}_1, \tau_1), \dots, (\mathcal{C}_{2^K}, 2^K \tau_1), (\mathcal{C}_1, \tau'_n + \tau_1), \dots, (\mathcal{C}_{2^K}, \tau'_n + 2^K \tau_1), \dots, (\mathcal{C}_1, (2m-1)\tau'_n + \tau_1), \dots, (\mathcal{C}_{2^K-1}, (2m-1)\tau'_n + (2^K-1)\tau_1)$ . The total service cost during  $T$  then is

$$(2m-1) \sum_{j=1}^{2^K} w(\mathcal{C}_j) + \sum_{j=1}^{2^K-1} w(\mathcal{C}_j) \leq 2m \sum_{j=1}^{2^K} w(\mathcal{C}_j). \quad (9)$$

Recall that  $\mathcal{D}_k = \{D_{k,1}, D_{k,2}, \dots, D_{k,q}\}$  is the set of  $q$  closed tours for the  $q$ -rooted TSP problem in graph  $G[R \cup V_0 \cup \dots \cup V_k]$  delivered by Algorithm 2. Let  $\mathcal{C}(\tau'_n) = \{(\mathcal{C}_1, \tau'_1), (\mathcal{C}_2, \tau'_2), \dots, (\mathcal{C}_{2^K}, \tau'_n)\}$ .

From the construction of  $\mathcal{C}(\tau'_n)$ , we can see that there are  $2^{K-1-k}$  identical charging schedulings in  $\mathcal{C}(\tau'_n)$  with each only containing the nodes in  $R \cup V_0 \cup V_1 \dots \cup V_k$ . Denote by  $w(\mathcal{D}_k)$  the cost of the charging scheduling  $\mathcal{D}_k$ , where  $0 \leq k \leq K-1$ . And there is one charging scheduling in  $\mathcal{C}(\tau'_n)$  containing the nodes in  $R \cup V_0 \cup \dots \cup V_K = R \cup V$ , denote by  $w(\mathcal{D}_K)$  the cost of the charging scheduling  $\mathcal{D}_K$ . We then rewrite the upper bound on the service cost in Inequality (9) as

$$2m \sum_{j=1}^{2^K} w(\mathcal{C}_j) = 2m(w(\mathcal{D}_K) + \sum_{k=0}^{K-1} 2^{K-1-k} w(\mathcal{D}_k)). \quad (10)$$

Denote by  $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \dots, D_{k,q}^*\}$  the set of the optimal  $q$  closed tours for the  $q$ -rooted TSP problem in graph  $G[R \cup V_0 \cup \dots \cup V_k]$ . Since  $\mathcal{D}_k$  is an approximate solution by Theorem 1,  $w(\mathcal{D}_k) \leq 2w(\mathcal{D}_k^*)$ ,  $0 \leq k \leq K$ . Also, by Lemma 3,  $w(\mathcal{D}_k^*) \leq \frac{OPT}{m2^{K-k}}$ . We have

$$\begin{aligned} & 2m(w(\mathcal{D}_K) + \sum_{k=0}^{K-1} 2^{K-1-k} w(\mathcal{D}_k)) \\ & \leq 4m\left(\frac{OPT}{m} + \sum_{k=0}^{K-1} 2^{K-1-k} \frac{OPT}{m2^{K-k}}\right) = 2(K+2)OPT. \end{aligned} \quad (11)$$

The time complexity analysis is straightforward, omitted.  $\square$

### 3.3 Heuristic algorithm with variable maximum charging cycles

So far we have developed an approximation algorithm for the service cost minimization problem, assuming that the maximum charging cycle of each sensor is fixed for the given monitoring period. This assumption however sometimes may be restrictive and unrealistic in some applications. In this subsection we devise a novel heuristic algorithm by removing this assumption.

#### 3.3.1 Heuristic algorithm

Within the period  $T$ , the energy consumption rates of sensors may dynamically change over time, resulting in the changes of sensor maximum charging cycles eventually. Recall that the base station maintains the updated energy information of each sensor, including its residual energy and energy consumption rate. Also, the sensor sends an updating request of its energy information to the base station if the variation of its maximum charging cycle is beyond a pre-defined threshold.

Assume that the base station receives the maximum charging cycle updates from some sensors at time  $t$ , this implies that the charging schedulings based on the previous maximum charging cycles of these sensors may not be applicable any more, otherwise these sensors will deplete their energy prior to their next chargings. For example, assume that a sensor has changed its maximum charging cycle from a longer one to a shorter one, it might be dead if the sensor is still charged according to its previous longer charging cycle since the sensor now can last for only a shorter cycle once it is fully charged.

The basic idea of the heuristic algorithm is as follows. When the base station receives maximum charging cycle updates, it checks whether the previous schedulings are still applicable for these updated maximum charging cycles. If so, nothing needs to be done. Otherwise, it re-computes a new series of schedulings, by first applying the approximation algorithm based on the updated maximum charging cycles, followed by modifications to the solution delivered by the approximation algorithm.

Assume that the previous maximum charging cycle of sensor  $v_i$  is  $\hat{\tau}_i(t-1)$  and it was charged at a charging cycle  $\hat{\tau}'_i(t-1)$  in the previous series of schedulings. At time  $t$ , the base station receives the maximum charging cycle updating of sensor  $v_i$ , which changes from  $\hat{\tau}_i(t-1)$  to  $\hat{\tau}_i(t)$ . The base station then checks the feasibility of the previous schedulings as follows. If  $\hat{\tau}'_i(t-1) \leq \hat{\tau}_i(t) < 2\hat{\tau}'_i(t-1)$ , the previous schedulings are still feasible as sensor  $v_i$  will be charged with a charging cycle  $\hat{\tau}'_i(t-1)$  no more than its current maximum charging cycle  $\hat{\tau}_i(t)$ . Otherwise ( $\hat{\tau}_i(t) < \hat{\tau}'_i(t-1)$  or  $\hat{\tau}_i(t) \geq 2\hat{\tau}'_i(t-1)$ ), we re-compute a new series of schedulings based on the updated maximum charging cycles since the previous schedulings are not feasible any more (i.e.,  $\hat{\tau}_i(t) < \hat{\tau}'_i(t-1)$ ), or though the schedulings still are feasible, they are not optimal in terms of the service cost (i.e.,  $\hat{\tau}_i(t) \geq 2\hat{\tau}'_i(t-1)$ ). In the following, we re-compute a new series of schedulings.

We first invoke the proposed approximation algorithm based on the updated maximum charging cycles. Let  $\hat{\tau}_1(t), \hat{\tau}_2(t), \dots, \hat{\tau}_n(t)$  be the updated maximum charging cycles of the  $n$  sensors. Assume that residual lifetimes of the  $n$  sensors are  $\hat{l}_1(t), \hat{l}_2(t), \dots, \hat{l}_n(t)$ , respectively. We further assume that the solution delivered by the approximation algorithm based on the updated maximum charging cycles consists of

$$\begin{aligned} &(\mathcal{C}_1, t + \hat{\tau}_1(t)), \quad (\mathcal{C}_2, t + 2\hat{\tau}_1(t)), \quad \dots, (\mathcal{C}_{2^K}, t + 2^K \hat{\tau}_1(t)), \\ &(\mathcal{C}_1, t + \hat{\tau}'_n(t) + \hat{\tau}_1(t)), (\mathcal{C}_2, t + \hat{\tau}'_n(t) + 2\hat{\tau}_1(t)), \dots, (\mathcal{C}_{2^K}, t + \hat{\tau}'_n(t) + 2^K \hat{\tau}_1(t)), \\ &\vdots \\ &(\mathcal{C}_1, t + x\hat{\tau}'_n(t) + \hat{\tau}_1(t)), \dots, (\mathcal{C}_y, t + x\hat{\tau}'_n(t) + y\hat{\tau}_1(t)), \end{aligned}$$

where  $t + x\hat{\tau}'_n(t) + y\hat{\tau}_1(t) < T$ ,  $t + x\hat{\tau}'_n(t) + (y+1)\hat{\tau}_1(t) \geq T$ , and  $x$  and  $y$  are positive integers. The most updated charging cycles of the  $n$  sensors in the solution are  $\hat{\tau}'_1(t), \hat{\tau}'_2(t), \dots, \hat{\tau}'_n(t)$ , where  $\hat{\tau}'_i(t) = 2^{\lfloor \log_2 \frac{\hat{\tau}_i(t)}{\hat{\tau}_1(t)} \rfloor} \hat{\tau}_1(t)$ .

Note that the solution delivered may not be feasible as different sensors may have different amounts of residual energy. This violates the condition of applying the approximation algorithm, that is, all sensors must be fully charged initially. The residual energy in some sensor  $v_i$  may not support its operation until its next charging time  $t + \hat{\tau}'_i(t)$ , i.e.,  $\hat{l}_i(t) < \hat{\tau}'_i(t)$ . Denote by  $V^a$  the set of sensors with  $\hat{l}_i(t) < \hat{\tau}'_i(t)$ . We then schedule the mobile chargers to replenish sensors in  $V^a$  to avoid their energy depletion, through adding a new charging scheduling  $(\mathcal{C}'_0, t)$  and modifying the first  $2^K$  schedulings from  $(\mathcal{C}_1, t + \hat{\tau}_1(t)), (\mathcal{C}_2, t + 2\hat{\tau}_1(t)), \dots, (\mathcal{C}_{2^K}, t + 2^K \hat{\tau}_1(t))$  to  $(\mathcal{C}'_1, t + \hat{\tau}_1(t)), (\mathcal{C}'_2, t + 2\hat{\tau}_1(t)), \dots, (\mathcal{C}'_{2^K}, t + 2^K \hat{\tau}_1(t))$ . Also, the charging schedulings delivered by the heuristic algorithm after the first  $2^K$  schedulings are the same as them delivered by the approximation algorithm. The rest is to construct the first  $2^K + 1$  charging schedulings.

Let  $V_t^a = \{v_i | v_i \in V^a \text{ \& } \hat{l}_i(t) < \hat{\tau}_1(t)\}$ , which implies that the residual lifetime of each sensor in  $V_t^a$  is less than  $\hat{\tau}_1(t)$  and  $V_t^a \subseteq V^a$ . We construct a scheduling  $(\mathcal{C}_0, t)$ , in which all sensors in  $V_t^a$  will be charged at time  $t$ . We then, like the node set partition in the approximation algorithm, partition the set  $V^a \setminus V_t^a$  into  $K+1$  disjoint sets  $V_0^a, V_1^a, \dots, V_K^a$  according to their residual lifetimes, where  $K = \lfloor \log_2 \frac{\hat{\tau}_n(t)}{\hat{\tau}_1(t)} \rfloor$  and a sensor  $v_i \in V^a \setminus V_t^a$  is contained in  $V_k^a$  if  $2^k \hat{\tau}_1(t) \leq \hat{l}_i(t) < 2^{k+1} \hat{\tau}_1(t)$ . Note that the

residual lifetime  $\hat{l}_i(t)$  of each sensor  $v_i$  in  $V_k^a$  at time  $t$  is no less than  $2^k \hat{\tau}_1(t)$  but no greater than its charging cycle  $\hat{\tau}_i'(t)$ , i.e.,  $2^k \hat{\tau}_1(t) \leq \hat{l}_i(t) < \hat{\tau}_i'(t)$ . To avoid the energy depletion of sensor  $v_i$ , we can add it into any one of the schedulings:  $\{(\mathcal{C}_0, t), (\mathcal{C}_1, t + \hat{\tau}_1(t)), (\mathcal{C}_2, t + 2\hat{\tau}_1(t)), \dots, (\mathcal{C}_{2^k}, t + 2^k \hat{\tau}_1(t))\}$ . However, to minimize the service cost, we add sensor  $v_i$  into a nearest scheduling  $\mathcal{C}_j$ . The detailed construction of the  $2^K + 1$  schedulings is as follows.

We construct the  $2^K + 1$  schedulings by iteratively invoking Algorithm 1 for the  $q$ -rooted minimum spanning forest problem. Denote by  $V(\mathcal{C}_j^{(k)})$  and  $V(\mathcal{C}_j^{(k+1)})$  the constructed node sets of scheduling  $\mathcal{C}_j'$  before and after iteration  $k$ , respectively, where  $0 \leq k \leq K$ . Note that  $\mathcal{C}_j^{(k)} = \{C_{j,1}^{(k)}, \dots, C_{j,q}^{(k)}\}$  and  $V(\mathcal{C}_j^{(k)}) = \bigcup_{l=1}^q V(C_{j,l}^{(k)})$ . After  $K + 1$  iterations, we let  $V(\mathcal{C}_j') = V(\mathcal{C}_j^{(K+1)})$ . We finally obtain scheduling  $\mathcal{C}_j'$  by applying Algorithm 2 for the  $q$ -rooted TSP problem in the induced graph  $G[V(\mathcal{C}_j')]$ . Consequently, each sensor in  $V_t^a \cup V_0^a \cup \dots \cup V_K^a = V^a$  will be charged in time. Initially, let  $V(\mathcal{C}_0^{(0)}) = V_t^a \cup R$  and  $V(\mathcal{C}_j^{(0)}) = V(\mathcal{C}_j)$ , where  $1 \leq j \leq 2^K$ . At iteration  $k$  ( $0 \leq k \leq K$ ), we first construct an auxiliary graph  $G^{(k)} = (V_k^a \cup R^{(k)}, E^{(k)}; w^{(k)})$  based on node sets  $V_k^a$  and  $V(\mathcal{C}_0^{(k)}), V(\mathcal{C}_1^{(k)}), \dots, V(\mathcal{C}_{2^k}^{(k)})$ , where there is a root  $r_j^{(k)}$  in  $R^{(k)}$  representing node set  $V(\mathcal{C}_j^{(k)})$ ,  $0 \leq j \leq 2^k$ , and  $E^{(k)} = V_k^a \times V_k^a \cup V_k^a \times R^{(k)}$ . Then,  $|R^{(k)}| = 2^k + 1$ . For each edge  $(u, v) \in V_k^a \times V_k^a$ ,  $w^{(k)}(u, v)$  is the Euclidean distance between nodes  $u$  and  $v$ . For each edge  $(u, r_j^{(k)}) \in V_k^a \times R^{(k)}$ ,  $w^{(k)}(u, r_j^{(k)})$  is the smallest Euclidean distance between node  $u$  and nodes in  $V(\mathcal{C}_j^{(k)})$ . We then obtain  $2^k + 1$  minimum cost rooted trees  $T_0^{(k)}, T_1^{(k)}, \dots, T_{2^k}^{(k)}$ , by invoking Algorithm 1 on  $G^{(k)}$ , where tree  $T_j^{(k)}$  contains root  $r_j^{(k)}$  and  $0 \leq j \leq 2^k$ . Note that each sensor in  $V_k^a$  is contained in a tree  $T_j^{(k)}$  and  $V_k^a = V(T_0^{(k)}) \cup V(T_1^{(k)}) \cup \dots \cup V(T_{2^k}^{(k)}) - R^{(k)}$ . Then, the sensors in tree  $T_j^{(k)}$  will be charged in scheduling  $(\mathcal{C}_j', t + j\hat{\tau}_1(t))$ . To this end, we let  $V(\mathcal{C}_j^{(k+1)}) = V(\mathcal{C}_j^{(k)}) \cup V(T_j^{(k)}) - \{r_j^{(k)}\}$  if  $0 \leq j \leq 2^k$ , otherwise ( $2^k + 1 \leq j \leq 2^K$ ),  $V(\mathcal{C}_j^{(k+1)}) = V(\mathcal{C}_j^{(k)})$ . We refer to this heuristic algorithm as *MinDis-var*.

**Theorem 3.** *There is a heuristic algorithm for the service cost minimization problem with variable maximum charging cycles, which takes  $O(\frac{\tau_{\max}}{\tau_{\min}} n^2 + \frac{T}{\tau_{\min}} n + \frac{\tau_{\max}^2}{\tau_{\min}^2})$  time, where  $n = |V|$ ,  $\tau_{\max} = \max_{i=1}^n \{\tau_i\}$ , and  $\tau_{\min} = \min_{i=1}^n \{\tau_i\}$ .*

## 4 Approximation Algorithm for the Minimum Number of Mobile Chargers Deployment Problem

In this section, we propose a novel approximation algorithm for the minimum number of mobile chargers deployment problem. We first detail a 5-approximation algorithm for the optimal  $p$ -closed tour problem in Subsection 4.1, which serves as a

subroutine of the proposed algorithm. We then present the approximation algorithm in Subsection 4.2.

#### 4.1 Algorithm for the $p$ -optimal closed tour problem

In this subsection we devise a 5-approximation algorithm for the  $p$ -optimal closed tour problem in a node and edge weighted metric graph  $G_s(V_s, E_s; h, w)$ . This algorithm will be used as a subroutine for the minimum number of mobile chargers deployment problem in Subsection 4.2. As a special case of the  $p$ -optimal closed tour problem when  $p = 1$  is the well-known TSP problem which is NP-hard, the  $p$ -optimal closed tour problem is NP-hard, too. In the following, we start by introducing a popular technique to transform a tree into a closed tour in  $G_s$ . We then introduce a novel tree decomposition. We finally present an approximation algorithm for the problem based on the tree decomposition.

##### 4.1.1 A closed tour derived from a tree

We first introduce the technique that transforms a tree in  $G_s$  to a closed tour by the following lemma.

**Lemma 4.** *Given a node and edge weighted metric graph  $G_s = (V_s, E_s; h, w)$  with sets  $V_s$  and  $E_s$  of nodes and edges,  $h : V_s \mapsto \mathbb{R}^{\geq 0}$  and  $w : E_s \mapsto \mathbb{R}^{> 0}$ , and the edge weight follows the triangle inequality, let  $T = (V_s, E_T; h, w)$  be a spanning tree of  $G_s$  rooted at  $r$ . Let  $C$  be the travelling salesman tour of  $G_s$  derived from  $T$  through performing the pre-order traversal on  $T$  and pruning, then the total cost  $WH(C)$  of  $C$  is no more twice the total cost  $WH(T)$  of  $T$ , i.e.,  $WH(C) \leq 2WH(T) = 2(\sum_{v \in V_s} h(v) + \sum_{e \in E_T} w(e))$ .*

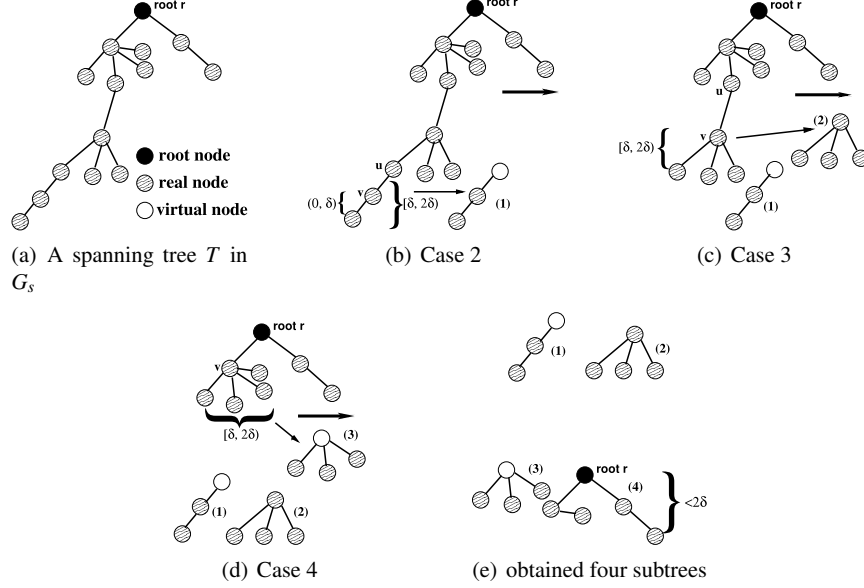
*Proof.* Let  $H(X)$  be the weighted sum of nodes in  $X$  and  $W(Y)$  the weighted sum of edges in  $Y$ . As the weighted sum  $W(C)$  of the edges in  $C$  is no more than  $2\sum_{e \in E_T} w(e)$ , and the weighted sum  $H(C)$  of nodes in  $C$  is the same as the one in  $T$ . Thus, the total cost of  $C$  is  $WH(C) = W(C) + H(C) \leq 2W(T) + H(T) \leq 2(W(T) + H(T)) = 2WH(T)$ .  $\square$

##### 4.1.2 Tree decomposition

Given a metric graph  $G_s = (V_s, E_s; h, w)$ , let  $T = (V_s, E_T; h, w)$  be a spanning tree in  $G_s$  rooted at node  $r$  and  $\delta \geq \max_{v \in V_s} \{h(v), 2w(v, r)\}$  a given value, then, both the node weight  $h(v)$  of any node  $v \in V_s$  and the edge weight  $w(e)$  of any edge  $e \in E_T$  in tree  $T$  are no more than  $\delta$ , i.e.,  $h(v) \leq \delta$  and  $w(e) \leq \delta$ . We decompose the tree into a set of subtrees such that the total cost of each subtree is no more than  $2\delta$  as follows.

Let  $(u, v)$  be a tree edge in  $T$ , where  $u$  is the parent of  $v$  and  $v$  is a child of  $u$ . Also, let  $T_v$  be a subtree of  $T$  rooted at node  $v$ . We perform a depth-first search on  $T$  starting from the tree root  $r$  until the total cost of the leftover tree rooted at  $r$  is

no less than  $2\delta$ , i.e.,  $WH(T_r) < 2\delta$ . Fig. 2 demonstrates an example of the tree decomposition procedure. Assume that node  $v$  is the node that is currently visited, we distinguish into the following four cases.



**Fig. 2** An illustration of the tree decomposition

Case 1. If  $WH(T_v) < \delta$  and  $WH(T_v) + w(u, v) < \delta$ , no action is needed, and the tree decomposition procedure continues.

Case 2. If  $WH(T_v) < \delta$  and  $WH(T_v) + w(u, v) \geq \delta$ . Then, we must have  $WH(T_v) + w(u, v) < 2\delta$ , since the weight  $w(u, v)$  of edge  $(u, v)$  is no more than  $\delta$ . A new tree  $T_v \cup \{(v, u')\}$  is created with a *virtual node*  $u'$  with  $h(u') = 0$ . Split the subtree  $T_v \cup \{(v, u')\}$  from the original tree, see Fig. 2(b).

Case 3. If  $\delta \leq WH(T_v) < 2\delta$ , split the subtree  $T_v$  from the original tree and remove edge  $(u, v) \in E_T$  from the original tree, see Fig. 2 (c).

Case 4. Let  $v_1^c, v_2^c, \dots, v_k^c$  be the  $k$  children of  $v$ . Let  $l$  be the maximum children index so that  $\delta \leq \sum_{j=1}^l (WH(T_{v_j^c}) + w(v_j^c, v)) < 2\delta$  with  $1 \leq l \leq k$ , then, a new subtree  $\cup_{j=1}^l (T_{v_j^c} \cup \{(v_j^c, v')\})$  rooted at the virtual node  $v'$  is created, which consists of these subtrees with  $h(v') = 0$ . Split off this subtree from the original tree, see Fig. 2 (d).

As a result, a set of subtrees is obtained by the tree decomposition on  $T$ , see Fig. 2 (e). The number of subtrees is bounded by the following lemma.

**Lemma 5.** Given a spanning tree  $T = (V_s, E_T; h, w)$  of a graph  $G_s = (V_s, E_s; h, w)$  with the total cost  $WH(T)$  and a value  $\delta \geq \max_{v \in V_s} \{2w(r, v), h(v)\}$ , the tree  $T$  can be decomposed into  $p$  subtrees  $T_1, T_2, \dots, T_p$  with  $WH(T_i) < 2\delta$  by the proposed tree-decomposition procedure,  $1 \leq i \leq p$ . Then,  $p \leq \lfloor \frac{WH(T)}{\delta} \rfloor$ .

*Proof.* Following the tree decomposition on  $T$ , subtrees with the total cost in  $[\delta, 2\delta)$  are split away from  $T$  until the total cost of the leftover tree including root  $r$  is less than  $2\delta$ . Suppose that  $T_1, T_2, \dots, T_p$  are the split trees with  $p \geq 2$ . From the subtree construction, we know that  $\delta \leq WH(T_i) < 2\delta$  for each  $i$  with  $1 \leq i \leq p-1$ . The only subtree with the total cost less than  $\delta$  is  $T_p$ . Note that prior to splitting  $T_{p-1}$ , the total cost of the remaining tree is at least  $2\delta$ . Therefore, the average total cost of  $T_{p-1}$  and  $T_p$  is no less than  $\delta$ . That is, the average total cost of all  $T_i$  is at least  $\delta$ . Thus,  $p \cdot \delta \leq WH(T)$ , i.e.,  $p \leq \frac{WH(T)}{\delta}$ . Since  $p$  is an integer,  $p \leq \lfloor \frac{WH(T)}{\delta} \rfloor$ .  $\square$

#### 4.1.3 Algorithm for finding $p$ -optimal closed tours

Given a metric graph  $G_s = (V_s, E_s, h, w)$  with root  $r$  and a positive integer  $p$ , we now devise an approximation algorithm for the  $p$ -optimal closed tour problem in  $G_s$  as follows.

Let  $T$  be a minimum spanning tree (MST) of  $G_s$  rooted at  $r$ . The basic idea of the proposed algorithm is that we first perform a tree decomposition on  $T$  with  $\delta = \max_{v \in V_s} \{WH(T)/p, 2w(v, r) + h(v)\}$  and we later show that  $\delta$  is a lower bound on the optimal cost of the  $p$ -optimal closed tour problem. As a result,  $p'$  subtrees are derived from such a decomposition, and  $p'$  closed tours are then derived from the  $p'$  subtrees. We finally show that  $p' \leq p$  and the maximum total cost of any closed tour among the  $p'$  closed tours is no more than  $5\delta$ .

Specifically,  $T$  is decomposed into no more than  $p'$  edge-disjoint subtrees, excepting the root node  $r$  which appears in one of these subtrees. Let  $T_1, T_2, \dots, T_{p'}$  be the  $p'$  trees obtained by decomposing  $T$ . It can be observed that each  $T_i$  contains at least one real node and at most one virtual node, where a node  $v$  is a *real node* if  $h(v) \neq 0$ ; otherwise, it is a virtual node. As a result, a forest  $\mathcal{F}$  consisting of all the trees is found through the tree decomposition, and the number of trees in  $\mathcal{F}$  is  $p' \leq \lfloor WH(T)/\delta \rfloor$  and the total cost of each subtree is no more than  $2\delta$  by Lemma 5.

For each  $T_i \in \mathcal{F}$ , if it does not contain the root  $r$ , then, a tree  $T'_i = T_i \cup \{(v_i, r)\}$  rooted at  $r$  is obtained by including node  $r$  and a tree edge  $(v_i, r)$  into  $T_i$ , where node  $v_i$  is a node in  $T_i$  and  $w(v_i, r) = \min_{v \in T_i} \{w(v, r)\}$ . The total cost  $WH(T'_i)$  of  $T'_i$  is

$$WH(T'_i) = WH(T_i) + w(v_i, r) \leq 2\delta + w(v_i, r) \leq 2.5\delta, \text{ as } w(v_i, r) \leq \delta/2.$$

Otherwise ( $T_i$  contains node  $r$ ),  $T'_i = T_i$  and  $WH(T'_i) = WH(T_i) \leq 2\delta$ . We thus obtain a forest  $\mathcal{F}' = \{T'_1, T'_2, \dots, T'_{p'}\}$ . From the trees in  $\mathcal{F}'$ ,  $p'$  edge-disjoint closed tours with each containing the root  $r$  can be derived. Let  $\mathcal{C}' = \{C'_1, C'_2, \dots, C'_{p'}\}$  be the set of  $p'$  closed tours obtained, by transforming each tree in  $\mathcal{F}'$  into a closed tour. For each  $C'_i$ , we have that  $WH(C'_i) \leq 2 \cdot WH(T'_i) \leq 5\delta$  by Lemma 4. As there are some  $C'_i$ s containing virtual nodes that are not part of a feasible solution to the problem, a feasible solution can be derived through a minor modification to the closed tours in  $\mathcal{C}'$ . That is, for each  $C'_i$ , if it contains a virtual node (as each  $C'_i$  contains at most one virtual node), a closed tour  $C_i$  with a less total cost than that of  $C'_i$  is obtained, by removing the virtual node and the two edges incident to the node from  $C'_i$  through short cutting, then  $WH(C_i) \leq WH(C'_i)$  as the edge weight follows the triangle in-



equality. Otherwise,  $C_i = C'_i$ . Clearly, each of the  $p'$  closed tours  $C_1, C_2, \dots, C_{p'}$  roots at  $r$ . The detailed algorithm is described in Algorithm 4.

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**Algorithm 4:** *finding closed tours rooted at  $r$  with each having the bounded total cost*

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**Input:** A metric graph  $G_s = (V_s, E_s; h, w)$ , a root  $r \in V_s$ , and a given value  $\delta \geq \max_{v \in V_s} \{h(v), 2w(v, r)\}$ .

**Output:** a set of node-disjoint closed tours covering all nodes in  $V_s$  with the shared root  $r$  so that the total cost of each tour is no more than  $5\delta$ .

- 1 Let  $T$  be an MST of  $G_s$  and  $WH(T)$  be the total cost of  $T$ ;
  - 2 Let  $\mathcal{T} = \{T_1, T_2, \dots, T_{p'}\}$  be the forest obtained by performing the tree decomposition on  $T$  with the given value  $\delta$ ;
  - 3 Let  $\mathcal{T}' = \{T'_1, T'_2, \dots, T'_{p'}\}$  be a forest, where  $T'_i = T_i \cup \{(r, v_i)\}$  is derived by adding root  $r$  and an edge with the minimum edge weight between a node  $v_i$  in  $T_i$  and  $r$  if  $r$  is not in  $T_i$ ; otherwise  $T'_i = T_i$ , where  $1 \leq i \leq p'$ ;
  - 4 Let  $\mathcal{C}' = \{C'_1, C'_2, \dots, C'_{p'}\}$ , where closed tour  $C'_i$  is derived from  $T'_i$ ;
  - 5 Let  $\mathcal{C} = \{C_1, C_2, \dots, C_{p'}\}$  be a set of closed tours, where  $C_i$  is derived by removing the virtual node from  $C'_i \in \mathcal{C}'$  if it does contain a virtual node. Otherwise,  $C_i = C'_i$ ;
  - 6 **return**  $\mathcal{C}$ .
- 

#### 4.1.4 Algorithm analysis

In the following, We show that Algorithm 4 delivers a 5-approximate solution. Specifically, we first show that Algorithm 4 delivers a feasible solution to the  $p$ -optimal closed tour problem. We then show that the total cost of each closed tour in the solution is no more than  $5\delta$ . We thirdly show that  $\delta (= \max_{v \in V_s} \{WH(T)/p, 2w(v, r) + h(v)\})$  is a lower bound on the optimal cost of the problem. Then, the total cost of each closed tour in the solution delivered by Algorithm 4 is no more than  $5\delta \leq 5OPT$ . We finally analyze the time complexity of Algorithm 4.

**Theorem 4.** *Given a metric graph  $G_s = (V_s, E_s; h, w)$  and an integer  $p \geq 1$ , there is a 5-approximation algorithm for finding  $p$ -optimal closed tours. The time complexity of the proposed algorithm is  $O(|V_s|^2)$ .*

*Proof.* We first show that Algorithm 4 delivers a feasible solution to the  $p$ -optimal closed tour problem. Recall that  $T$  is an MST of  $G_s$ . Since  $\delta = \max_{v \in V_s} \{WH(T)/p, 2w(v, r) + h(v)\}$ ,  $\delta \geq \max_{v \in V_s} \{2w(v, r), h(v)\}$ . A solution  $\mathcal{C}$  which consists of  $p'$  closed tours rooted at  $r$  can be obtained, by applying Algorithm 4 on  $T$ , and

$$\begin{aligned} p' &\leq \lfloor WH(T)/\delta \rfloor \leq WH(T)/\delta \\ &= \frac{WH(T)}{\max_{v \in V_s} \{WH(T)/p, 2w(v, r) + h(v)\}} \leq \frac{WH(T)}{WH(T)/p} = p, \end{aligned} \quad (12)$$

by Lemma 5. Thus,  $\mathcal{C}$  is a feasible solution.

We then show that the total cost of each closed tour in  $\mathcal{C}$  is no more than  $5\delta$ . As each  $C_i \in \mathcal{C}$  is derived from a  $C'_i \in \mathcal{C}'$ , we have  $WH(C_i) \leq WH(C'_i) \leq 2 WH(T'_i) \leq 2 \cdot 2.5\delta = 5\delta$  by Lemma 4.

We thirdly prove that  $\delta$  is a lower bound on the optimal cost of the problem. Given a node and edge weighted metric graph  $G_s = (V_s, E_s; h, w)$  with root  $r$ , an integer  $p \geq 1$ , partition the nodes in  $V_s$  into  $p$  disjoint subsets  $X_1, X_2, \dots, X_p$ , and let  $C_j$  be the closed tour containing all nodes in  $X_j$  and the root  $r$ . The optimal partitioning is a partitioning such that the maximum value  $\max_{1 \leq j \leq p} \{WH(C_j)\}$  is minimized. Let  $OPT$  be the total cost of the maximum closed tour in the optimal solution. We show that  $\delta \leq OPT$  as follows.

Let  $C_1^*, C_2^*, \dots, C_p^*$  be the  $p$  closed tours in the optimal solution with the shared root  $r$ . Then,  $WH(C_i^*) \leq OPT$ . Let  $e_i$  be the maximum weighted edge in  $C_i^*$ . Then, a tree  $T' = \cup_{i=1}^p C_i^* \setminus \cup_{i=1}^p \{e_i\}$  rooted at  $r$  can be obtained by removing  $e_i$  from each tour  $C_i^*$ . We then have

$$WH(T') = \sum_{i=1}^p (WH(C_i^*) - w(e_i)) \leq \sum_{i=1}^p WH(C_i^*) \leq p \cdot OPT. \quad (13)$$

It can be seen that  $T'$  is a spanning tree in  $G_s$ . Since  $T$  is an MST of  $G_s$ ,  $WH(T) \leq WH(T')$ . We thus have

$$\frac{WH(T)}{p} \leq \frac{WH(T')}{p} \leq OPT. \quad (14)$$

On the other hand, each node  $v \in V_s$  must be contained by one closed tour  $C_i^*$  in the optimal solution. Since tour  $C_i^*$  contains node  $v$  and the depot  $r$ , then the total cost of  $C_i^*$ ,  $WH(C_i^*)$ , is at least  $2w(v, r) + h(v)$ . Thus,

$$2w(v, r) + h(v) \leq WH(C_i^*) \leq OPT, \quad \forall v \in V_s. \quad (15)$$

Combing inequalities (14) and (15), we have

$$\delta = \max_{v \in V_s} \left\{ \frac{WH(T)}{p}, 2w(v, r) + h(v) \right\} \leq OPT. \quad (16)$$

The time complexity analysis of Algorithm 4 is straightforward, omitted.  $\square$

## 4.2 Approximation algorithm for the minimum number of mobile chargers deployment problem

In this subsection we provide an approximation algorithm for the minimum number of mobile chargers deployment problem. As each mobile charger consumes energy on travelling and charging sensors per tour, the total amount of energy consumed by the mobile vehicle is bounded by its energy capacity  $IE$ .

#### 4.2.1 Algorithm

The basic idea of the proposed approximation algorithm is to reduce the minimum number of mobile chargers deployment problem into a  $p$ -closed tour problem, by bounding the total cost of each closed tour. A solution to the latter in turn returns a solution to the former as follows.

Recall that we assume that the base station knows both the residual energy  $RE_i$  and the energy consumption rate  $\rho_i$  of each sensor  $v_i \in V_s$ , and  $\mu$  is the wireless charging rate of a mobile vehicle. Assume that there are sufficient numbers of fully charged mobile vehicles available at the depot. Then, a mobile vehicle takes  $\tau_i = \frac{B_i - RE_i}{\mu}$  time to charge sensor  $v_i$  to its full capacity  $B_i$  when it approaches the sensor. We thus construct a node and edge weighted metric graph  $G_s = (V_s, E_s; h, w)$ , where  $V_s$  is the set of sensors to be charged in this round. There is an edge in  $E_s$  between any two to-be-charged sensor nodes. For each edge  $(u, v) \in E_s$ , its weight is  $w(u, v) = \eta \cdot d(u, v)$  which is the amount of energy consumed by a mobile vehicle travelling along the edge, where  $\eta$  is the energy consumption rate of a mobile vehicle for travelling per unit length and  $d(u, v)$  is the Euclidean distance between sensor nodes  $u$  and  $v$ .

For each node  $v_i \in V_s$ , its weight  $h(v_i)$  ( $= B_i - RE_i = \mu \cdot \tau_i$ ) is the amount of energy needed to charge sensor  $v_i$  to reach its full capacity  $B_i$ . We assume that  $IE \geq \max_{v \in V_s} \{2w(v, r) + h(v)\}$ ; otherwise, there are no feasible solutions to the problem, which will be shown by Lemma 6 later. The detailed algorithm is described in Algorithm 5. We refer to this algorithm as `NMV_without_Eloss`.

#### 4.2.2 Algorithm analysis

We analyze the approximation ratio of the proposed algorithm, Algorithm 5, and its time complexity as follows. We start by Lemma 6, which says that there must be a feasible solution to the problem if and only if  $IE \geq \max_{v \in V_s} \{2w(v, r) + h(v)\}$ ; otherwise, there are no solutions to the problem. Thus, in the rest of our discussions, we assume that  $IE \geq \max_{v \in V_s} \{2w(v, r) + h(v)\}$ .

**Lemma 6.** *Given a metric graph  $G_s = (V_s, E_s; h, w)$  and an energy capacity  $IE$  of each mobile charging vehicle, there is a feasible solution to the minimum number of mobile chargers deployment problem in  $G_s$  if and only if  $IE \geq \max_{v \in V_s} \{2w(v, r) + h(v)\}$ , where  $r$  is the depot of charging vehicles.*

*Proof.* If  $IE \geq \max_{v \in V_s} \{2w(v, r) + h(v)\}$ , we can derive a feasible solution to the problem, by dispatching one charging vehicle to charge only one of the  $n = |V_s|$  sensors. Thus,  $n$  charging vehicles are deployed. On the other hand, assume that there is a feasible solution  $\mathcal{C} = \{C_1, C_2, \dots, C_p\}$  to the problem, where  $p$  charging vehicles are deployed to fully charge the  $n$  sensors and  $C_j$  is the charging tour of the  $j$ -th charging vehicle with  $1 \leq j \leq p$ . It is obvious that  $WH(C_j) \leq IE$  for  $1 \leq j \leq p$ . Consider a sensor  $v_i \in V_s$  such that  $v_i = \arg \max_{v \in V_s} \{2w(v, r) + h(v)\}$ . Let  $C_j$  be the charging tour containing sensor  $v_i$  in the solution. Since tour  $C_j$  must

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**Algorithm 5:** finding the optimal number of mobile vehicles and their closed tours (NMV\_without\_Eloss)

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**Input:** A metric graph  $G_s = (V_s, E_s; h, w)$ , a root  $r$ , and  $IE$  with

$$IE \geq \max_{v \in V_s} \{2w(r, v) + h(v)\}.$$

**Output:**  $p$ -node-disjoint closed  $r$ -rooted tours  $C_1, C_2, \dots, C_p$  covering all nodes in  $V_s$  such that  $WH(C_i) \leq IE$ .

- 1 Let  $T$  be an MST of  $G_s$ . Denote by  $W(T)$  and  $H(T)$  the total costs of the edges and nodes in  $T$ , respectively;
  - 2 **if**  $IE \geq 2 \cdot W(T) + H(T)$  **then**
  - 3     One mobile vehicle suffices by Lemma 4; EXIT;;
  - 4 **end if**
  - 5  $A \leftarrow \max_{v \in V_s} \{2w(v, r)\}$ ;
  - 6 **if**  $IE/5 \geq A$  **then**
  - 7      $\delta \leftarrow IE/5$ ; /\* $\delta$  is the average subtree cost after tree decomposition\*/;
  - 8 **else**
  - 9      $\delta \leftarrow \frac{IE-A}{4}$ ;
  - 10 **end if**
  - 11 Perform the tree decomposition using  $\delta$ . If there is a node  $v$  with  $h(v) > \delta$ , then the node itself forms a tree;
  - 12 Let  $\mathcal{C} = \{C_1, C_2, \dots, C_p\}$  be the solution by applying Algorithm 4 for the tree decomposition on  $T$  with the given  $\delta$ ;
  - 13 **return**  $\mathcal{C}$  as a solution of the problem and  $p = |\mathcal{C}|$ .
- 

contain sensor  $v_i$  and depot  $r$ , the total cost of the tour,  $WH(C_j)$ , must be no less than  $2w(v_i, r) + h(v_i)$ , i.e.,  $WH(C_j) \geq 2w(v_i, r) + h(v_i)$ . Then,  $IE \geq 2w(v_i, r) + h(v_i) = \max_{v \in V_s} \{2w(v, r) + h(v)\}$ .  $\square$

**Theorem 5.** Given a metric graph  $G_s = (V_s, E_s; h, w)$  and the energy capacity  $IE$  of each mobile charging vehicle with  $IE \geq \max_{v \in V_s} \{2w(v, r) + h(v)\}$ , there is an approximation algorithm, Algorithm 5, with an approximation ratio of 8 for the minimum number of mobile chargers deployment problem in  $G_s$  if  $IE \geq 2A$ ; otherwise, the approximation ratio of the algorithm is  $4(1 + \frac{A}{h_{min}}) = O(1)$ . The algorithm takes  $O(|V_s|^2)$  time, where  $r$  is the depot of charging vehicles,  $A = \max_{v \in V_s} \{2w(v, r)\}$ , and  $h_{min} = \min_{v \in V_s} \{h(v)\}$ .

*Proof.* We first show that Algorithm 5 can deliver a feasible solution  $\mathcal{C} = \{C_1, C_2, \dots, C_p\}$ . Recall that  $A = \max_{v \in V_s} \{2w(v, r)\}$ , which is the maximum energy consumption of a charging vehicle on one round trip between a sensor  $v$  and the depot  $r$  in the sensor network. We distinguish it into three cases.

Case 1. If  $IE \geq 2 \cdot W(T) + H(T)$ , then there is a closed tour  $C$  including all nodes in  $V_s$  derived from  $T$  and the total cost of  $C$ ,  $WH(C)$  ( $\leq 2 \cdot W(T) + H(T) \leq IE$  by Lemma 4), is no more than the energy capacity of a mobile vehicle  $IE$ . Hence, one mobile charging vehicle suffices for charging all nodes in  $V_s$ .

Case 2. If  $IE/5 \geq A$ , then  $\delta = IE/5$ , and the total cost of each closed tour in the solution is no more than  $5\delta = IE$  by Theorem 4.

Case 3 ( $IE/5 < A \leq IE$ ). Following Algorithm 5, we set  $\delta = \frac{IE-A}{4}$ . Clearly,  $w(v, r) \leq A/2$  for any node  $v \in V_s$  since  $A = \max_{v \in V_s} \{2w(r, v)\}$ . Then, the total cost of each closed tour  $C$  in the solution is analyzed as follows. (i)  $C$  contains only one sensor node  $v \in V_s$ . The total cost of  $C$  thus is  $WH(C) = 2w(r, v) + h(v) \leq IE$  by Lemma 6 and the input condition of the algorithm. (ii)  $C$  consists of multiple sensor nodes and is derived from a tree  $T_i$ . Then, the total cost of tour  $C$  in the solution is  $WH(C) \leq 2 \cdot (2\delta + w(v_0, r)) \leq 4 \cdot \frac{IE-A}{4} + 2w(v_0, r) \leq IE - A + A = IE$ , where  $w(v_0, r) = \min_{v \in T_i} \{w(v, r)\}$  and  $T_i$  is the tree from which  $C$  is derived. Thus, the solution is a feasible solution of the problem.

We then analyze the approximation ratio of the proposed algorithm. Assume that the minimum vehicles needed is  $p_{min}$ . With a similar discussion in Theorem 4, a lower bound on the value of  $p_{min}$  is

$$p_{min} \geq \lceil \frac{WH(T)}{IE} \rceil. \quad (17)$$

Let  $p$  be the number of vehicles delivered by the proposed algorithm. We show the approximation ratio by the following four cases.

Case 1. If  $IE \geq 2 \cdot W(T) + H(T)$ , only one mobile vehicle suffices, and this is an optimal solution.

Case 2. If  $IE/5 \geq A$ , we have  $\delta = IE/5$ . Then,  $\frac{p}{p_{min}} \leq \frac{|WH(T)/\delta|}{|WH(T)/IE|} \leq \frac{WH(T)/\delta}{WH(T)/IE} = IE/\delta = 5$  by Lemma 5.

Case 3 ( $IE/5 < A \leq IE/2$ ). We have  $\delta = \frac{IE-A}{4}$ . Then,  $\frac{p}{p_{min}} \leq \frac{|WH(T)/\delta|}{|WH(T)/IE|} = \frac{WH(T)/\delta}{WH(T)/IE} = \frac{IE}{\delta} = \frac{4 \cdot IE}{IE-A} = \frac{4}{1-A/IE} \leq \frac{4}{1-A/2A} = 8$ , by Lemma 5, Eq. (17), and  $IE \geq 2A$ .

Case 4 ( $IE/2 < A < IE$ ). We have  $\delta = \frac{IE-A}{4}$ . Let  $h_{min} = \min_{v \in V_s} \{h(v)\}$ , which is the minimum amount of energy for fully charging an energy-critical sensor  $v$  in the sensor network. Then,  $IE \geq \max_{v \in V_s} \{2w(r, v) + h(v)\} \geq 2w(r, v_i) + h(v_i) = A + h(v_i) \geq A + h_{min}$ , where  $v_i = \arg \max_{v \in V_s} \{2w(r, v)\}$ . The approximation ratio for Case 4 then is  $\frac{p}{p_{min}} \leq \frac{|WH(T)/\delta|}{|WH(T)/IE|} \leq \frac{WH(T)/\delta}{WH(T)/IE} = \frac{IE}{\delta} = \frac{4 \cdot IE}{IE-A} = 4(1 + \frac{A}{IE-A}) \leq 4(1 + \frac{A}{A+h_{min}-A}) = 4(1 + \frac{A}{h_{min}}) = O(1)$ , as each of the to-be-charged sensors has consumed a large portion of its energy already and  $h_{min}$  thus is proportional to the battery capacity of each sensor, the ratio  $\frac{A}{h_{min}}$  is usually a constant, where  $A$  is the maximum energy consumption of a charging vehicle on one round trip between a sensor and the depot  $r$  and  $h_{min}$  is the minimum amount of energy for fully charging an energy-critical sensor. Therefore, the approximation ratio for Case 4 is a constant. Notice that Case 4 in practice rarely happens, since the energy capacity of a charging vehicle cannot be used just for its travel without charging sensors, or its energy is only enough to charge one or two sensors per tour.

In summary, the approximation ratio of Algorithm 5 is no more than 8 when  $IE \geq 2A$ ; otherwise ( $\max_{v \in V_s} \{2w(w, r) + h(v)\} \leq IE < 2A$ ), its approximation ratio is  $4(1 + \frac{A}{h_{min}}) = O(1)$ . The dominant time of Algorithm 5 is the invoking of Algorithm 4, which takes  $O(|V_s|^2)$  time by Theorem 4.  $\square$

## 5 Performance Evaluation of the Algorithms for the Service Cost Minimization Problem

In this section, we evaluate the performance of the proposed algorithms for the service cost minimization problem through experimental simulations. We also study the impact of important parameters on the algorithm performance, including network size, data aggregation, and the ratio of the maximum data generation rate to the minimum data generation rate.

### 5.1 Simulation environment

We consider a WSN consisting of from 100 to 500 sensors in a  $1,000m \times 1,000m$  square area, in which sensors are randomly deployed. The base station is located at the center of the square. The battery capacity  $B_i$  of each sensor  $v_i$  is  $10.8 \text{ kJ}$  [27]. The data sensing rate  $b_i$  of each sensor  $v_i$  is randomly chosen from an interval  $[b_{min}, b_{max}]$ , where  $b_{min} = 1 \text{ kbps}$  and  $b_{max} = 10 \text{ kbps}$  [27]. The coefficient  $\alpha$  in Eq. (2) is 2. Furthermore, we assume that each sensor  $v_i$  performs data aggregations on both pass-by traffic and self-sensed data with a data aggregation factor  $\theta$ , i.e., the data transmission rate  $b_i^{Tx}$  of sensor  $v_i$  is  $b_i^{Tx} = \theta \cdot (b_i^{Rx} + b_i)$ , where  $b_i^{Rx}$  and  $b_i$  are the data reception rate and data sensing rate of sensor  $v_i$ , respectively, and  $\theta$  is constant with  $0 < \theta \leq 1$  [14]. The default value of  $\theta$  is 1.

There are 5 depots in the WSN (i.e.,  $q = 5$ ) and there is a mobile charger at each depot. To reduce the total travelling distance of the  $q$  mobile chargers, one depot is co-located with the base station, as the most energy-consuming sensors in a WSN usually are close to the base station for relaying data from other remote sensors. The rest of  $q - 1$  depots are randomly distributed in the area. The entire monitoring period  $T$  is one year, which is partitioned into equal time slots with each lasting  $\Delta T$  ( $\Delta T$  typically is much shorter than  $T$ , e.g.,  $\Delta T$  is one month). We assume that the data sensing rate  $b_i$  of each sensor  $v_i \in V$  does not change within each time slot  $\Delta T$ . Even if it does change within the time slot, the difference can be neglected.

To evaluate the performance of the proposed algorithms `MinDis` and `MinDis-var` against the state-of-the-art algorithms, we implement three benchmark algorithms of sensor charging `Periodic` [27, 30, 31, 29], `OnDemand`, and `Partition` [28, 41], which are described as follows. In algorithm `Periodic`, the base station periodically dispatches the  $q$  mobile chargers to charge every sensor in the network with charging period being  $\tau_{min}$ . The charging tours of the  $q$  chargers will be found by applying Algorithm 2. In algorithm `OnDemand`, each sensor sends a charging request to the base station when its residual energy is below a given energy threshold. Having received a set of such requests, the base station then dispatches the  $q$  mobile chargers to charge the sensors whose estimated residual lifetimes are less than a given threshold  $\Delta l$  with  $\Delta l = \tau_{min}$ . The charging tours of the  $q$  mobile chargers are finally obtained by applying Algorithm 2 for the  $q$ -rooted TSP problem in the induced graph of the to-be-charged sensors. Finally, in algorithm `Partition`, the monitoring region is divided into  $q$  subregions, in other words, the

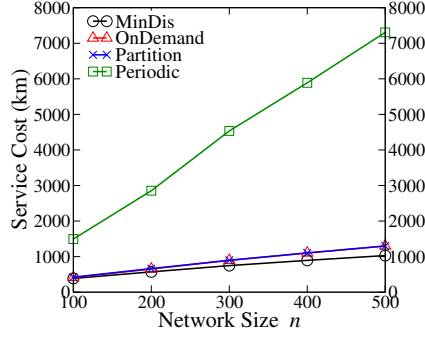
sensors in the network are first partitioned into  $q$  disjoint sets  $V_1, V_2, \dots, V_q$  with each set corresponding to the sensors in its subregion, where a sensor  $v_i$  is contained in set  $V_j$  if depot  $r_j$  is its nearest depot among the  $q$  depots. Then, the sensors in  $V_j$  will be charged by only the mobile charger located at depot  $r_j$ , where  $1 \leq j \leq q$ . Each sensor  $v_i \in V_j$  sends a charging request to the base station when it will deplete its energy soon. Once receiving the request, the base station dispatches the mobile charger at depot  $r_j$  to charge a subset  $V'_j$  of sensors of  $V_j$  with the residual lifetime of each sensor in  $V'_j$  being less than a given threshold  $\Delta l_j$ , i.e.,  $V'_j = \{v_i \mid v_i \in V_j, l_i < \Delta l_j\}$ , and the charging tour of the charger is a shortest closed tour visiting the sensors in  $V'_j$  and depot  $r_j$ , where  $\Delta l_j = \tau_{min}^j$  and  $\tau_{min}^j$  is the shortest maximum charging cycle of sensors in set  $V_j$ , i.e.,  $\tau_{min}^j = \min_{v_i \in V_j} \{\tau_i\}$ .

It must be mentioned that each value in all figures is the average of the results by applying each mentioned algorithm to 100 different network topologies with the same network size.

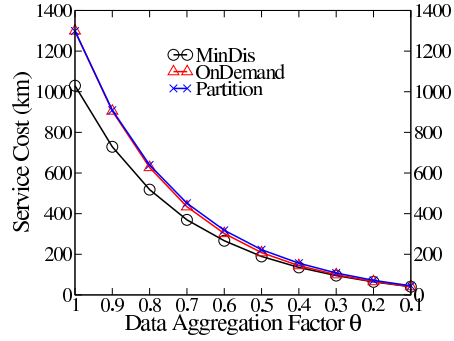
## 5.2 Performance with fixed maximum charging cycles

We first evaluate the performance of the proposed approximation algorithm *MinDis* against algorithms *OnDemand*, *Partition*, and *Periodic* by varying network size  $n$ , assuming that maximum charging cycles within  $T$  are fixed. Fig. 3 shows that the service costs delivered by algorithms *MinDis*, *OnDemand*, and *Partition* are much less than that by algorithm *Periodic*. For example, Fig. 3 demonstrates that the service cost by algorithm *MinDis* only about from 15% to 25% of the cost by algorithm *Periodic*, and the costs by algorithms *OnDemand* and *Partition* are from 19% to 28% of that by algorithm *Periodic*. Also, it can be seen from Fig. 3 that the proposed algorithm *MinDis* delivers a solution with the least service cost of mobile chargers, while the service costs delivered by algorithms *OnDemand* and *Partition* are almost identical and the one by algorithm *OnDemand* is only marginal better than that by algorithm *Partition*, ranging from 0.3% to 1.5% improvement. In the following, we only compare the performance of algorithms *MinDis*, *OnDemand*, and *Partition*, and omit the performance of algorithm *Periodic*, since the service cost delivered by the algorithm is much higher than that by the three algorithms.

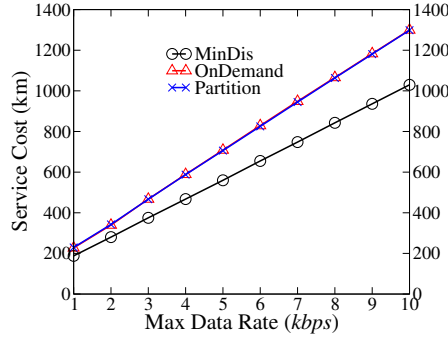
We then examine the impact of the data aggregation factor  $\theta$  on the performance of the three algorithms, by decreasing  $\theta$  from 1.0 to 0.1. Fig. 4 clearly presents that the service costs by algorithms *MinDis*, *OnDemand*, and *Partition* decrease when  $\theta$  becomes smaller and the service costs by the three algorithms are almost identical when  $\theta = 0.1$ . The rationale behind the phenomenon is that the data transmission rates of sensors can be greatly reduced by a small data aggregation factor  $\theta$  while the sensor energy consumption on data transmission is usually the dominant one [14]. As a result, the maximum charging cycles of sensors becomes longer with a smaller value of  $\theta$  and the service cost of mobile chargers thus is significantly reduced.



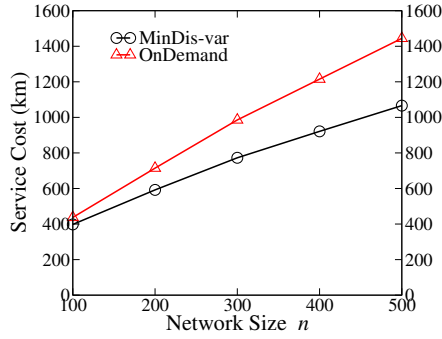
**Fig. 3** Performance of algorithms MinDis, OnDemand, Partition, and Periodic by varying the network size from 100 to 500 sensors.



**Fig. 4** Performance of algorithms MinDis, OnDemand, and Partition by decreasing the data aggregation factor  $\theta$  from 1.0 to 0.1 when  $n = 500$ .



**Fig. 5** Performance of algorithms MinDis, OnDemand, and Partition by varying the maximum data rate  $b_{max}$  from 1 kbps to 10 kbps when  $b_{min} = 1$  kbps and  $n = 500$ .



**Fig. 6** Performance of algorithms MinDis-var and OnDemand by varying the network size when  $\Delta T$  is one month.

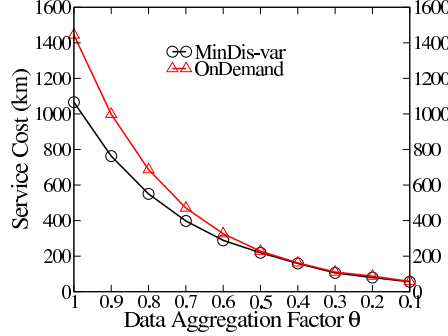
We finally study the impact of the maximum data rate  $b_{max}$  on the algorithm performance, by varying  $b_{max}$  from 1 kbps to 10 kbps when  $b_{min} = 1$  kbps. Fig. 5 demonstrates that the service cost by algorithm MinDis is only from 79% to 82% of the service cost by algorithm OnDemand and their performance gap increases when  $b_{max}$  becomes larger. Furthermore, Fig. 5 clearly shows that the service costs by the three algorithms increase with the increase of  $b_{max}$ . This is because that the energy consumption rates of sensors becomes higher when the maximum data rates of sensors  $b_{max}$  increases. As a result, sensors must be charged more frequently, which incurs more service cost of the mobile chargers.

In the following, we omit the performance of algorithm Partition, since the service costs by algorithms OnDemand and Partition are almost identical, which have already been shown in figures 3, 4, and 5.

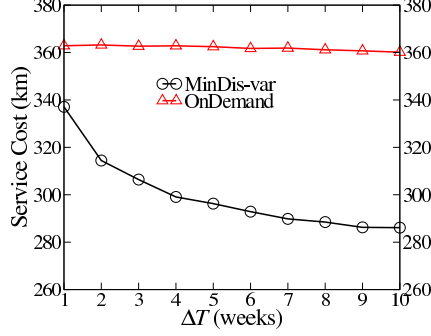


### 5.3 Performance with variable maximum charging cycles

We first investigate the performance of the proposed heuristic algorithm MinDis-var against algorithm OnDemand with variable maximum charging cycles. Fig. 6 and Fig. 7 illustrate the performance of both algorithms, by varying network size  $n$  and the data aggregation factor  $\theta$ , respectively. It can be seen that algorithm MinDis-var is still very competitive as it did under fixed maximum charging cycles.



**Fig. 7** Performance of algorithms MinDis-var and OnDemand by decreasing the data aggregation factor  $\theta$  from 1.0 to 0.1 when  $\Delta T$  is one month and  $n = 500$ .



**Fig. 8** Performance of algorithms MinDis-var and OnDemand by varying  $\Delta T$  from 1 week to 10 weeks when  $b_{min} = 1 \text{ kbps}$ ,  $b_{max} = 2 \text{ kbps}$ , and  $n = 500$ .

We finally study the impact of the dynamics of maximum charging cycles on the algorithm performance, by varying parameter  $\Delta T$  from 1 week (i.e., extremely dynamic) to 10 weeks (i.e., rather stable). Fig. 8 shows that the service cost by algorithm MinDis-var decreases with the increase of the stability of the sensor maximum charging cycles (a larger  $\Delta T$ ), while the service cost by algorithm OnDemand almost does not change with the increase of  $\Delta T$ . We also note that algorithm MinDis-var significantly outperforms algorithm OnDemand even when the maximum charging cycles are stable only in a short time slot  $\Delta T$  (e.g.,  $\Delta T =$  one week), which indicates that algorithm MinDis-var can quickly adapt to the changes of maximum charging cycles.

## 6 Performance Evaluation of the Algorithm for the Minimum Number of Mobile Chargers Deployment Problem

In this section, we evaluate the performance of the proposed algorithm for the minimum number of mobile chargers deployment problem through experimental simulations. We also investigate the impact of several important parameters on the algorithm performance including the network size  $n$ , the variance of energy consumption rates, the energy capacity  $IE$  of mobile charging vehicles, and the critical lifetime interval parameter  $\alpha$ .

## 6.1 Simulation environment

We consider a wireless rechargeable sensor network consisting of from 100 to 500 sensors that are randomly deployed in a  $500m \times 500m$  square. The battery capacity  $B_i$  of each sensor  $v_i \in V_s$  is set to be 10.8 *kiloJoules* ( $kJ$ ), by referring to a regular *NiMH* battery [27]. A base station is located at the center of the square, and a depot of mobile vehicles is co-located with the base station. The energy capacity of each mobile charging vehicle  $IE$  ranges from 1,000  $kJ$  to 5,000  $kJ$ . We assume that each of them travels at a constant speed of  $s = 5$   $m/s$  with energy consumption rate of  $\eta = 0.6$   $kJ/m$  [30]. The energy charging rate of each charging vehicle is  $\mu = 5$  *Watts* [13]. The default value of  $\alpha$  is 5.

We consider two different distributions of energy consumption rates of sensors: the linear distribution and the random distribution. In *the linear distribution*, the energy consumption rate  $\rho_i$  of sensor  $v_i$  is proportional to its distance to the base station. The nearest and farthest sensors to the base station have the maximum energy consumption rates  $\rho_{max}$  and the minimum energy consumption rates  $\rho_{min}$ , respectively, where  $\rho_{min} = 1$   $mJ/s$  and  $\rho_{max} = 10$   $mJ/s$ . The linear distribution models the energy consumptions of sensors in WSNs where the main energy consumption of sensors is on the data transmission and relays. Sensors close to the base station must relay the sensing data for other remote sensors, thus consuming much more energy than the others. Furthermore, by adjusting the energy consumption ratio of each sensor from  $\rho_{max}$  to  $\rho_{min}$ , this model can be used to model data aggregations at relay sensor nodes, i.e., a smaller ratio  $\frac{\rho_{max}}{\rho_{min}}$  implies a higher data aggregation. On the other hand, in *the random distribution*, the energy consumption rate  $\rho_i$  of each sensor  $v_i \in V_s$  is randomly chosen from a value interval  $[\rho_{min}, \rho_{max}]$ . The random distribution captures the energy consumption of heterogeneous sensors. For example, video camera sensors in multimedia sensor networks typically consume plenty of energy on image processing [3]. Thus, the energy consumption rates of sensors in such sensor networks do not closely correlated with the distances between the sensors and the base station. We further assume that the energy charging rate  $\mu$  of each mobile vehicle is several orders of magnitude of the energy depletion rate of sensors, i.e.,  $\mu \gg \max_{v_i \in V} \{\rho_i\}$ . A fully charged sensor can survive from 10 days up to 4 months. We put one year as our monitoring period of the sensor network. Each value in figures is the mean of the results by applying each mentioned algorithm to 50 different network topologies with the same network size.

To evaluate the performance of the proposed algorithms, we have also implemented three benchmarks  $LB_{optimal}$ , algorithm *Heuristic*, and algorithm *minMCP* [8, 21], in which  $LB_{optimal}$  is a lower bound on the minimum number of mobile chargers which is an approximate estimation of the optimal solution, i.e.,  $LB_{optimal} = \lceil WH(T)/IE \rceil$  by Eq. (17), where  $WH(T)$  is the total cost of the MST  $T$  of the metric graph  $G_s$  induced by the to-be-charged sensors, and  $IE$  is the energy capacity of each mobile charging vehicle. Algorithm *Heuristic* is described as follows. Given  $n$  to-be-charged sensors  $v_1, v_2, \dots, v_n$  indexed by their appearance in the area, we assume that the depot is the origin, and index the sensors in anti-clockwise order. Algorithm *Heuristic* assigns the vehicles to the sensors

one by one until all sensors are charged. Specifically, assume that the first  $K - 1$  mobile vehicles have been assigned to sensors  $v_1, v_2, \dots, v_{i-1}$  already. We now assign the  $K$ th mobile vehicle to charge the sensors in the sequence  $v_i, v_{i+1}, \dots, v_n$ . Initially,  $K = 1$  and  $i = 1$ . The set of sensors charged by vehicle  $K$  will be  $v_i, v_{i+1}, \dots, v_j$  if the total cost of a shortest closed tour  $C_K$  including depot  $r$  and sensors  $v_i, v_{i+1}, \dots, v_j$  is no more than the energy capacity  $IE$  while the total cost of a shortest closed tour  $C'_K$  including depot  $r$  and sensors  $v_i, v_{i+1}, \dots, v_j, v_{j+1}$  is larger than  $IE$ , i.e.,  $WH(C_K) \leq IE$  and  $WH(C'_K) > IE$ , where  $i \leq j \leq n$ . This procedure continues until all  $n$  sensors are charged.

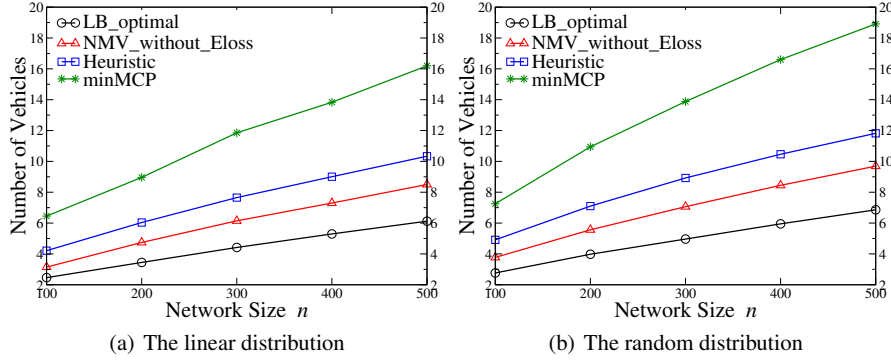
To compare our work with the two closely related works [21, 8], we adopt a variant of algorithm `minMCP` in [21, 8] since the total energy consumption of some of the closed tours delivered by their algorithms may violate the energy capacity constraint  $IE$ , and the amount of energy consumed on each such a tour can be up to  $IE(1 + \epsilon)$  with  $\epsilon > 0$  being a constant. To ensure that the energy consumption of any charging tour is no greater than the energy capacity  $IE$  of each mobile vehicle when applying algorithm `minMCP`, we set the energy capacity of mobile vehicles as  $\frac{IE}{1+\epsilon}$  when invoking the algorithm. Thus, the total energy consumption of a charging vehicle per tour will be no more than  $\frac{IE}{1+\epsilon} \cdot (1 + \epsilon) = IE$ , and we set  $\epsilon = 0.1$  in all our experiments in the default setting.

## 6.2 Performance evaluation of algorithms

We evaluate the performance of algorithms `NMV_without_Eloss`, `NMV_with_Eloss`, `Heuristic` and `minMCP` as follows, where algorithm `NMV_without_Eloss` does not take into account the sensor energy consumption during each charging tour, while algorithm `NMV_with_Eloss` does take such sensor energy consumption into consideration.

We first evaluate the performance of algorithms `NMV_without_Eloss`, `Heuristic`, and `minMCP` under the assumption that sensor energy consumption rates follow linear and random distributions, by varying the network size from 100 to 500 sensors. Fig. 9(a) plots their performance curves, from which it can be seen that the solution delivered by algorithm `NMV_without_Eloss` is fractional of the optimal one. Specifically, the number of mobile vehicles delivered by it is around 35% more than the lower bound `LP_optimal`, while the number of mobile vehicles by it is about 20% and 45% less than that by algorithms `Heuristic` and `minMCP`, respectively. The rationale behind is as follows. Given a set of to-be-charged sensors, algorithm `Heuristic` first sorts the sensors in anti-clockwise order, where the depot is the origin. The algorithm then assigns the mobile vehicles to sensors one by one until all sensors are charged. There may be some cases that some sensors charged by a mobile charging vehicle are far away from each other. Then, the charging vehicle consumes more energy on traveling, rather than on charging the sensors. As a result, more charging vehicles are needed. In contrast, the proposed algorithm `NMV_without_Eloss` will schedule a mobile charging vehicle to replenish a set of sensors whose locations are close to each other. Therefore, less charging vehicles

are required. Fig. 9(b) indicates that the four algorithms have the similar behaviors under both linear and random distributions of energy consumption rates.

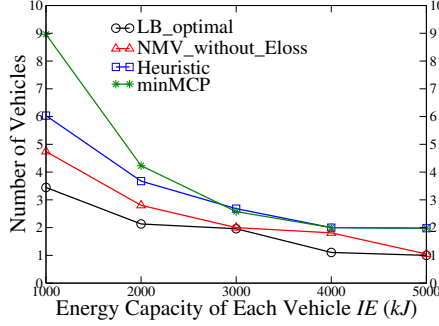


**Fig. 9** Performance of algorithms NMV\_without\_Eloss, Heuristic, and minMCP by varying network size under two different distributions of energy consumption rates when  $IE = 1,000$  kJ,  $\rho_{min} = 1$  mJ/s, and  $\rho_{max} = 10$  mJ/s.

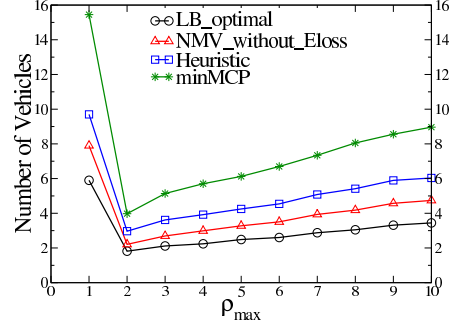
We then study the impact of the energy capacity of mobile charging vehicle  $IE$  on the performance of algorithms NMV\_without\_Eloss, Heuristic, and minMCP by varying  $IE$  from 1,000 kJ to 5,000 kJ. Fig. 10 shows that with the growth of the energy capacity  $IE$ , the number of mobile charging vehicles delivered by algorithm NMV\_without\_Eloss decreases, and the gap between the solution and the lower bound of the optimal solution becomes smaller and smaller, which implies that the performance of algorithm NMV\_without\_Eloss is near-optimal. On the other hand, the number of vehicles delivered by algorithm NMV\_without\_Eloss is up to 50% less than that by algorithm Heuristic.

We finally investigate the impact of the variance among energy consumption rates of sensors on the performance of algorithms NMV\_without\_Eloss, Heuristic, and minMCP, by varying  $\rho_{max}$  from 1 mJ/s to 10 mJ/s while fixing  $\rho_{min}$  at 1 mJ/s. Fig. 11 indicates that the number of mobile vehicles needed by each of the three algorithms NMV\_without\_Eloss, Heuristic, and minMCP decreases, followed by slowly growing. The rationale behind is that when the variance is quite small (i.e., the gap between  $\rho_{max}$  and  $\rho_{min}$  is small), the solution delivered by algorithm NMV\_without\_Eloss will include almost all sensors in each charging round, thus, a large number of mobile vehicles are required. With the increase on the variance, the number of to-be-charged sensors in each charging round significantly decreases. On the other hand, when the maximum energy consumption rate  $\rho_{max}$  becomes large, the average energy depletion rate of the sensors will be faster, the solution by algorithm NMV\_without\_Eloss will include more sensors to be charged per charging round as more sensors are within their critical life-times. In the following, we do not compare the performance of algorithm minMCP, since its performance is the worst one among the four algorithms LB\_optimal,

NMV\_without\_Eloss, Heuristic, and minMCP, which has been shown in Fig. 9–Fig. 11.

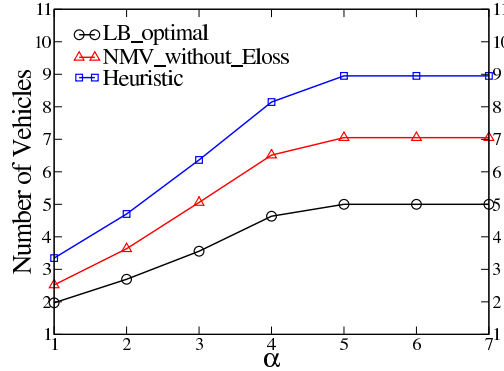


**Fig. 10** Performance of algorithms NMV\_without\_Eloss, Heuristic, and minMCP by varying the energy capacity of each mobile vehicle  $IE$  when  $n = 200$ ,  $\rho_{min} = 1 \text{ mJ/s}$ , and  $\rho_{max} = 10 \text{ mJ/s}$ .



**Fig. 11** Performance of algorithms NMV\_without\_Eloss, Heuristic, and minMCP by varying the maximum energy consumption rate  $\rho_{max}$  from  $1 \text{ mJ/s}$  to  $10 \text{ mJ/s}$  when  $n = 200$ ,  $IE = 1,000 \text{ kJ}$ , and  $\rho_{min} = 1 \text{ mJ/s}$ .

### 6.3 The impact of $\alpha$ on algorithmic performance



**Fig. 12** Performance of algorithms NMV\_without\_Eloss and Heuristic by varying  $\alpha$  when  $n = 200$ ,  $IE = 1,000 \text{ kJ}$ ,  $\rho_{min} = 50 \text{ mJ/s}$ , and  $\rho_{max} = 100 \text{ mJ/s}$ .

We now evaluate the impact of critical lifetime interval parameter  $\alpha$  on the performance of the proposed algorithms by varying the value of  $\alpha$  from 1 to 7. A smaller  $\alpha$  implies that more frequent schedulings are needed, and less numbers of mobile vehicles are employed per charging round. With the growth of  $\alpha$ , more and more sensors will be included in  $V_s$ , and more sensors will be charged by mobile charging vehicles per charging round. Fig. 12 implies that with the growth of  $\alpha$ , more charging vehicles are needed by algorithms NMV\_without\_Eloss and

Heuristic in each charging round, as more sensor nodes fall in the defined critical lifetime interval. However, it is interesting to see that no more mobile vehicles are required when the value of  $\alpha$  is greater than 6, since all sensors will be charged in each charging round.

## 7 Conclusions

In this chapter we studied the use of multiple mobile chargers to charge sensors in a wireless sensor network so that none of the sensors will run out of energy for a given monitoring period, for which we first formulated the novel service cost minimization problem of finding a series of charging schedulings of mobile chargers so that the total travelling distance of the mobile chargers for the monitoring period is minimized, and the problem of using the minimum number of mobile chargers to charge sensors such that none of the sensors will run out of energy, subject to the energy capacity constraint imposed on each mobile charger, while maintaining the perpetual operations of sensors. As these optimization problems are NP-hard, we then devised an approximation algorithm for the service cost minimization problem with a provable approximation ratio if the maximum charging cycle of each sensor is fixed in the given monitoring period. Otherwise, we developed a novel heuristic solution through modifications to the approximate solution. We further devised an approximation algorithm for the minimum number of mobile chargers deployment problem with a provable performance guarantee. We finally evaluated the performance of the proposed algorithms through extensive experimental simulations, and experimental results showed that the proposed algorithms are very promising, and the solutions obtained are fractional of the optimal ones.

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