Collusion-Resistant Repeated Double Auctions for Relay Assignment in Cooperative Networks

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***Abstract*—Cooperative communication effectively enhances the channel capacity of wireless networks by allowing some single-antenna nodes to relay data for other nodes. In such a communication scheme, choosing appropriate relay nodes is critical to maximize the overall network performance. In this paper, we consider the assignment problem of relay nodes in a cooperative wireless network, where physical relay infrastructures and relay supporting services (relay assignment) are independently operated by different selfish entities, each of which is driven by its own benefit. We first formulate the problem as a repeated double auction by taking into account the benefits of all entities in the system. That is, we consider a system consisting of a set of source-to-destination pairs, relay nodes, group agents, and the auctioneer, where source nodes are grouped into different groups and each group is represented by a group agent. The source nodes and group agents seek opportunities to maximize their own benefits through untruthful bidding, colluding with each other, and so on. We then show that these behaviors will jeopardize the social benefit of all entities in the system. To mitigate the effect of such behaviors, we devise a truthful repeated double auction that is able to bound the collusion probability of each entity. We finally conduct experiments by simulations to evaluate the performance of the proposed auction mechanism. Empirical results show that the proposed auction is effective in collusion-resistance with bounded collusion probabilities. To our best knowledge, this is the first auction mechanism for relay assignment in wireless networks that is truthful, collusion-resistant, budget-balance and individual-rational.**

***Index Terms*—Cooperative wireless communications; relay assignment; game theory; repeated double auction; collusion resistance.**

I. INTRODUCTION

WIRELESS channels often suffer from time-varying fading caused by multi-path propagation and Doppler shifts, resulting in significant performance degradation. Recently, one important technique that exploits spatial diversity achieved by employing multiple transceiver antennas has been shown to be very effective in coping with channel fadings. However, in reality equipping each wireless node with multiple antennas may not be feasible, as the footprints of multiple antennas may not fit in the wireless node. To enhance spatial diversities among wireless nodes, cooperative communications, having each node equipped with a single antenna and exploiting the spatial diversity via antennas of some relay nodes, have exhibited great potentials in the improvement of both data rates and qualities [9]. Various types of networks particularly the mobile cellular networks are hungering for high-rate and quality-guaranteed communication techniques to cater ever increasing demands of multimedia data services. However, deploying more communication infrastructures (base stations) to existing 3G/4G wireless networks has been shown to be very costly and thus is not be applicable to small cell phone carriers. In contrast, cooperative communication technology does not require adding any extra infrastructures into existing networks but offers great flexibilities. With the cooperative communication, cell phone carriers can economically enhance their network coverage and data rates by leasing their infrastructures from other carriers.

The main obstacle to applying cooperative communications to wireless cellular networks is the lack of incentives from wireless nodes to relay data for others. Most existing studies focused on devising auction mechanisms to incentive wireless nodes to relay data for other nodes [22], [26]. These mechanisms jointly consider the benefits of sellers (relay-holding carriers) and buyers (relay-requesting carriers) by assuming that the auctioneers are willing to participate if their budgets are well balanced (always being non-negative). However, in realistic cellular networks, the auctioneers usually hope to maximize their budgets since they are independent entities that provide marketing and billing services (relay assignment auctions) for infrastructure carriers. For example, according to a report by Australian Communications and Media Authority [1], all three major cell phone carriers Telstra, Optus and Vodafone in Australia have their own Mobile Virtual Network Operators. They are the independent entities operating marketing and billing services on the behalf of three of them to maximize their own benefits. These mentioned mechanisms thus may not always be applicable to the scenarios where the auctioneer is rational. This raises an important question, how to give each selfish entity an incentive to encourage it to participate in the trading while considering the revenue of the auctioneer. The incentive compatible double auction [2] is an appropriate mechanism to address this question, as it considers not only the revenues of buyers and sellers but also the revenue of the auctioneer, which is referred to as 'the social-welfare' of the system in this paper.

Since all participating entities in the system are selfish, each aims to maximize its own benefit through strategically manipulating its own bid or forming collusion groups to manipulate the auction. The negative effects of such manipulations are that the benefits of the sellers and the auctioneer will be ruined, thereby reducing their willingness to participate the auction. Therefore, designing an auction for relay assignment to resist

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such a revenue-jeopardizing manipulation poses two important challenges: one is truthfulness, which intuitively means that reporting true valuation as a bid is a dominant strategy for all participants. Another is collusion-resistance, which means that manipulating the auction through forming collusion groups is prohibited. In this paper we aim to design a truthful and collusion-resistant auction for relay assignment.

The main contributions of this paper are as follows. We first propose a two-phase auction model for the relay assignment problem in cooperative wireless networks. We then devise a repeated multi-heterogeneous-item double auction by jointly considering the benefit of each entity in the system. We analytically show that the proposed auction is truthful, and has a promising probability lower bound on the number of colluding agents. We finally conduct experiments by simulations to evaluate the performance of the proposed repeated double auction, and the results demonstrate that the proposed auction is very promising in terms of social-welfare and network capacity.

The remainder of the paper is organized as follows. We first summarize literatures on relay assignment in Section II. We then introduce the network model, the auction model, and a motivation example to demonstrate the need of collusion-resistant auctions in Section III. We thirdly devise a novel, repeated double auction and analyze its economic properties in Section IV. We finally present the numerical results and conclude in Sections V and VI.

II. Related Work

Since the initialization of three-terminal cooperative channel model in [11], lots of efforts have been taken in the research of wireless cooperative networks [3], [6], [9], [18], [19], [27], [29]. A non-negligible portion of these studies focus on the problem of relay assignment in wireless cooperative networks. For example, Shi *et al.* [18] studied the relay assignment problem by developing an Optimal Relay Assignment algorithm (ORA) through adopting a 'linear marking' mechanism. Yang *et al.* [27] considered maximizing the total channel capacity of source nodes in a cooperative wireless network where nodes transmit their data through orthogonal channels (OFDMA) to mitigate channel interference effects. They reduced the Relay Assignment Problem (RAP) to the Maximum Weighted Bipartite Matching (MWBM) problem. Zhang *et al.* [29] aim to minimize channel interferences and maximize average channel capacities in the relay assignment problem through migrating interferences in transmissions from relays to destinations. They assume that interference only occurs in links between relay nodes and destination nodes.

Relay nodes are usually reluctant to relay data for other nodes, since they need to consume their own energy and other resources. Therefore, several studies focus on designing methodologies that stimulate nodes to serve as relay nodes. For example, Shastry *et al.* [17] addressed the issue of stimulating cooperative diversity in cooperative wireless networks by proposing a pricing game that converges to a Nash Equilibrium. Wang *et al.* [22] devised a two-level Stackelberg game by jointly considering the utilities of selfish buyers and sellers. In the top level, source nodes as the buyers aim to maximize their utilities, while in the bottom level, the power allocation of source nodes is determined. Ren *et al.* [14] considered a cooperative wireless network consisting of multiple source-destination pairs and a single relay node. They designed a compensation framework to provide incentives to relay nodes to relay data for others. However, they only considered one relay node which seems not be applicable to multiple relay nodes. Yang *et al.* [28] introduced the 'virtual currency' concept to encourage relay nodes to promote the system performance by providing the incentives, or to punish the relay nodes when they are reducing the system performance. Huang *et al.* [8] investigated the relay assignment problem by incorporating fairness and energy efficiency. A truthful relay assignment auction, TASC, is devised in [26]. They successfully applied the McAfee double auction for relay assignment problem, by incorporating an optimal pre-allocation method while meeting the truthfulness.

Auction-based mechanisms perform very well in wireless cooperative networks as surveyed by the mentioned studies [8], [14], [17], [22], [25], [26], [28]. However, to apply auction-based incentive mechanisms to realistic wireless cooperative networks, we also need to consider revenues of auctioneers who are selfish entities and provide relay assignment services for relay nodes. Unfortunately little attention to this important issue has been addressed, and none of these mentioned studies jointly considered the revenues of all participating entities including the auctioneers. In addition, if collusion-resistance must be enforced, the existing auction mechanisms may not meet this property. In contrast, in this paper we consider applying repeated double auctions to the relay assignment problem. Such auctions guarantee not only truthfulness but also collusion-resistance.

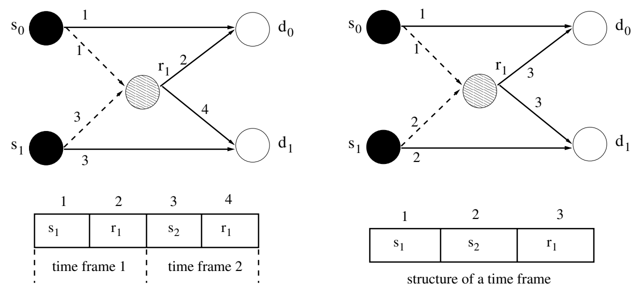
There are various collusion patterns in double auctions including buyer-seller collusions, auctioneer-seller collusions, auctioneer-buyer collusions, auctioneer-auctioneer collusions, seller-seller collusions, and buyer-buyer collusions. However, some of these patterns will not occur in relay assignment auctions for wireless cooperative networks. The reasoning is as follows. (i) If collusions between buyers (source nodes) and sellers (relay nodes) do occur, this implies that the auctioneer fails to fulfill the asks of sellers. Then, the sellers may conduct the relay assignment auctions among themselves. As a result, the problem of concern is reduced to the relay assignment problem in [8], [14], [17], [22], [26], [27], [28]. (ii) If collusions between auctioneers and sellers occur, the auctioneers and the sellers may form illegal agreements by raising relay capacity prices, which can happen only when the buyers cannot evaluate the relay capacity. However, the channel capacity of relay nodes in wireless cooperative networks can be properly evaluated. (iii) If the final trading prices are determined by the bids of buyers only, collusions between auctioneers and buyers will happen. In this paper, such collusions will be considered as trading prices in double auctions are jointly determined by bids and asks. (iv) If collusions among sellers occur, such collusions are referred to as oligopoly in economics, and pricing will be the major concern, which is out the scope of this paper. (v) As we assume only a single auctioneer available, there is no need to consider the collusions among auctioneers.

In this paper, we focus on buyer-buyer collusions, where buyers (group agents and source nodes) strategically form illegal groups to promote their revenues. This pattern of collusion can be further classified into the bidding ring collusion, the loser collusion, and the sublease collusion [7]. The bidding ring collusion can be avoided by setting a reserve price [26]. The pre-condition for the sublease collusion is that all products should be sub-leasable. That is why the sublease collusion is prevalent in spectrum auctions [24]. However, in cooperative communications, relay nodes are not sub-leasable, because relaying data needs the cooperation of relay nodes (overhearing) while a relay node can prohibit sub-leasing of its relaying services by refusing overhearing. We therefore only consider the loser collusion, where the losers in an auction seek chances to collude with others to promote their revenues.

III. System Model and Problem Definition

A. Network model

We consider cooperative wireless networks consisting of sets of source nodes $S = \{s_i \mid 1 \le i \le N\}$, relay nodes $R = \{r_j \mid 1 \le j \le J\}$, and destination nodes $D = \{d_i \mid 1 \le i \le N\}$. In such networks, relay nodes enhance communications between source nodes and destinations by overhearing source nodes and transmitting overheard data to their destinations. To enhance the overheard data, they may adopt different modes, Amplify-and-Forward (AF) and Decode-and-Forward (DF) [11]. A typical cooperative communication (CC) mechanism in cooperative wireless networks proceeds in a frame-by-frame fashion, assuming time is divided into equal time frames and each time frame t is further divided into two time slots. That is, a source node transmits its data in the first time slot of each frame, and a relay node overhears the transmission due to the broadcast nature of wireless communication. The relay node then transmits the data to a destination node in the second time slot of each time frame, as illustrated by Fig. 1(a). However, such mechanism reduces the throughput of cooperative wireless networks greatly, since source nodes lose a lot of transmission opportunities by allowing relay nodes to transmit right after each transmission of source nodes [16]. Network coding technique (NC) has potential to further enhance the network throughput by reducing wasted chances of source-node transmissions [16]. Specifically, each relay node temporarily stores overheard data from multiple source nodes first, and then encodes all the overheard data before broadcasting them to the destination nodes. An example is shown in Fig. 1(b), where source nodes s_0 and s_1 transfer their data in the first 2 time slots, and then relay node r_1 broadcasts encoded and enhanced data to destination nodes at time slot 3. Thus, only three time slots are needed. In contrast, if adopting traditional frame-by-frame fashion without network coding, it needs four time slots as shown in Fig. 1(a). Motivated by this scenario, we assume that source nodes are divided into groups, and each group is assigned with a single relay node. Denote by $G = \{g_k \mid 1 \le k \le K\}$ and $|g_k|$ the set of groups and the number of source nodes in each group g_k, respectively. Clearly, $K \le N$. It must be mentioned that session grouping of cooperative communications with network coding is out the scope of this paper, we thus assume grouping information



(a) Frame-by-frame cooperative communications

(b) Cooperative communications with network coding

Fig. 1. Cooperative communications in cooperative wireless networks.

of source nodes are given *a prior*, and can be obtained by applying existing grouping methods in [4], [13], [15].

In the following, we define the channel capacity of cooperative communications. Given the transmission power P_{s_i} of source node s_i, the SNR at destination node d_i [21] is $SNR_{s_i,d_i} = P_{s_i}/(N_{d_i} \cdot ||s_i, d_i||^\alpha)$, where N_{d_i} is the white noise at node d_i and α is the path loss factor of wireless channels which is a constant between 2 and 4, and $||s_i, d_i||$ is the Euclidean distance between s_i and d_i.

We adopt the cooperative communication model with analog network coding [16], where each group of source nodes is assigned a single relay node. Denote by W the amount of bandwidth a relay node can utilize. Let N_{r_j} be the white noise at relay node r_j. Given the transmission power P_{r_j} of relay node r_j, the channel capacity from source node $s_i \in g_k$ to destination node d_i through relay node r_j can be calculated by a simplified channel capacity model in [16] as follows.

$$C(s_i, r_j, d_i) = \frac{W}{|g_k|+1} \log_2 \Big(1 + SNR_{s_i,d_i} + \frac{SNR_{s_i,r_j} \cdot SNR_{r_j,d_i}}{|g_k|\sigma + SNR_{r_j,d_i} + \sigma \sum_{s_i \in g_k} SNR_{s_i,r_j}}\Big), \quad (1)$$

where $\sigma = 1 + \frac{(|g_k|-1)N_{r_j}\alpha_r^2}{N_{d_i}} + (|g_k| - 1)\alpha_r^2$ with $\alpha_r^2 = \frac{P_{r_j}}{|g_k|N_{r_j} + \sum_{s_{i'} \in g_k} P_{s_{i'}}}$.

B. Auction model

Double auctions have a wide application in economics to jointly fulfill the needs of buyers and sellers [10]. Each buyer submits a bid and each seller offers an ask. The auctioneer in a double auction will clear the market ultimately. Items for such auctions usually are homogeneous [10]. This type of auctions have been successfully applied to cooperative communications by adding the support for heterogeneous relay services [27]. In such applications, source nodes and relay nodes are considered as buyers and sellers, respectively. Source nodes (relay nodes) directly submit their bids (asks) to an auctioneer. The auctioneer first decides a matching of source-node bids and relay-node asks, which maximizes the total capacity achieved. It then decides the winning source and relay nodes by a similar method used in McAfee double auctions [10]. However, within our model source nodes are partitioned into different groups. Each group will be assigned a single relay node. Due to heterogeneous relaying services of relay nodes, source nodes in each group usually have different preferences in the choice of relay nodes. To model the differences, we assume that each group is represented by a group agent that makes decisions in representation of the source nodes in the group.

With a little abuse of notations, we also use $G = \{g_k \mid 1 \leq k \leq K\}$ to denote the set of group agents. Instead of directly submitting bids to the auctioneer, each source node $s_i \in g_k$ in our auction model submits its bids for relay nodes to its group agent g_k. Group agent g_k then participates in the double auction to complete relay nodes for its group members by recalculating and submitting bids to the auctioneer, and it is allowed to charge a feasible amount from its source nodes as its return of serving as a representative. In addition, the relay quality and data transmission requirements of the source nodes vary over time. Relay nodes are typically allocated to groups of source nodes dynamically. Therefore, we assume that double auctions are repeatedly carried out in the beginning of each time frame t. Transmissions from source nodes to destination nodes occur in the rest time slots of the time frame. A similar model assumption like this with a different application scenario has also been adopted in [5]. This assumption is based on the fact that auction mechanisms usually incur less communication overheads than that of source-node data transmissions. However, source-node data transmissions after each auction may be reduced when the number of source nodes in each group is quite large. To avoid this transmission reduction, one approach is to restrict the number of source nodes in each group. In addition, we also assume that the 'fluctuation' of the value of each relay node from the current time frame to the next one can be learned by various prediction methods [20], [5]. In an auction, each bidder only knows its own bid but nothing of the bids and the asks. In the end of each auction, the auctioneer only reveals the winner identities and payments not their exact bids.

To be specific, we now elaborate on each double auction carried out at the beginning of each time frame t. In the beginning of each auction, each source node s_i calculates a bid for each relay node based on its valuation on the relay node, and submits its bids for all relay nodes to its group agent. Let $b_i(t) = \{b_{i,j}(t) \mid 1 \leq j \leq J\}$ be the bid set of source node s_i, where $b_{i,j}(t)$ is the bid of source node s_i for relay node r_j at time frame t. To calculate $b_{i,j}(t)$, source nodes usually consider the achieved channel capacity by r_j as an important metric, as a higher channel capacity implies a larger system throughput and a higher channel reliability. The true valuation on relay node r_j by s_i thus is defined as $v(s_i, r_j, t) = C(s_i, r_j, d_i)$. Then, the bid of s_i for r_j is calculated by $b_{i,j}(t) = \beta_i \cdot v(s_i, r_j, t)$, where β_i is a private value representing the preference of s_i on the channel capacity, which is a constant with $\beta_i > 0$. Notice that, in truthful auctions, $\beta_i = 1$, meaning that the bid of s_i equals its real valuation, i.e., $b_{i,j}(t) = v(s_i, r_j, t)$. Let $p_i(t)$ denote the payment by source node s_i to its group agent g_k for the achieved network capacity at time frame t. Thus, the utility of source node s_i can be represented by $u(s_i) = v(s_i, r_j, t) - p_i(t)$, which is the gross benefit (valuations on the relay nodes) taken out its payment.

Having received the bids from its members, a group agent g_k then calculates its bid set $B_k(t) = \{B_{k,j}(t) \mid 1 \leq j \leq J\}$, where $B_{k,j}(t)$ is the bid of g_k for relay node r_j at time frame t. We here calculate $B_{k,j}(t)$ based on all the bids of source nodes in g_k for r_j, $\{b_i(t) \mid s_i \in g_k\}$, by adopting a truthful group winner selection of [23], which will be described later. Having been decided by the auctioneer, winning group agents will pay for the relay service. Let $p_k(t)$ denote the payment by a wining group agent g_k to the auctioneer at time frame t, the budget of g_k at time frame t is defined as

$$\Psi_{g_k}(t) = \sum_{s_i \in S_{g_k}^{win}(t)} p_i(t) - p_k(t), \tag{2}$$

where $S_{g_k}^{win}(t)$ is the set of winning source nodes in group g_k at time frame t.

Each relay node r_j has an ask for its relaying service. Let $A_j(t)$ be the ask of relay node r_j, and $p_a^{r_j}(t)$ the payment received by r_j from its clients at time frame t. The social welfare that the auctioneer aims to maximize for a time period starting from the very first time frame and ending at the last time frame t thus is

$$V(t) = \sum_{\tau=0}^{t} \Big(\sum_{g_k \in G^{win}(\tau)} p_k(t) - \sum_{r_j \in R^{win}(\tau)} p_a^{r_j}(t) \Big), \tag{3}$$

where $G^{win}(\tau) \subseteq G$ and $R^{win}(\tau) \subseteq R$ denote the sets of winning group agents and relay nodes at time frame τ, respectively. Then, $G^{win}(\tau) = R^{win}(\tau)$, as each relay node is assigned to a single group agent only. Therefore, to maximize $V(t)$, the auctioneer first needs to maximize the number of trading pairs of relay nodes and group agents. Assuming that there are abundant relay nodes in the system, the auctioneer only needs to guarantee the number of participating group agents by making their budgets non-negative. Also, a larger $p_k(t)$ of group agent g_k can lead to a larger $V(t)$. To increase $p_k(t)$ is to increase the bids of source nodes in group g_k, as $p_k(t)$ is related to the bids of the source nodes. As a result, the expected revenues of source nodes need to be guaranteed, otherwise less and less source nodes are willing to participate the auction.

To decide $G^{win}(t) \subseteq G$ and $R^{win}(t) \subseteq R$ at time frame t, the auctioneer firstly performs a maximum weight matching between group agents and relay nodes due to the heterogeneity of relay services offered by relay nodes [27]. The weight assigned to an edge between a buyer and a seller is the bid of the buyer for the seller product. Denote by a bijective function $\mathcal{A}(*)$ of asks and bids to represent the maximum matching. For example, if the buyer with bid $B_{i,j}(t)$ and the seller with ask $A_j(t)$ is matched, then $A_j(t) = \mathcal{A}(B_{i,j}(t))$ and $B_{i,j}(t) = \mathcal{A}(A_j(t))$. Matching $\mathcal{A}(*)$ here aims to maximize the total bids of all group agents. It is equivalent to maximizing the channel capacity since bids of group agents is proportional to the channel capacity by definition. Then, the auctioneer decides the winning pairs of matched group agents and relay nodes in $\mathcal{A}(*)$.

C. Economic properties

The economic properties of an auction determine the auction efficiency. *Collusion-resistance* is one of such properties that do not allow agents to collude with each other, since in some scenarios agents can form coalitions with each other to promote their own revenues through paying the cooperating agents ‘side payments’ as a return. To make collusion-resistance tractable, we assume that all buyers (source nodes and group agents) are ϵ-greedy [20]. That is, with probability $w \cdot \epsilon$, a group agent or a source node seeks to form collusion groups with others to promote its revenue, where w is the unfulfilled percentage of the expected revenue of the buyer and ϵ is a given threshold. A buyer thus would not collude with others either passively or actively when its expected revenue is fully satisfied, i.e., $w = 0$. The rationale behind is that each source node needs to find a relay node to relay its data through a bid for each relay node. Such a bid, in some extent, represents the expected revenue (channel capacity) that can be achieved by a relay node. If one of its bids is accepted by the auctioneer, the source node (the buyer) is assigned the relay node that it bids for. It thus will not seek to collude with others. For each group agent, in real implementations of our model, it can be a source node with the least (or no) data relaying needs. Such a source node voluntarily serves as the group agent, expecting to achieve some revenues. We thus assume that each group agent will not actively manipulate the auction as long as its expected revenue is fulfilled.

Let $Pr(s_i, t)$ and $Pr(g_k, t)$ be the probabilities that source node s_i and group agent g_k collude with other entities in the system at time frame t. Intuitively, if the expected revenue of a source node or a group agent can be fulfilled to some extent by providing some incentives, its collusion probability can be reduced accordingly even if it is a loser. Let $I_{s_i}(t)$ and $I_{g_k}(t)$ denote the amounts of incentives for losing source node s_i and group agent g_k in the auction at time frame t, respectively. Suppose relay node r_j is matched to group agent g_k by matching $\mathcal{A}$. Then, the collusion probabilities of each source node s_i and each group agent g_k can be calculated as follows.

$$Pr(s_i, t) = w_i \cdot \epsilon = \frac{[b_{i,j}(t) - I_{s_i}(t)]^+}{b_{i,j}(t)} \cdot \epsilon, \tag{4}$$

and

$$Pr(g_k, t) = w_k \cdot \epsilon = \frac{[(B_{k,j}(t) - \mathcal{A}(B_{k,j}(t))) - I_{g_k}(t)]^+}{B_{k,j}(t) - \mathcal{A}(B_{k,j}(t))} \cdot \epsilon, \tag{5}$$

where $[x-y]^+$ is $x-y$ if it is not negative, and zero otherwise.

By equations (4) and (5), to achieve collusion resistance, the auctioneer has to fulfil the expected revenue by each loser. However, by doing so the auctioneer will not have a balanced budget. Thus, if the collusion is unavoidable, the only thing that the auctioneer can do is to minimize the collusion chances. A cost-effective method thus is to limit the number of colluding agents under a pre-defined threshold. A number that exceeds this threshold may make relay nodes more reluctant to relay data for others, and group agents refuse to serve as the representatives of their group members. Let N_c denote the pre-defined threshold and κ the percentage that the number of colluding agents exceeds the threshold, the collusion resistance is then defined as follows.

Definition 1: An auction is a $(N_c,\ \kappa,\ p)$-collusion-resistant auction when the probability that the number of colluding group agents X exceeds a given threshold N_c by κ percentage is bounded by probability p, i.e., $Pr[X \geq (1+\kappa)N_c] \leq p$. Note that when $N_c = 0$, $\kappa = 0$ and $p = 0$, the auction becomes ideal, and no group agents collude with each other.

TABLE I
GROUP BIDS AT TIME FRAME t

	r_1	r_2	r_3	r_4	r_5	r_6	r_7
$B_1(t)$	10	0	0	3	0	0	0
$B_2(t)$	0	0	2	4	3	0	8
$B_3(t)$	6	0	0	4	0	0	0
$B_4(t)$	0	2	0	10	4	6	0
$B_5(t)$	0	0	8	0	0	9	4

In addition to collusion-resistance, truthfulness, budget balance and individual rationality are also important properties of an auction, as described in the following.

- *Truthfulness* implies that the dominant strategy of each agent is to submit truthful valuations of the products. It can be achieved by unrelated bids of participating agents to their payments.
- *Budget Balance* means that the total payment received from the buyers is no less than the total payment to the sellers in an auction.
- *Individual Rationality* means that no winning buyer will be charged more than its bid, and no winning seller will be paid less than its ask.

D. An example of collusion resistance

Collusions among entities in a system can cause a great loss of social welfare of the system. We here use an example to demonstrate the necessity of collusion resistance in the auction design for relay assignment. For the sake of clarity, in our examples we assume that the fluctuation of values of relay nodes between two consecutive time frames is zero. Consider a scenario where there are seven relay nodes, $R = \{r_j \mid 1 \leq j \leq 7\}$, and source nodes are divided into five groups, $G = \{g_k \mid 1 \leq k \leq 5\}$. Within each group, the source node with the lowest bid is sacrificed (considered as a loser), and its bid equals to the payment of each of the rest source nodes. Then, given the bid set $b_i(t)$ of each source node s_i in group g_k, the group bid $B_{k,j}$ is $B_{k,j} = \min\{b_i(t)\} \cdot (|b_i(t)| - 1)$. We do not discriminate the payments of source nodes, and all winning source nodes in each group g_k have identical payments.

Given group bids at time frame t, Table I shows the optimal relay assignment based on the maximum weight matching [27]. It can be seen that r_1 is the best choice for both g_1 and g_3, and there may exist some ‘side payments’ exchange between them in order to maximize their own revenues.

To take a deep insight on this collusion, we first list the bids of source nodes in group g_1 and g_3 at time frame t in Table II, where both g_1 and g_3 are competing for r_1, and g_1 succeeds. Let l_i^k be the source node s_i that loses the auction in group g_k. Following Table I, group agent g_3 has not been assigned relay node r_1 at time frame t. Thus, it may send a side payment to lure the loser l_6^1 in group g_1 to lower its bid. If loser l_6^1 agrees to form a collusion group with g_3, it will consequently leads to the loss of g_1 at time frame $t+1$. It must be mentioned that loser l_6^1 has a high probability to form

TABLE II
THE BIDS OF $s_i \in g_1$ FOR RELAY NODE r_1, AND $s_i \in g_5$ FOR RELAY NODE r_5 AT TIME FRAME t.

	$b_{1,1}$	$b_{2,1}$	$b_{3,1}$	$b_{4,1}$	$b_{5,1}$	$b_{6,1}$	$b_{7,1}$
$b_{i,1},\ s_i \in g_1$	3	4	9	7	8	2	0
$b_{i,1},\ s_i \in g_3$	5	6	4	2	0	0	0

TABLE III
GROUP BIDS AT TIME FRAME $t + 1$

	r_1	r_2	r_3	r_4	r_5	r_6	r_7
$B_1(t+1)$	5	0	0	3	0	0	0
$B_2(t+1)$	0	0	2	4	3	0	8
$B_3(t+1)$	6	0	0	4	0	0	0
$B_4(t+1)$	0	2	0	10	4	6	0
$B_5(t+1)$	0	0	8	0	0	9	4

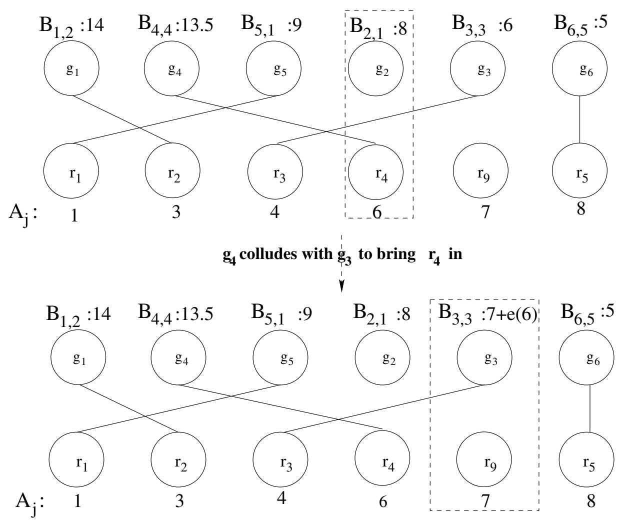


Fig. 2. An example of collusion among group agents.

such collusion groups when it is unable to increase its bid in time frame $t + 1$.

For example, if the bids of source nodes in g_1 for relay node r_1 are changed to $b_{i,1}(t + 1) = \{3, 4, 9, 7, 8, 1, 0\}$, with $i \in [1, 7]$. The group bid $B_{1,1}(t+1) = \min\{\ b_{i,1}(t+1) \mid s_i \in g_1\} \cdot (|g_1| - 1) = 5$. The optimal assignment matrix at time frame $t + 1$ is shown in Table III. As a result, though these colluding source nodes (nodes in g_3 and l_6^1) achieved their own benefits, the sum of bids received by all relay nodes decrease by 4 (from 37 to 33).

However, this is not yet the whole story of collusions if the group agents are also ϵ-greedy. Fig. 2 is an example of collusion between two group agents. In this example, the value above each group agent is its bid, while the value below each relay node is its ask. The auctioneer first matches relay nodes and group agents according to the optimal relay assignment in [27]. The matched bids and asks are then sorted in non-decreasing and non-increasing orders, respectively. Let x be the largest index in the sorted sequence satisfying $B_x > A_x$, then the relay node with ask A_x and the group agent with bid B_x will be sacrificed in this round auction. The winners are these relay nodes whose asks are lower than A_x, while the winning group agents are those ones whose bids are higher than B_x according to McAfee double auction [10], [27]. The payments by the winners are A_x and B_x, respectively. As illustrated in the top relay assignment of Fig. 2, the sacrificed ones are g_4 and r_4 ($x = 4$). Since g_4 does not receive its expected revenue, it may collude with g_3 by luring g_3 to bid $7 + e$ at the next time frame $t + 1$, where e is a very small constant. Following the mentioned rule, the revenue received by the auctioneer at time frame $t+1$, $e \cdot |G^{win}(t+1)|$, will be very small. This example indicates that the truthful property of the auction cannot guarantee its social welfare when collusions occur. Motivated by these examples, in this paper we aim to devise an auction for a relay assignment problem that is not only truthful but also collusion-resistant.

The symbols of this paper are summarized in Table IV.

E. Problem definition

Given a cooperative wireless network with multiple source-to-destination pairs and a set of relay nodes, assume that relaying services are leasing to source nodes periodically, the relay assignment problem is to design an auction such that the revenues of all the entities in the system are considered, while meeting individual rationality, budget balance, truthfulness and collusion resistance.

IV. A REPEATED DOUBLE AUCTION FOR RELAY ASSIGNMENT

In this section, we devise a repeated double auction for the relay assignment problem. We start by the process of the proposed auction and the design rationale. We then describe the detailed design that consists of two collusion-resistant stages: inter-group and intra-group winners selection. Finally, we show that the proposed, repeated double auction is collusion-resistant, truthful, individual-rational, and budget balanced.

A. Overview

Static relay assignment that assigns relay nodes to source nodes once during an entire of auction period is very effective in reducing the overhead incurred per time frame, compared with dynamic relay assignment that assigns relay nodes dynamically at each time frame. However, static resource assignment ignores the dynamics of relay nodes and may under-utilize relaying services. Instead, we focus on dynamic relay assignment by proposing a repeated double auction that allocates relay nodes to group agents periodically.

Although budget-balanced, truthful auctions for relay assignment have been shown to be feasible in general cooperative communications, the non-collusion-resistant characteristic may not be acceptable to the auctioneer when it rationally maximizes its own budget through resisting revenue-jeopardizing behaviors. In contrast, inspired by the negative effect of collusion on the social-welfare, as illustrated in the previous section, we here focus on the design of a truthful, collusion-resistant double auction.

To meet the truthfulness, we adopt the truthful double auction of [2], where truthfulness is achieved through adopting a Trade-Reduction method, in which some trade pairs are sacrificed to guarantee the truthfulness, while the winner payment is proportional to the bid of sacrificed agents.

TABLE IV
SYMBOLS

Symbols	Meaning
s_i / d_i	source/destination node i
r_j	relay node j
g_k	group agent k
$\|\|s_i, d_i\|\|$	Euclidean distance between s_i and d_i
P_{s_i}/P_{r_j}	transmission power of source node s_i / relay node r_j
N_{d_i}/N_{r_j}	white ambient noise at destination node d_i / relay node r_j
α	path loss factor of wireless channels
$C(s_i, r_j, d_i)$	channel capacity from s_i to d_i through r_j
$b_{i,j}(t)$	the bid of s_i for relay node r_j at time frame t
$B_{k,j}(t)$	the bid of group agent g_k for relay node r_j at time frame t
$A_j(t)$	ask of relay node r_j for its relaying services at time frame t
β_i	private value of source node s_i
$v(s_i, r_j, t)$	valuation of r_j by s_i at time frame t
$p_i(t)$ / $p_k(t)$	payment of s_i/g_k at time frame t
$u(s_i)$	utility of s_i
$p_a^{r_j}(t)$	payment that the auctioneer needs to pay to relay node r_j
$S_{g_k}^{win}(t)$	set of winning source nodes in group g_k
$G^{win}(t)$ / $R^{win}(t)$	set of winning group agents/relay nodes at time frame t
$V(t)$	accumulative social welfare
$\mathcal{A}(*)$	a bijective function that represents the maximum weight matching from bids to asks
ϵ	greedy degree of a group agent
$Pr(s_i, t)$ / $Pr(g_k, t)$	probability that s_i and g_k collude with others
N_c	threshold of the number of colluding agents
$I_{g_k}(t)/I_{s_i}(t)$	incentives for source nodes and group agents
w_i / w_k	unfulfilled percentage of the expected revenue of s_i and g_k
κ	percentage of the number of colluding group agents exceeding N_c
p	probability of exceeding N_c by κ

Meeting collusion-resistance is to fulfill the expected revenues of losers, and it is reasonable that the auctioneer provides small incentives to the losers to avoid the huge loss of its own revenue. The winning source nodes then transmit their data to the allocated relay nodes in the rest of the time frame. Also, to prevent the losers using the relay nodes for transmitting their own data, we assume that the relay nodes will perform a verification process prior to performing any data transmission, and will refuse enhancing the data overheard from the losers in its group.

B. Intra-group winner selection

We decompose the intra-group winner selection stage into three steps as follows. In the first step, each source node s_i submits its bid set $b_i(t) = \{b_{i,j}(t) \mid 1 \leq j \leq J\}$ consisting of bids for all relay nodes to its group agent g_k. Since group agent g_k has not been assigned a single relay node in this step, it cannot decide the winning source nodes within its group. However, it can decide candidate winners in its group by assuming it has been assigned with a relay node. Specifically, group agent g_k determines the candidate winning source nodes that competing for each relay node r_j through sacrificing the source node with the minimum bid. The rest of source nodes in group g_k are considered as the candidate winners.

The second step is to calculate the payment for each candidate winner if it is selected as a winner ultimately. To guarantee truthfulness within each group g_k, the payment of winners in g_k is equal to the bid of the loser (the source node with the minimum bid), i.e., $\min\{b_{i,j}(t) \mid s_i \in g_k\}$.

In the third step, for each given relay node r_j, each group agent g_k decides its bid set $B_{k,j}(t)$ for r_j by calculating a pre group bid $B_{k,j}^{pre}$ that is defined as follows.

$$B_{k,j}^{pre}(t) = (|g_k| - 1) \cdot \min\{\ b_{i,j}(t) \mid s_i \in g_k\}. \quad (6)$$

TABLE V
GROUP BIDS AT TIME FRAME $t + 1$

	r_1	r_2	r_3	r_4	r_5	r_6	r_7
$B_1(t)$	8	0	0	3	0	0	0
$B_2(t)$	0	0	2	4	3	0	8
$B_3(t)$	6	0	0	4	0	0	0
$B_4(t)$	0	2	0	10	4	6	0
$B_5(t)$	0	0	8	0	0	9	4

Note that this is also the payment received by g_k from its source nodes if r_j is assigned to group g_k. Resisting collusions of a loser is to provide a monetary incentive that equals its expected revenue. We thus entitle each group agent g_k to provide a portion from $B_{k,j}^{pre}(t)$ as the incentive that will pay for the losers. The portion is extracted proportionally to the bids of winning source nodes. Suppose r_j is assigned to group g_k, the candidate winners of group g_k will be the final winners in $S_{g_k}^{win}(t)$. Then, the incentive can be calculated by

$$I_{s_i}(t) = \sum_{s_i \in S_{g_k}^{win}(t)} p_i(t) \cdot \frac{b_{i,j}(t)}{\sum_{s_i \in S_{g_k}^{win}(t)} b_{i,j}(t)}. \quad (7)$$

The final group bid by group g_k is then calculated by

$$B_{k,j}(t) = B_{k,j}^{pre}(t) - I_{s_i}(t). \quad (8)$$

To illustrate the intra-group winner selection, we revisit the example used in Section III-D. Recall that the collusion between loser l_6^1 in group g_1 and group agent g_3 results in that group agent g_1 becomes a loser in time frame $t+1$ (see Table III). To resist such a collusion, g_1 may extract some incentives from its pre-bid $B_{1,1}^{pre}(t)$ for the loser l_6^1. According to Table II and Eq. (6), $B_{1,1}^{pre}(t) = (|g_1| - 1) \cdot b_{6,1} = 10$. By deducting the amount of the incentive $b_{6,1}$ $(= 2)$ for the loser l_6^1, the bid $B_{1,1}(t)$ of g_1 becomes 8 $(10 - 2)$. From Table V, we can see that g_1 still is a winner at time frame $t + 1$.

The intra-group winner selection stage is detailed in algorithm 1

Algorithm 1: *Intra-group winner selection stage at time frame t*

Input: R, G
Output: $S^{win}_{g_k}(t)$
Each source node s_i in each group submits its bid set $b_i(t) = \{b_{i,j}(t) \mid 1 \leq j \leq J\}$ for all relay nodes in R;
for *each relay node* $r_j \in R$ **do**
  All source nodes that competing r_j except the one with the minimum bid in group g_k are considered as candidate winners;
  The payment of these candidate winners in group g_k is the minimum bid, i.e., $\min\{b_{i,j} \mid s_i \in g_k\}$;
  Each group agent g_k calculates its pre bid $B^{pre}_{k,j}(t)$ by $B^{pre}_{k,j}(t) = (|g_k| - 1) \cdot \min\{\, b_{i,j}(t) \mid s_i \in g_k\}$;
  Each group agent g_k calculates the incentive for the loser (source node with minimum bid) by $I_{s_i}(t) = \sum_{s_i \in S^{win}_{g_k}(t)} p_i \cdot \frac{b_i(t)}{\sum_{s_i \in S^{win}_{g_k}(t)} b_i(t)}$;
  Each group calculate its bid $B_{k,j}$ for r_j by $B_{k,j} = (|g_k| - 1) \cdot \min\{\, b_{i,j}(t) - I_{s_i}(t) \mid s_i \in g_k\}$;
end
Group agents submit their bids to the auctioneer;
When assigned a relay node, each group agent g_k will finally determine winner set $S^{win}_{g_k}(t)$;
Only winners in $S^{win}_{g_k}(t)$ transfer payment to group agent g_k;

C. Inter-group winners selection

In the inter-group winner selection stage, each group agent first submits its bid calculated by Eq. (8) to the auctioneer. Assume there is an edge between group agent g_k and relay node r_j if g_k has a bid for r_j, and its bid $B_{k,j}(t)$ is the weight of the edge. The auctioneer then calculates a weighted maximum matching $\mathcal{A}$ between group agents and relay nodes [27]. For simplicity, in the following we omit the second subscript of $B_{k,j}(t)$ and let $B'_k(t) = B_{k,j}(t)$. Let $\langle A_{n_1}(t), A_{n_2}(t), \ldots, A_{n_J}(t)\rangle$ be an increasing sequence of asks and $\langle B'_{m_1}(t), B'_{m_2}(t), \ldots, B'_{m_K}(t)\rangle$ a decreasing sequence of bids in $\mathcal{A}$, and let x and y be the largest indexes in the two sequences such that $B'_{m_x}(t) \geq A_{n_y}(t)$. The winning matched pairs in $\mathcal{A}$ are the pairs with bids higher than $B'_{m_x}(t)$ and asks lower than $A_{n_y}(t)$. Having the winner set at time frame t, $G^{win}(t)$ and $R^{win}(t)$ are then determined.

To resist collusions among the group agents, a similar method used in intra-group winner selection stage is adopted. That is, we provide incentives to losers. It can be seen from Eq. (5) that the probability that a group agent colludes with others will drop significantly if the percentage of its expected revenue is promoted. In a series of auctions, if the auctioneer can adjust the incentives dynamically, then, the number of collusions can be controlled. To this end, the auctioneer sets an incentive according to its current budget and the number of colluding group agents when proceeding the auction at the next time frame. In other words, the auctioneer should dynamically promote this incentive to limit the number of potential colluding group agents under a tolerable threshold N_c without exceeding its budget. Let $I_{g_k}(t)$ and $I_{g_k}(t-1)$ denote the incentives for the losers at time frames t and $t-1$, respectively. Then, we have

$$I_{g_k}(t) = I_{g_k}(t-1) + \vartheta \cdot V(t), \tag{9}$$

where ϑ is a constant with $0 \leq \vartheta \leq 1$ that represents the percentage that the auctioneer draws part of its revenue for the incentive, whose value is continuously adjusted by the auctioneer over time in response to the number of colluding group agents in each auction. This readjustment procedure is a process of reinforcement learning [20]. The uniform incentives at time frame t are provided to these group agents that are not in $G^{win}(t)$ and have bids higher than $B'_{m_x}(t)$. The detailed algorithm is presented in Algorithm 2.

Algorithm 2: *Inter-group winner selection stage at time frame t*

Input: $R, G, \{B_{k,j} \mid 1 \leq k \leq K, 1 \leq j \leq J\}, N_c, \vartheta$
Output: $G^{win}(t), R^{win}(t), \{p_k(t) \mid g_k \in R^{win}(t)\}, \{p^{r_j}_a(t) \mid r_j \in R^{win}(t)\}, I_{g_k}(t)$
$\mathcal{A} \leftarrow OPRA(R, G, \{B_{k,j} \mid 1 \leq k \leq K, 1 \leq j \leq J\})$ by the algorithm in [27];
$G^{win}(t) \leftarrow \emptyset$; $R^{win}(t) \leftarrow \emptyset$;
Sort group bids and asks under $\mathcal{A}$ into non-increasing sequence, $\langle B'_{m_1}(t), B'_{m_2}(t), \ldots, B'_{m_K}(t)\rangle$, and non-decreasing sequence, $\langle A_{n_1}(t), A_{n_2}(t), \ldots, A_{n_J}(t)\rangle$;
Find the largest x and y s.t. $B'_{m_x}(t) \geq A_{n_y}(t)$;
The winning trading pairs in $\mathcal{A}$ of group agents in $G^{win}(t)$ and relay nodes in $R^{win}(t)$ are the ones with bids higher than $B'_{m_x}(t)$ and asks lower than $A_{n_y}(t)$;
$p_k(t) \leftarrow B'_{m_x}(t)\ \forall g_k \in G^{win}(t)$;
$p^{r_j}_a(t) \leftarrow A_{n_y}(t),\ \forall r_j \in R^{win}(t)$;
Let N_L be the number of losers whose bids are greater than $B'_{m_x}(t)$;
$I_{g_k}(t) \leftarrow I_{g_k}(t-1) + \vartheta \cdot V(t)$, where ϑ is used to tune the incentive for group agents in the next time frame;
if $N_L = 0$ **then**
  $\vartheta \leftarrow 0$;
elseif $N_L \leq N_c$ **then**
  $\vartheta \leftarrow (1-\gamma)\vartheta$;
else
  $\vartheta \leftarrow \gamma + (1-\gamma)\vartheta$;
Assign each group agent whose bid is lower than $B'_{m_x}(t)$ a uniform incentive $I_{g_k}(t)$;

D. Auction analysis

To analyze the correctness of the proposed auction, we need to analyze any possible collusions of the auction. Generally, only losers in the proposed auction seek to collude with others, as the colluding probability of winners are zero by Eq. (4) and Eq. (5). In the rest of this subsection, we show the correctness of the intra-group and inter-group winner selection stages.

We start with the intra-group winner selection stage by showing that the loser (source node with the minimum bid) and the winners (other source nodes in the group) will not collude with others either passively or actively in the proposed auction by the following lemma.

Lemma 1: The intra-group winner selection stage is collusion-resistant.

Proof: First notice that winners will not collude with others either passively or actively, as their expected revenues are satisfied by Eq. (4). The rest is to show losers in the intra-group winner selection stage will not collude with others. To this end, we only need to show the expected revenues of the losers can be satisfied, since losers will not collude with others as long as their revenues are satisfied.

The expected revenue of a loser (a source node) in a group agent g_k is its bid, i.e., $\min\{\, b_{i,j}(t) \mid s_i \in g_k\}$, which equals the payment of each winner, $p_i(t)$, by the intra-group winner selection rule. If no incentive for the loser is provided, the total payment that group agent g_k received from its winners will be $(|g_k|-1)\cdot p_i(t)$ by Eq. (6). To resist collusions of the loser, g_k needs to extract a portion of its received payments as the incentive to satisfy the expected revenue of the loser, $\min\{\, b_{i,j}(t) \mid s_i \in g_k\}$. Since $p_i(t) = \min\{\, b_{i,j}(t) \mid s_i \in g_k\}$ and $|g_k|-1 \geq 1$, $(|g_k|-1)\cdot p_i(t) \geq \min\{\, b_{i,j}(t) \mid s_i \in g_k\}$. This means that each group agent will always be able to afford its loser's expected revenue. The lemma then follows. ■

We then analyze the collusions initiated by group agents in the inter-group winner selection stage. By algorithm 2, a group agent loses the auction is because its bid is lower than $B'_{m_x}(t)$, or its matched relay node asks higher than $A_{n_y}(t)$. We first analyze that any two losing group agents with bids lower than $B'_{m_x}(t)$ will not collude with each other no matter which one initiates the collusion by Lemma 2.

Lemma 2: It is impossible that there is a collusion among losing group agents with bids less than $B'_{m_x}(t)$.

Proof: We show this by contradiction. Suppose that group agents g_k and g_c are losers at the auction of time frame t, and their bids at time frame t are both less than $B'_{m_x}(t)$. If they can greatly promote their bids in time frame $t+1$, they do not need to collude with others, as high bids enable them to win. That is, they want to win in time frame $t+1$ in spite of their low bids. Therefore, their bids at time frame $t+1$ are also lower than $B'_{m_x}(t+1)$ at time frame $t+1$. Without loss of generality, we assume that g_k initiates the collusion. Then, group agent g_k has to provide some incentives in terms of monetary or relay capacity in return. However, incentives in terms of relay capacity are impossible, since source nodes that have not been assigned to a relay node can not use the relaying services of the relay node. Thus, g_c will not share the relay capacity of the relay node who ask $\mathcal{A}(B_k)$ with g_k. Also, if g_k provides some monetary incentives, it needs to pay much more than its bid because $p_k(t+1) \geq B_k(t+1)$ by algorithm 2. Group agent g_k thus can not persuade g_c collude with it due to fail in providing some incentives. The lemma holds. ■

Therefore, collusions only happen between losing group agents with bids lower than $B'_{m_x}(t)$ and the ones with bids higher than $B'_{m_x}(t)$. To resist such collusions, one naive method is to provide incentives to fulfill the expected revenues of all group agents. This simple method however may incur the budget imbalance of the auctioneer by Lemma 3.

Lemma 3: If the auctioneer provides incentives to all losers to fulfil their expected revenues, then the budget of the auctioneer will be imbalanced.

Proof: Let r_l be the relay node that matched with g_l in algorithm 2. Suppose $B'_l(t)$ is the bid of a losing group agent g_l. By Lemma 2, $B'_l(t) \geq B'_{m_x}(t)$, which means that g_l lose the game because r_l asks higher than $A_{n_y}(t)$. Then, $A_l(t) = \mathcal{A}(B'_l(t))$. To be winners in time frame $t+1$, g_l may collude with other losers to bring r_l into set $R^{win}(t+1)$.

The utility the auctioneer received at time frame t is $|G^{win}(t)| \cdot |B'_{m_x}(t) - A_{n_y}(t)|$. If g_l colludes with others, the auctioneer can afford its expected revenue only when $|G^{win}(t)|\cdot|B'_{m_x}(t) - A_{n_y}(t)| \geq B'_l(t) - \mathcal{A}(B'_l(t))$. Obviously, this does not hold, since $B'_l(t) \geq B'_{m_x}(t)$ and $A_{n_y} \leq \mathcal{A}(B'_l(t))$. ■

As it is impossible to fulfil the expected revenues of all losers, a rational way is to setup a uniform incentive $I_{g_k}(t)$ for the losers whose bids are greater than $B'_{m_x}(t)$ to prevent these group agents from collusions at time frame t, this type of incentive schema is referred to as *the minimum guaranteed revenue* by the auctioneer in order to attract bidders.

We now show that the inter-group winner selection stage is (N_c, κ, p)-collusion-resistant by Lemma 4. To this end, we first give some notations that facilitate the proof. For collusions among the group agents, we assume that each group agent g_k has a probability $Pr(g_k)$ to collude with the other group agents. According to Eq. (5), there is not any chance for any group agent in $G^{win}(t)$ to collude with others, as its collusion probability is zero ($Pr(g_k \in G^{win}(t)) = 0$). Denote by X_k an *i.i.d* event of whether group agent g_k colludes or not. Then, $X_k = 1$ when g_k colludes with others, otherwise $X_k = 0$. It can be seen that $\langle X_1, X_2, \ldots, X_K\rangle$ is a Poisson sequence. Then, $Pr(X_k) = Pr(g_k)$. Denote by X the event representing the number of colluding group agents, then $X = \sum_{g_k \in \{g_i | B_i > B'_{m_x}\}} X_k$. Let μ be the expected number of colluding group agents, then $\mu = E[X] = \sum_{g_k \in \{g_i | B_i > B'_{m_x}\}} E(X_k)$. Without loss of generality, we assume that $N_c \geq \mu$, which indicates that the number of colluding agents is well controlled. We then have

Lemma 4: The number of colluding group agents which exceeds the given threshold N_c by κ percentage is bounded by $\left(\frac{e^{\zeta-1}}{\zeta^\zeta}\right)^\mu$ where $\zeta = \frac{(1+\kappa)\cdot N_c}{\mu}$ when $N_c \geq \mu$, or $2^{-(1+\kappa)N_c}$ when $N_c \geq 6\mu$.

Proof: Let g_k be a loser with collusion probability $Pr(g_k)$. Since $N_c \geq \mu$ and $\delta = \zeta - 1 > 0$, we have $Pr[X \geq (1+\kappa)\cdot N_c] = Pr[X \geq (1+\delta)\cdot\mu]$. According to the Chernoff bound [12], the probability that the number of colluding group agents exceeding the given threshold N_c by κ percentage is bounded by $Pr[X \geq (1+\kappa)\cdot N_c] \leq \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu$. Substitute δ with $\zeta - 1$, we then have $Pr[X \geq (1+\kappa)\cdot N_c] \leq \left(\frac{e^{\zeta-1}}{\zeta^\zeta}\right)^\mu$. In a special case where $N_c \geq 6\mu$, we have $(1+\kappa)N_c \geq 6\mu$ since $0 \leq \kappa \leq 1$. Then, $Pr[X \geq (1+\kappa)\cdot N_c] \leq 2^{-(1+\kappa)\cdot N_c}$. The lemma follows. ■

Note that when the uniform incentive is fixed, the colluding probability of each group agent is fixed, too, which means that μ must be fixed as well. Thus, μ can be treated as a constant.

In summary, we have shown that the intra-group winner selection stage is collusion-resistant, and the inter-group winner selection stage is (N_c, κ, p)-collusion-resistant. We thus have the following theorem.

Theorem 1: The proposed repeated double auction is $(N_c, \kappa, \left(\frac{e^{\zeta-1}}{\zeta^\zeta}\right)^\mu)$-collusion-resistant, individually rational, budget-balanced, and truthful.

Proof: By Lemma 1 and Definition 1, the intra-group winner selection stage of algorithm 2 is collusion-resistant ((1, 0, 1)-collusion-resistant). By Lemma 4, the inter-group winner selection stage is $(N_c, \kappa, \left(\frac{e^{\zeta-1}}{\zeta^\zeta}\right)^\mu)$-collusion-resistant. Since $\left(\frac{e^{\zeta-1}}{\zeta^\zeta}\right)^\mu < 1$, the proposed auction thus is $(N_c, \kappa, \left(\frac{e^{\zeta-1}}{\zeta^\zeta}\right)^\mu)$-collusion-resistant. The rest is to show that proposed auction is individually rational, budget balanced, and truthful.

We start by showing that the proposed auction is individually rational as follows. For each winning source node s_i, we have $b_{i,j} \geq \min\{b_{i,j}(t) \mid s_i \in g_k\} = p_i(t)$; for each winning relay node $r_j \in R^{win}$, we have $A_j(t) \leq A_{n_y} = p_a^{r_j}(t)$; and for each group agent g_k, we have $B_{k,j}(t) \geq B'_{m_x}(t) = p_k(t)$.

We then show that the proposed auction is budget balanced. From the perspective of group agents, their budgets in the end of the auction period depends on the value of $B_{k,j}(t) - B'_{m_x}(t)$. Since each group agent g_k has already extracted the incentive for the loser from the payments of the winners in the group according to Eq. (7), and $B_{k,j}(t) - B'_{m_x}(t) > 0$, this implies that the budget of each group agent is balanced. Let $\mathcal{L}$ be the set of losing group agents. From the perspective of the auctioneer, the utility is $|G^{win}|(B'_{m_x}(t) - A_{n_y}(t)) - |\mathcal{L}| \cdot I_{g_k}(t)$. Although there is a minimum incentive for losers, the auctioneer will not pay the incentives to the losers if $|G^{win}|(B'_{m_x}(t) - A_{n_y}(t)) - |\mathcal{L}| \cdot I_{g_k}(t) \leq 0$. Therefore, the budget of the auctioneer in each auction is balanced, too.

Finally, we show the truthfulness of the proposed auction. Since we adopt the common truthful method that a portion of the participants are sacrifices, the proof of truthfulness is straightforward and can be found in [23], [26], [30]. In the intra-group winner selection stage, it is obvious that the payments of the winners are independent of their bids. While in the inter-group winner selection stage, the selection process is truthful, because the winner determination is bid-monotonic and ask-monotonic [2], which mean if a group agent wins, it will also win by bidding higher, and if a relay node wins, it will also win by asking lower. Furthermore, the calculation of payments is both bid-independent for group agents and ask-independent for relay nodes. Therefore, the proposed auction is truthful. ■

V. Numerical Results

In this section we evaluate the performance of the proposed repeated double auction and investigate the impact of several parameters on the performance. We first study the impact of collusions of the proposed auction for the following three scenarios: (i) both source nodes and group agents do not seek to collude with the others; (ii) only the source nodes seek to collude with the other source nodes; and (iii) only group agents seek to collude with the other group agents. For the sake of convenience, we denote the repeated auction without collusion resistance under these three scenarios as RDA-WOC, RDA-SC and RDA-GC, respectively. We then examine the positive effect of the proposed collusion-resistant double auction (RDA-CR) by comparing it with RDA-SC and RDA-GC. Finally, we study the performances of auction RDA-CR through varying parameters ϵ and ϑ.

A. Simulation environment

We consider a cooperative wireless network with 200 source nodes that are randomly deployed in a 100 m × 100 m square region. The bandwidth of all channels is 22 MHz. The transmission range and transmission power for each node are 20 meters and 1 $Watt$, respectively. The path loss exponent is 4 and the noise at each destination node is $10^{-10} dBs$. Source nodes are geographically grouped into 25 distinct groups. Accordingly, the number of group agents is set to 25. Source nodes intending to transmit their data at each time frame are chosen randomly. The network capacity achieved by each source node is calculated by Eq. (1). Parameters ϵ and ϑ of the auctioneer will vary in most of our experiments.

There are 25 relay nodes that are selected based on the generated network. Specifically, we assume that there is an edge between two source nodes if they are within the transmission range of each other. Intuitively, relay nodes should be scattered in the whole network and cover as many source nodes as possible. We thus use a maximal independent set of nodes in the network as the candidate relay nodes. Notice that the cardinality of the maximal independent set may be smaller than 25. If this happens, we add other relay nodes into the set iteratively until its size reaches 25.

B. Impact of collusions

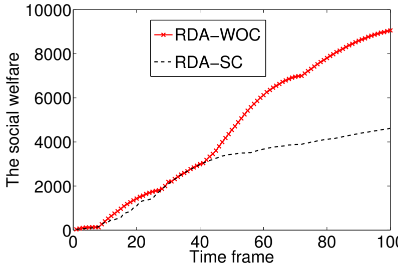
To confirm the negative impact of collusions on the social welfare, we first consider a case where there are 25 group agents with 0-greedy ($\epsilon = 0$) and 200 source nodes with 0-greedy ($\epsilon = 0$) for RDA-WOC, we then study a case where there are 25 group agents with 0-greedy and 200 source nodes with 0.3-greedy for RDA-SC, and we finally deal with a case where there are 25 group agents with 0.3-greedy and 200 source nodes with 0-greedy for RDA-GC. For each of these cases, we evaluate the performance of auctions RDA-WOC, RDA-SC and RDA-GC as follows.

Fig. 3(a) plots the social welfare curves when auctions RDA-WOC and RDA-SC are applied. It can be seen that auction RDA-SC reduces the social welfare of the auctioneer, due to source node collusions, a loser in one group submits a very low bid for a specific relay node, which consequently reduces the bid of its corresponding group agent for that relay node, according to our intra-group winner selection algorithm. Consequently, the social welfare will decrease accordingly.

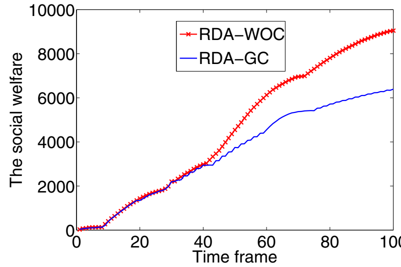
The variation curves of social welfare are plotted in Fig. 3(b) when group agents are allowed to collude with each other. It can be seen that the collusions among the group agents will reduce the social welfare as time goes, as such collusions can reduce either the number of trading pairs between group agents and relay nodes or the net-income of the auctioneer from each trading pair, i.e., $p_k(t) - p_a^{r_j}(t)$, thereby reducing the social welfare by Eq. (3). Fig. 3(c) shows the average network capacity achieved by all source nodes over 100 time frames, from which it can be seen that collusion behaviors of group agents and source agents all cause serious degradations in network capacity, compared with the scenario no agents collude with others (RDA-WOC).

C. Comparison with non-collusion-resistant algorithms

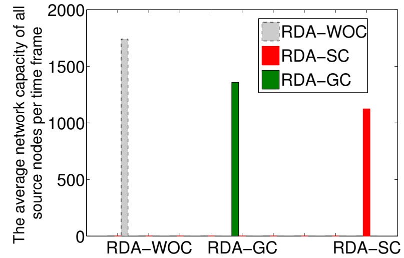
To examine the positive effect of auction RDA-CR against auctions RDA-SC and RDA-GC by fixing the number of source nodes to 200, the number of group agents to 25, and ϵ to 0.3. For auction RDA-CR, the incentive parameter ϑ and the learning parameter γ are set to 0.1 and 0.2, respectively. Fig. 4(a) plots the social welfare curves delivered by auctions RDA-CR, RDA-SC and RDA-GC, respectively. It can be seen that RDA-CR outperforms both RDA-SC and RDA-GC over



(a) Impact of source-node collusion on the social welfare

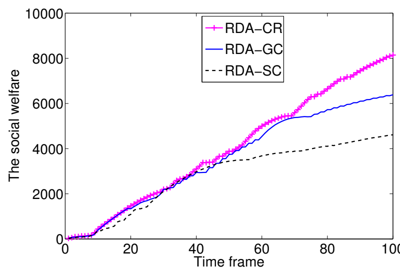


(b) Impact of group-agent collusion on the social welfare

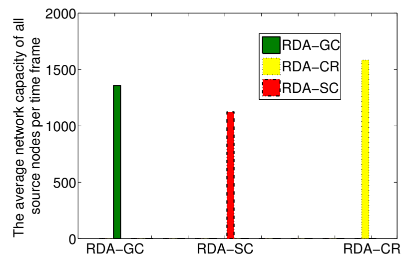


(c) Impact of group-agent collusion on the average network capacity over 100 time frames

Fig. 3. Impact of collusions on the social welfare and network capacity.

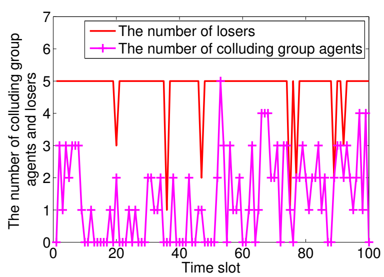


(a) The social welfare comparison of auctions RDA-CR, RDA-SC and RDA-GC

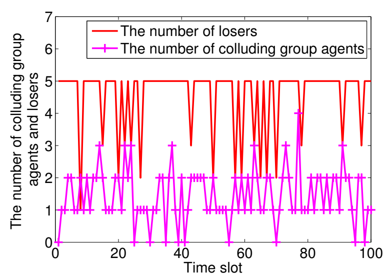


(b) The network capacity comparison of auctions RDA-CR, RDA-SC and RDA-GC

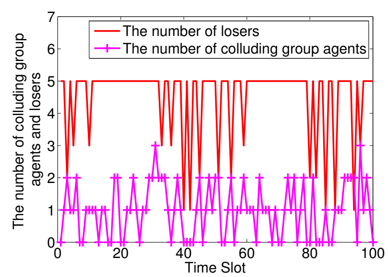
Fig. 4. The performance enhancement of collusion-resistant algorithms.



(a) $\vartheta = 0.1$



(b) $\vartheta = 0.2$



(c) $\vartheta = 0.3$

Fig. 5. Impact of ϑ on the number of colluding group agents.

time, as the colluding probabilities of group agents in RDA-CR are reduced by providing all the losers with a uniform incentive. Therefore, the social welfare increases as less group agents choose to collude with each other. Fig. 4(b) plots the average network capacity curves by all source nodes over 100 time slots, and it can be seen that the network capacity can be improved if collusion-resistance is imposed.

D. Impact of parameters ϑ and ϵ

In order to investigate the performance of auction RDA-CR under different sets of group agents with various ϵ, the number of source nodes and group agents are fixed at 200 and 25, respectively. The threshold of colluding number of group agents, N_c, is set to 2. The value of ϑ varies from 0.1 to 0.3, and the value of ϵ varies from 0.2 to 0.6. Fig. 5(a) indicates that there are a largest number of time frames in which the number of colluding group agents exceeds N_C when ϑ is small in comparison with Figures 5 (b) and (c), because a lower ϑ implies a lower incentive $I_{g_k}(t)$, which results in more losers to collude with the others.

The impact of parameter ϵ on the social welfare is plotted in Fig. 6, from which it can be seen that the greater the value of ϵ, the higher the probability of that group agents collude with others. Consequently, a lower the social welfare will be the result, which can be observed from Fig. 6 where the curve with $\epsilon = 0.2$ has the highest social welfare while the one with $\epsilon = 0.6$ has the lowest social welfare.

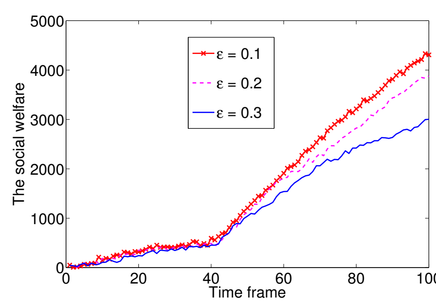


Fig. 6. Impact of parameter ϵ on the social welfare.

VI. Conclusion

In this paper we dealt with the relay assignment problem in a wireless cooperative network to prompt the social welfare among source nodes, relay nodes, group agents and the auctioneer, by mitigating negative effects on the collusions among the entities, for which we first devised a truthful and collusion-resistant repeated double auction, RDA-CR, which can guarantee that the number of colluding group agents is controlled under a tolerable threshold, by providing uniform incentives and dynamically adjusting the incentives over time. We then showed that RDA-CR not only controls the collusion probability of each group agent but also meets several important economic properties of auctions including the individual rationality, budget balanced, and truthfulness. We finally conducted extensive experiments by simulations to validate the analytical results of the proposed auction.

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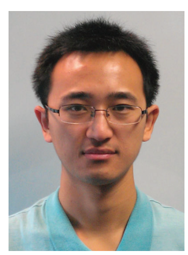

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