

# Approximation Algorithms for Charging Reward Maximization in Rechargeable Sensor Networks via a Mobile Charger

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**Abstract**—Wireless energy transfer has emerged as a promising technology for wireless sensor networks to power sensors with controllable yet perpetual energy. In this paper, we study sensor energy replenishment by employing a mobile charger (charging vehicle) to charge sensors wirelessly in a rechargeable sensor network, so that the sum of charging rewards collected from all charged sensors by the mobile charger per tour is maximized, subject to the energy capacity of the mobile charger, where the amount of reward received from a charged sensor is proportional to the amount of energy charged to the sensor. The energy of the mobile charger will be spent on both its mechanical movement and sensor charging. We first show that this problem is NP-hard. We then propose approximation algorithms with constant approximation ratios under two different settings: one is that a sensor will be charged to its full energy capacity if it is charged; another is that a sensor can be charged multiple times per tour but the total amount of energy charged is no more than its energy demand prior to the tour. We finally evaluate the performance of the proposed algorithms through experimental simulations. The simulation results demonstrate that the proposed algorithms are very promising, and the solutions obtained are fractional of the optimum. To the best of our knowledge, the proposed algorithms are the very first approximation algorithms with guaranteed approximation ratios for the mobile charger scheduling in a rechargeable sensor network under the energy capacity constraint on the mobile charger.

**Index Terms**—Rechargeable wireless sensor networks, wireless energy transfer, mobile chargers, approximation algorithms, combinatorial optimization problem, sensor energy replenishments.

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## I. INTRODUCTION

WIRELESS Sensor Networks (WSNs) have shown great potential in various applications, from military surveillance to environmental monitoring, to disaster reliefs, and to home automation [10], [26], [29]. Energy efficiency is critical for a WSN to achieve a long lifetime. Currently, most sensor nodes in a WSN are powered by batteries. The batteries add significant size, cost to the system, and may also be hazardous to the environment. One solution to this is to allow each sensor scavenge energy from surrounding energy sources such as solar, vibration, temperature variations, wind, and biochemical processes [7], [14], [18]. The drawback of this approach, however, lies in its high reliance on unpredictable environmental conditions [13]. For example, solar-powered nodes may fail to work if there is insufficient sunshine. *Rechargeable Sensor Networks* – consisting of sensors with rechargeable batteries and a mobile charger (or mobile charging vehicle or drone) to charge the sensors – is emerging as a promising solution to prolong their network lifetimes [4], [17], [20], [25], [27].

Most existing studies on rechargeable sensor networks assumed that each mobile charging vehicle (or mobile charger) has sufficient energy to charge all sensors in a WSN [17], [22], [24], [30]. However, in a large sensor network, the amount of energy carried by a mobile charger may not be enough to charge all nearly-expired sensors in a single tour when there are a large portion of sensors to be charged. Existing approaches thus are not applicable to rechargeable sensor networks with energy-limited mobile charging vehicles. New algorithms for scheduling the charging tours of a mobile charger must be devised, to determine which sensors should be charged if not all of sensors can be charged during the charging tour of the mobile charger [11].

In this paper we study a fundamental sensor charging problem. Given a set of energy-critical sensors and the energy capacity of the mobile charger, how to determine which sensors should be charged? and how to fairly charge the sensors through finding a charging tour for the mobile charger? where charging fairness means that a sensor under a critical energy level should be charged first compared with another sensor with plenty of residual energy. In this paper we will address the mentioned challenges by devising approximation algorithms with provable approximation ratios for the charging reward maximization problems.

### A. Novelty of This Work

The novelty of our work lies in three aspects. First, unlike most existing studies in literature that assumed either all sensors can be charged by the mobile charger per tour without any energy capacity constraint on the mobile charger, or each sensor can be charged to its full energy capacity without taking into account its residual energy, we are the first to formulate novel charging reward maximization problems in rechargeable sensor networks, where more prizes can be collected by charging a sensor with less residual energy than that by charging a sensor with much residual energy, and a sensor can be charged multiple times per tour (due to other energy-critical sensors to be charged during the course), while it may not be charged to its full energy capacity at each time. Second, this work builds a connection between the prize collection and the urgency of sensor charging. It assigns a node a certain amount of prizes to model its urgency to be charged before its energy depletion, i.e., the amount of prizes assigned to a sensor is inversely proportional to its residual energy. Finally, existing studies of traditional reward maximization problems [1], [2] in an undirected graph assumed that *the prize* at each node and *the length* of each edge are two independent metrics, i.e., maximizing the reward collected from the nodes while bounding the length of a traveling path by a given value. In this paper, both the prize at each node and the length of each edge are the functions of energy, and they are closely related to each other. If more energy is spent on the travelling path, then less energy will be available for sensor charging, thereby resulting in less prizes collected from the charged sensors, or vice versa.

To the best of our knowledge, this is the first approximation algorithm with a constant approximation ratio for scheduling a mobile charger to charge a set of sensors to maximize the sum of collected prizes (or to maximize the survival lifetimes of these sensors), under the energy capacity constraint imposed on the mobile charger.

### B. Contributions

The main contributions of this paper are as follows. We first formulate novel optimization problems of using a mobile charger to wirelessly charge a set of sensors under the energy capacity constraint on the mobile charger, with an objective of maximizing the total prize collected from all charged sensors per tour, we term the problems as the fully and partially charging reward maximization problems respectively, depending on whether each sensor can be charged multiple times by the mobile charger per its tour. We then show that the problems are NP-hard, and instead devise approximation algorithms with constant approximation ratios for them. A key technique in the design of approximation algorithms is a non-trivial reduction that reduces each of the problems into an orienteering problem in an auxiliary, undirected graph, and a solution to the latter will return a solution to the former, through a series of transformations. In addition to providing analytical solutions with provably performance guarantees, we also conduct empirical evaluation on the performance of the proposed approximation algorithms through experimental simulations. Simulation results demonstrate that the proposed

algorithms significantly outperform other heuristics, and the solutions delivered by the proposed algorithms are fractional of the optimum.

The remainder of the paper is organized as follows. Section II reviews related studies. Section III introduces the system model, notations and notions, and problem definitions. Section IV shows that the problem of concern is NP-hard. Sections V and VI devise constant approximation algorithms for the fully charging reward maximization problem and partially charging reward maximization problem, respectively. Section VII evaluates the performance of the proposed algorithms, and Section VIII concludes the paper.

## II. RELATED WORK

With the advance in the wireless energy transfer technology based on strongly magnetic resonances [8], wireless energy replenishments have been adopted for the lifetime prolongation of WSNs in literature [9], [17], [21], [30]. Although the adoption of this technology is still in its infancy stage, several studies have been conducted recently. Most of these studies made use of a mobile charger to replenish energy to sensors and to collect sensing data from the charging sensors simultaneously [9], [17], [19], [21], [30]. For example, Shi *et al.* [17] and Xie *et al.* [21] conducted a theoretical study on the efficient usage of the wireless charging technique for WSNs, by employing a wireless charging vehicle to periodically charge sensors such that the network can operate perpetually. Zhao *et al.* [30] proposed a joint design of energy replenishment and data gathering by exploiting sink mobility, and provided an adaptive solution that jointly selects the sensors to be charged and finds an optimal data gathering scheme such that the network utility can be maximized. Li *et al.* [9] argued that the mentioned charging schemes so far only passively replenish sensors that are deficient in energy supply, and cannot fully leverage the strength of wireless energy transfer technology. They instead proposed a ‘charging-aware’ routing protocol (J-RoC) that incorporates dynamic energy consumption rates of sensors into the design of data routing protocols. Although this schema can pro-actively guide routing activities and charge energy to sensors, this makes the design and management of routing protocols more complicated. For example, the deployed routing protocols in a sensor network sometimes are required to be updated periodically due to the security concern of sensing data.

In contrast, several recent works studied passive energy replenishments to sensors. For example, Xu *et al.* [22] considered the problem of scheduling  $K$  mobile chargers to replenish a set of sensors such that the length of the longest charging tour among the  $K$  chargers is minimized, for which they proposed constant approximation algorithms. Liang *et al.* [11], [12] considered an optimization problem of minimizing the number of mobile chargers to charge a set of sensors, assuming that the energy capacity of each mobile charger is limited. Ren *et al.* [15] provided a novel charging paradigm and proposed efficient sensor charging algorithms, considering the requirements of dynamic sensing and transmission behaviors of sensors. Ye and Liang [28] considered sensor

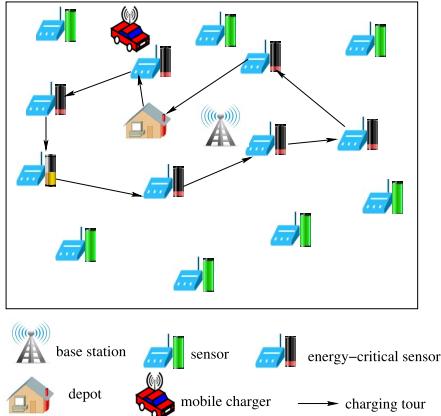


Fig. 1. A rechargeable sensor network.

charging with stringent charging deadlines, by formulating the problem as a charging utility maximization problem. He *et al.* [5] examined a mobile charging problem, using a Nearest-Job-Next with preemption, and provided analytical results on the number of sensor charging requests served and the charging latency at each charged sensor, assuming that sensor charging request rates follow a Poisson distribution. Their solution however cannot guarantee all sensors to be charged prior to their energy depletion. Given a set of to-be-charged sensors with different residual lifetimes, they also provided an adaptive algorithm to schedule a mobile charger to charge a proportion of the sensors before their energy depletions, with the objective to maximize the total amount of energy charged to sensors minus the total traveling energy cost of the charger [20]. In addition, Xu *et al.* [23] argued that it may take a long time to charge a sensor to its full energy capacity, instead they aim to minimize the depletion period of each sensor by charging each sensor with an amount of energy to its 'satisfied' energy level. Thus, a mobile charger can charge as many sensors as possible.

In the same spirit of the study in [23], we here assume that a sensor can be partially charged to a certain energy level and the sensor can be charged multiple times per charging tour. Also, a sensor with less residual energy should be urgently charged compared with a sensor with plenty of residual energy. We aim to find a charging tour for a mobile charger such that the total reward collected from all charged sensors on its closed tour is maximized while the total energy consumption on the tour is upper bounded by the energy capacity of the mobile charger.

### III. PRELIMINARIES

In this section we first introduce the system model, notations and notions, and then precisely define the problems.

#### A. System Model

We consider a large wireless sensor network  $G_s = (V_s, E_s)$  deployed in a monitoring region for environmental monitoring or event detection, where  $V_s$  is a set of sensors and a base station. There is an edge in  $E_s$  between each pair of sensors or a sensor and the base station if they are within the transmission range of each other. Each sensor  $v \in V_s$

sends its sensing data or requests to the base station via multi-hop relays. Also, each sensor  $v \in V_s$  is powered by a rechargeable battery with energy capacity  $B_v$ , and it consumes energy when performing sensing, data processing, and data transmissions and receptions. Each sensor can be charged by a wireless mobile charger (MC) if needed. Without loss of generality, we assume that there is sufficient energy supply to the base station. Fig. 1 illustrates a wireless rechargeable sensor network.

To maintain the long-term operation of  $G_s$ , its sensors will be charged at certain time points by a mobile charger which is located at a depot  $r$ . To this end, sensors in  $G_s$  will send their charging requests to the base station, and the mobile charger will be scheduled to respond the charging requests. Assume that the mobile charger has a full energy capacity  $IE$  that can be used for charging a nearby sensor (e.g., 2.6 meters within the location of the mobile charger [6]) with a fixed charging rate  $\mu$ , and for its mechanical movement at a constant speed  $s$ . We further assume that the energy consumption rate of the mobile charger per unit length is  $\xi$  when it travels at a constant speed  $s$ . The mobile charger will start from depot  $r$  when performing charging duties and return to the depot after finishing its charging tour to recharge itself for the next charging tour. Since the energy capacity of the mobile charger is limited, it will consume energy on its own travel and sensor charging during a charging tour, its total energy consumption thus is upper bounded by its energy capacity  $IE$ .

#### B. Charging Tours and Charging Rewards

Sensors in  $G_s$  can send their charging requests to the base station, it is the base station to schedule the mobile charger to charge these sensors. A sensor will issue a charging request to the base station when its residual energy is below a given threshold. The base station will respond to sensor charging requests periodically. That is, if the mobile charger is on its charging tour at the moment, all requests received at the base station during this period will be considered in the next charging tour. We assume that there is a server deployed at the base station, the next charging tour of the mobile charger will be delivered by running the designed scheduling algorithm on the server, and the base station dispatches the mobile charger to charge the requested sensors.

Denote by  $G = (V, E; \pi_0, l')$  a weighted undirected graph with a distance function on its edges  $l' : E \mapsto \mathcal{R}^+$ , and a *prize* or *reward* on its nodes  $\pi_0 : V \mapsto R^+$ . Let  $\pi_0(v)$  be the prize at node  $v$ , and  $r$  be a special node called the *root* or the depot of the mobile charger. For a path  $P_{u,v}$  in  $G$  from node  $u$  to node  $v$ , let  $l'(P_{u,v})$  be the length of path  $P$ , i.e.,  $l'(P_{u,v}) = \sum_{e \in P_{u,v}} l'(e)$ , where  $l'(e)$  is the length of edge  $e$ .

Let  $V_c \subseteq V_s$  be the set of sensors that send their charging requests to the base station. We assume that a charging request from a sensor  $v$  is expressed by a triplet  $(id_v, RE_v, R(v))$ , where  $id_v$  is the sensor ID,  $RE_v$  is the residual energy of  $v$ , and  $R(v)$  is the charging request time of  $v$ . Denote by  $G_c = (V_c, E_c; \pi_0, l')$  the induced metric graph by the nodes in  $V_c$  of to-be-charged where  $\pi_0(v)$  is the amount of energy charged to sensor  $v$  with  $0 \leq \pi_0(v) \leq B_v - RE_v$ . Let  $n = |V_c|$ .

A mobile charger that initially stays at a depot  $r$  and will return to the depot after finishing its charging tour, we term such a charging tour as *a closed tour*. The mobile charger is powered by batteries with limited capacity  $IE$ . The amount of energy spent by the charger on both its travelling and sensor charging in each closed tour should be no more than its energy capacity  $IE$ .

### C. Problem Definitions

We first define *the charging utility maximization problem* with travelling distance constraint. Given an undirected metric graph  $G_c = (V_c, E_c; \pi, l)$  where  $V_c$  is the set of sensors requested to be charged ( $V_c \subseteq V_s$ ) with  $l : E_c \mapsto Z^+$ , assume that there is a mobile charger located at a depot  $r \in V_c$  with the total traveling distance being bounded by an integer  $L$ . Associated with each sensor  $v \in V_c$ , there is a positive integer prize  $1 \leq \pi(v) \leq n^2$  to model the gain by charging the sensor at a tour of the mobile charger. In other words, a sensor with less residual energy will have a larger prize as it needs to-be-charged urgently and will be charged with more energy. The problem is to find a closed tour  $C$  for the mobile charger such that the sum of the prizes collected from the sensors in the closed tour  $C$ ,  $\sum_{v \in C} \pi(v)$ , is maximized, while the total traveling distance of the mobile charger,  $l(C)$  is no greater than  $L$ , i.e.,  $\sum_{e \in C} l(e) \leq L$ .

Notice that the charging utility maximization problem is a subproblem of the two optimization problems in this paper that are defined as *Definition 1* and *Definition 2*, respectively.

*Definition 1:* Given a sensor network  $G_s = (V_s, E_s)$  and a subset  $V_c$  of sensors requested to be charged ( $V_c \subseteq V_s$ ), assume that there is a mobile charger at depot  $r$  with energy capacity  $IE$  that will be used for its traveling and sensor charging, *the fully charging reward maximization problem* in  $G_s$  is to find a closed tour  $C$  for the mobile charger such that the sum of prizes collected from all charged sensors in  $C$ ,  $\sum_{v \in C} \pi(v)$ , is maximized, subject to that the total amount of energy consumed on sensor charging and the traveling of the mobile charger is no greater than its energy capacity  $IE$ , i.e.,  $\sum_{v \in C} (B_v - RE_v) + \sum_{e \in C} l'(e) \cdot \xi \leq IE$ , assuming that each sensor in  $C$  will be charged to its full energy capacity, where the prize assigned to a sensor is proportional to the amount of energy it will be charged, or inversely proportional to the residual energy of the sensor, and  $\xi$  is the amount of energy consumption of traveling unit length by the mobile charger.

In the problem definition 1, we assume that once a sensor is charged by the mobile charger, it will be charged to its full energy capacity. In practice, due to the charging duration and the total energy capacity constraint on the mobile charger, it is highly desirable that the mobile charger can charge as many lifetime-critical sensors as possible in order to minimize the number of dead sensors or shorten their expiration periods per tour. Thus, a sensor can be charged to a certain level of energy at each time, and can be charged multiple times by the mobile charger per tour. We then define a generalized sensor charging optimization problem as follows.

*Definition 2:* Given a sensor network  $G_s = (V_s, E_s)$  and a subset  $V_c$  of sensors requested to be charged ( $V_c \subseteq V_s$ ),

assume that there is a mobile charger at depot  $r$  with energy capacity  $IE$  that will be used for its traveling and sensor charging, *the partially charging reward maximization problem* in  $G_s$  is to find a closed tour  $C$  for the mobile charger such that the sum of prizes collected from all charged sensors in  $C$ ,  $\sum_{v \in C} \pi(v)$ , is maximized, subject to that the total amount of energy consumed on sensor charging and traveling is no more than its energy capacity  $IE$ , assuming that a sensor can be partially charged at each time and can be charged multiple times per tour. However, the total amount of energy charged to a sensor by the mobile charger per tour is no greater than the energy demand of the sensor.

Notice that every sensor in the closed tour must be fully charged in the problem *Definition 1*. In contrast, every sensor in the problem *Definition 2* is allowed to be partially charged each time, and it can be multiple charged per tour. Thus, the mobile charger can charge more energy-critical sensors (i.e., can collect more prizes) during one charging tour.

### D. Approximation Algorithm

We say an algorithm for a maximization optimization problem is an  $\alpha$ -approximation algorithm if the ratio of the approximate solution to the optimal solution is no less than  $\frac{1}{\alpha}$ , where  $\alpha$  is a constant with  $\alpha \geq 1$ .

## IV. NP-HARDNESS

In this section we show that the decision version of the fully charging reward maximization problem in a wireless rechargeable sensor network is NP-hard, so is the partially charging reward maximization problem, since the former is a special case of the latter. We thus only show that the fully charging reward maximization problem is NP-hard.

*Theorem 1:* The decision version of the fully charging reward maximization problem in  $G_s = (V_s, E_s)$  with the set  $V_c$  ( $V_c \subseteq V_s$ ) of sensors to be charged is NP-hard.

*Proof:* The proof is contained in the Supplementary Material.  $\square$

## V. APPROXIMATION ALGORITHM FOR THE FULLY CHARGING REWARD MAXIMIZATION PROBLEM

In this section we deal with the fully charging reward maximization problem under the total energy capacity constraint on the mobile charger. We approach this problem through *a novel reduction* by reducing it to the charging utility maximization problem with travelling distance constraint in another auxiliary graph  $G'' = (V', E'; \pi, l)$  with  $\pi : V' \mapsto Z^{\geq 0}$  and  $l : E' \mapsto Z^{\geq 0}$ . We first devise an efficient solution to the charging utility maximization problem with travelling distance constraint. We then elaborate on the non-trivial reduction. A solution to the latter in turn will return a feasible solution to the former.

### A. Approximation Algorithm for the Charging Utility Maximization Problem

We start with an approximation algorithm for the orienteering problem [1]. We then devise an approximation algorithm for the charging utility maximization problem, by reducing it to the orienteering problem, and a feasible solution to the latter then returns a feasible solution to the former.

Given an undirected graph  $G = (V, E; \pi, l)$  with  $\pi : V \mapsto Z^+$  and  $l : E \mapsto Z^+$ , a source node  $s \in V$ , a destination  $t \in V$ , and a non-negative integer  $L > 0$ , the *orienteeering problem* in  $G$  is to find a path  $P_{s,t}$  from node  $s$  to node  $t$  such that the total reward collected from the nodes in  $P_{s,t}$ ,  $\sum_{v \in P_{s,t}} \pi(v)$ , is maximized, while the length  $\sum_{e \in P_{s,t}} l(e)$  of path  $P_{s,t}$  is no greater than  $L$ , i.e.,  $\sum_{e \in P_{s,t}} l(e) \leq L$ .

The mentioned orienteeering problem is NP-hard, and there is an approximation algorithm with an approximation ratio of 3 for it due to Bansal *et al.* [1], which is an improvement of the result in [2] if the edge weights in  $G$  abide by the triangle inequality. Denote by  $T_{\text{ori}}(|V|, |E|)$  the time complexity of their approximation algorithm for the orienteeering problem in a graph  $G(V, E)$  with  $|V|$  nodes and  $|E|$  edges in the rest of this paper.

We now study the charging utility maximization problem with travelling length constraint  $L$ , which is the key ingredient in the development of approximation algorithms for the fully charging reward maximization problem under the energy capacity constraint  $IE$  on the mobile charger. We show how to reduce the charging utility maximization problem into the orienteeering problem, and an approximate solution to the latter will return an approximate solution to the former. Denote by  $G_c = (V_c, E_c; \pi_0, l')$  a metric graph induced by the set  $V_c$  of sensors to be charged with  $\pi_0 : V_c \mapsto R^+$  and  $l' : E_c \mapsto R^+$ , we assume that  $\pi_0(v) = B_v - RE_v$  for each  $v \in V_c$ . We further assume that the depot  $r$  of the mobile charger is in  $V_c$  for the sake of convenience.

Without loss of generality, we adopt the similar scaling approach as in [1] and [2]. That is, we assume that prizes at nodes in  $G_c(V_c, E_c)$  are integers in the range  $\{1, 2, \dots, n^2\}$  – this allows us to “guess” the reward collected by the optimal solution by trying out all integer values less than  $n^3$ . We can make this assumption by scaling the prizes down such that the maximum reward is exactly  $n^2$ , this guarantees that the optimal solution gets at least  $n^2$  reward. We then round the prize value of each node down to a nearest integer, losing an additive amount of at most  $n$  in total, which is a negligible multiplicative factor. Since the new reward obtained by the optimal solution is at least  $n^2$ , we only lose a factor of at most 2 in approximation.

The idea behind the proposed algorithm is to treat the depot  $r$  as two virtual nodes  $r$  and  $r'$ , and if there is any edge incident to node  $r$  there is an edge incident to node  $r'$  too. We assume that the resulting graph is still  $G_c$  after adding node  $r'$  and its incident edges. We then convert the amount of energy charged to a sensor into a prize assigned to the node. The problem then becomes finding a path from  $s (= r)$  to  $t (= r')$  in  $G_c$  such that the total reward collected from the nodes in the path is maximized, while the length of the path is no greater than  $L$ . The detailed algorithm for the charging utilization maximization problem is given in Algorithm 1.

#### B. An Approximation Algorithm for the Fully Charging Reward Maximization Problem

We deal with the fully charging reward maximization problem by reducing it to the charging utility maximization problem, and an approximate solution to the

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**Algorithm 1** finding a closed tour  $C$  in  $G_c$  rooted at  $r$  with a total length of the mobile charger  $L$  for the charging utility maximization problem

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**Input:** An undirected metric graph  $G_c = (V_c, E_c; l)$ , a depot  $r \in V_c$  and its virtual copy  $r' \in V_c$ ,  $l : E_c \mapsto Z^+$ , a given integer  $L \in Z^+$ , assume that each sensor  $v \in V_c$  is expressed as  $(id_v, RE_v, 0)$ . Let  $\Delta_{\max} = \max\{B_v - RE_v \mid v \in V_c\}$  and  $\Delta_{\min} = \min\{B_v - RE_v \mid v \in V_c\}$  be the maximum and minimum amounts of energy charged to the sensors in  $V_c$ , assuming that  $\Delta_{\min} \geq \Delta_{\max}/n^2$  with  $|V_c| = n$ .

**Output:** A closed tour  $C = P_{r,r'}$  including the source node  $r$  so that the award collected from all charged nodes in  $P_{r,r'}$  is maximized, while the length of the closed tour  $C$  by the mobile charger is no greater than  $L$ .

- 1:  $\delta \leftarrow \frac{\Delta_{\max}}{n^2}$ ; /\* convert the amount of energy charged to each sensor into an integer prize \*/
  - 2: Calculate the prize  $\pi(v)$  of each node  $v \in V_c$ , if  $RE_v \neq 0$ , then  $\pi(v) \leftarrow \lfloor \frac{(B_v - RE_v)}{\delta} \rfloor$ ; otherwise,  $\pi(v) \leftarrow M$ , where  $M$  is the given maximum prize that can be collected from a sensor, which is a value no more than  $n^2$ , e.g.,  $M = n^2$ ; /\* a node with a large prize implies less residual energy left and more energy will be charged to it \*/
  - 3: Find a maximum utility path  $P_{r,r'}$  in  $G$  such that the sum of prizes of the nodes in  $P_{r,r'}$  is maximized, while the length of the path  $P_{r,r'}$  is no greater than  $L$ , by applying the approximation algorithm for the orienteeering problem due to Bansal *et al.* [1];
  - 4: **return** path  $P_{r,r'}$ , and the total utility of prize in  $P_{r,r'}$ ,  $\sum_{v \in P_{r,r'}} \pi(v)$ .
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latter in turn returns an approximate solution to the former.

We first construct an auxiliary undirected, weighted graph  $G' = (V', E'; \pi, w)$  from  $G_c = (V_c, E_c; \pi_0, l')$  as follows. For each node  $v \in V_c$  in  $G_c$ , three corresponding nodes  $v_0, v_1$  and  $v_2$  are added to  $V'$ , and two edges  $(v_1, v_0)$  and  $(v_0, v_2)$  are added to  $E'$ . Each of these two edges is assigned a weight  $\frac{B_v - RE_v}{2}$ , which is half the amount of energy needed to charge node  $v$  to its full energy capacity if sensor  $v$  will be charged in the current tour of the mobile charger. The prize on  $v_0$  is  $\pi(v_0) = \lfloor \frac{B_v - RE_v}{\delta} \rfloor$  while  $\pi(v_1) = \pi(v_2) = 0$ , where  $\Delta_{\min} = \min_{v \in V_c} \{B_v - RE_v\}$ ,  $\Delta_{\max} = \max_{v \in V_c} \{B_v - RE_v\}$ , and  $\delta = \frac{\Delta_{\max}}{n^2}$ . In our discussion we assume that  $\Delta_{\min} \geq \Delta_{\max}/n^2$ . For the depot  $r$  of the mobile charger which is a special node, assign  $\pi(r_0) = \pi(r_1) = \pi(r_2) = 0$  and  $w(r_1, r_0) = w(r_0, r_1) = 0$ . For each edge  $(u, v) \in E_c$ , we add two edges  $(v_2, u_1)$  and  $(u_2, v_1)$  in  $G'$ , and assign each of the edges a weight  $l'(u, v) \cdot \xi$ , which is the amount of energy consumed by the mobile charger travelling along edge  $(u, v)$  and  $\xi$  is its energy consumption rate per unit length travelling.

The intuition behind the construction of  $G'$  is as follows. As we need to charge a sensor  $v$  with the amount of energy  $A$ , we create three virtual nodes  $v_0, v_1$ , and  $v_2$  in the auxiliary graph  $G'$  for sensor  $v$  such that the corresponding prize of  $A$  is assigned to  $v_0$ , and we set both  $v_1$  and  $v_2$  with prizes of zeros. Also, we have an edge between  $v_0$  and  $v_1$  ( $v_2$ ). Since

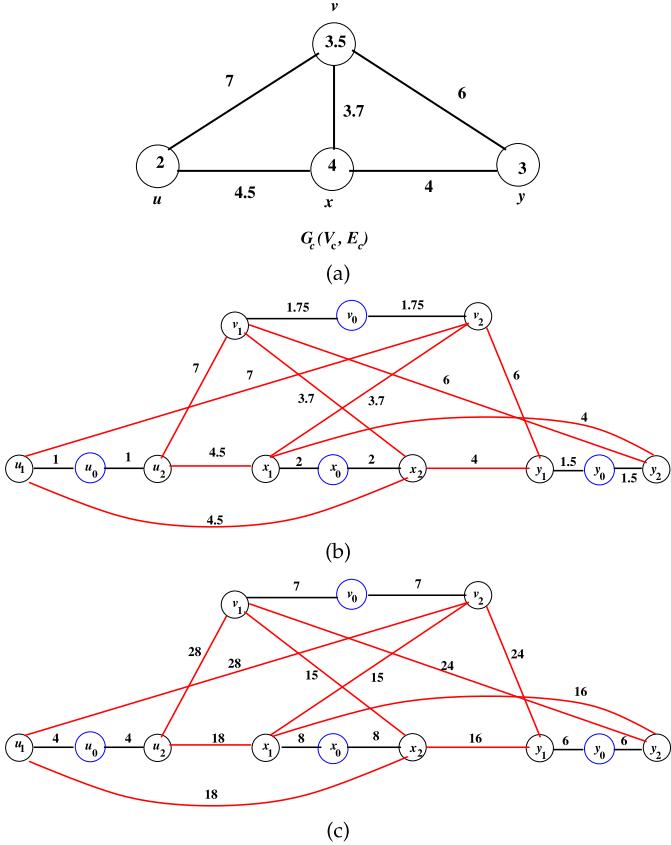


Fig. 2. The constructions of  $G'$  and  $G''$  from  $G_c$ , where the real number in each node or link in  $G_c$  represents the amount of energy to be charged to the sensor or the amount of energy consumed by traveling along the link, and the real number on each edge of  $G'$  is the amount of energy consumed to charge the sensor and traveling along the edge, while the integer length of each edge in  $G''$  is obtained by first dividing the edge weight by  $\delta$  and then rounding the real value upto a nearest integer no less than the real number. (a) The network  $G_c(V_c, E_c; \pi_0, l')$ . (b) The auxiliary graph  $G'(V', E'; \pi, w)$  of  $G_c$ . (c) The auxiliary graph  $G''(V', E'; \pi, l)$  of  $G'$ .

the charging tour of the mobile charger is a closed tour, this implies that the mobile charger must pass through both nodes  $v_1$  and  $v_2$  in its charging tour in order to collect the prize at node  $v_0$ . As the amount of energy charged to sensor  $v$  is  $A$ , the two edges  $(v_1, v_0)$  and  $(v_0, v_2)$  in  $G'$  are assigned the identical weight  $A/2$ . We treat the subgraph consisting of nodes  $v_0, v_1$ , and  $v_2$  for a sensor node  $v$  as a widget, we then connect the widgets of different sensor nodes in  $G_c(V_c, E_c)$  together to form the auxiliary graph  $G'$ . That is, for each edge  $(u, v) \in E_c$ , we add two edges  $(u_2, v_1)$  and  $(u_1, v_2)$  in  $G'$  to connect the two widgets derived by sensor nodes  $u$  and  $v$ , and each of the edges is assigned a value which is the amount of traveling energy consumed by the mobile charger on the edges. Thus, any closed tour in  $G'$  including the mobile charger will be a candidate solution to the fully charging reward maximization problem if the total energy consumption in the tour is upper bounded by  $IE$ .

Fig. 2 illustrates the construction of  $G'$  from the original graph  $G_c$ . In Fig 2(a), the number within each node is the amount of energy to be charged to it and the weight on each edge is the amount of energy consumed of the mobile charger traveling along the edge. Fig 2(b) is the resulting graph  $G'$  by transforming the energy consumption on sensor charging

and the traveling of the mobile charger of  $G_c$  into a graph with edge weights only. Meanwhile, the amount of energy charged to a node now is converted into a prize value for the node. In this example, we have  $\Delta_{max} = 4$ ,  $\Delta_{min} = 2$ ,  $n = 4$ , and  $\delta = \frac{\Delta_{max}}{n^2} = \frac{4}{4^2} = 1/4$ . The prizes on nodes  $v, u, x$  and  $y$  are  $\pi(v_0) = \lfloor \frac{B_v - RE_v}{\delta} \rfloor = \lfloor \frac{3.5}{1/4} \rfloor = 14$ ,  $\pi(u_0) = \lfloor \frac{B_u - RE_u}{\delta} \rfloor = \lfloor \frac{2}{1/4} \rfloor = 8$ ,  $\pi(x_0) = \lfloor \frac{B_x - RE_x}{\delta} \rfloor = \lfloor \frac{4}{1/4} \rfloor = 16$ , and  $\pi(y_0) = \lfloor \frac{B_y - RE_y}{\delta} \rfloor = \lfloor \frac{3}{1/4} \rfloor = 12$ , respectively. The prize of each of the rest of nodes in  $G'$  is set to zero. Clearly, the edge weights of  $G'$  meet the triangle inequality.

The weight of each edge in  $G'$  however is the amount of energy needed, which is a real value, not an integer. The rest is how to convert this real energy value into another integer length in another auxiliary graph  $G''$  that has the same topological structure as  $G'$ . Fig 2(c) is the resulting graph  $G''$  by converting the real weight value of each edge in  $G'$  into a nearest integer length no less than the real value obtained through dividing the real weight by  $\delta$ , as the approximation algorithm due to Bansal *et al.* [1] is only applicable to a graph with integer edge weights and the edge weights meet the triangle inequality. For example, the weights of edges  $(v_1, x_2)$  and  $(v_2, x_1)$  in  $G'$  are 3.7, we are given  $\delta = 1/4$ , then the lengths of edges  $(v_1, x_2)$  and  $(v_2, x_1)$  in  $G''$  are  $l(v_1, x_2) = l(v_2, x_1) = \lceil \frac{w(v_1, x_2)}{\delta} \rceil = \lceil \frac{3.7}{1/4} \rceil = \lceil 14.8 \rceil = 15$ .

Following the discussion in the previous subsection, we add a virtual copy  $r'$  of the depot  $r$  and its incident edge into  $G_c$ . Notice that  $G_c(V_c, E_c)$  meets the triangle inequality in terms of the length of each edge. It can be seen that  $G'$  meets the triangle inequality in terms of the energy weight on each edge, and the auxiliary graph  $G''$  that has the identical topological structure as  $G'$  meets the triangle inequality in terms of the integer length of each edge. The fully charging reward maximization problem then reduces to the problem of finding a maximum utility path (or a closed tour  $C$ )  $P_{r_0, r'_0}$  in  $G''$  from the depot  $r_0$  to a node  $r'_0$  such that the total prize collected from the nodes in  $P_{r_0, r'_0}$  is maximized, subject to that the length of path  $P_{r_0, r'_0}$  is bounded by  $L$  (or  $IE$  in terms of the energy metric). The detailed algorithm is described in Algorithm 2.

### C. Analysis of the Proposed Algorithms

We analyze the performance and the time complexities of the proposed algorithms.

**Lemma 1:** Given an undirected graph  $G_c(V_c, E_c; \pi_0, l)$  with  $\pi_0 : V_c \mapsto R^{>0}$  and  $l : E_c \mapsto Z^+$  where  $\pi_0(v) = B_v - RE_v$  for each  $v \in V_c$ , its edge weight meeting the triangle inequality, and a given integer value  $L > 0$ , there is an approximation algorithm, i.e., Algorithm 1, for the charging utility maximization problem with travelling distance constraint  $L$ , which delivers an approximate solution with an approximation ratio of 4. The time complexity of the proposed algorithm is  $T_{ori}(|V_c|, |E_c|)$ , where  $T_{ori}(|V|, |E|)$  is the time complexity of the approximation algorithm of Bansal *et al* [1] for the orienteering problem in a graph with  $|V|$  nodes and  $|E|$  edges.

*Proof:* The proof is contained in the Supplementary Materials.  $\square$

**Algorithm 2** finding a closed tour  $C$  for the mobile charger for the fully charging reward maximization problem

**Input:** An undirected metric graph  $G = (V_c, E_c; \pi_0, l')$ , a depot  $r \in V$ , assume that each requested sensor  $v \in V_c$  is expressed as  $(id_v, RE_v, 0)$ , Let  $\Delta_{max}$  and  $\Delta_{min}$  be the maximum and minimum amounts of energy to be charged to the sensors in  $V_c$  and  $\Delta_{min} \geq \frac{\Delta_{max}}{n^2}$ . If a sensor has an amount  $\epsilon \geq 0$  of energy, it is assumed that it runs out energy already, where  $n = |V_c|$ .

**Output:** A closed tour  $C = P_{r_0, r'_0}$  including the root  $r$  so that the total charging reward collected from the nodes in  $C$  is maximized, while the total amount of energy consumption of the mobile charger is no greater than  $IE$ .

- 1:  $\delta \leftarrow \frac{\Delta_{max}}{n^2}$ ;
- 2: Calculate the prize  $\pi(v)$  of each node  $v \in V_c$ , if  $RE_v \geq \epsilon$ , then  $\pi(v) \leftarrow \lfloor \frac{(B_v - RE_v)}{\delta} \rfloor = \lfloor \frac{\pi_0(v)}{\delta} \rfloor$ ; otherwise  $\pi(v) \leftarrow n^2$ ;
- 3: Construct the auxiliary graph  $G'(V', E'; \pi, w)$  from graph  $G_c$ ;
- 4: An auxiliary graph  $G'' = (V', E'; \pi, l)$  is derived from  $G'$  where  $l(e) = \lceil w(e)/\delta \rceil$  for each edge  $e \in E'$ ;
- 5:  $L \leftarrow \lfloor \frac{IE}{\delta} \rfloor$ ;
- 6: Find a maximum utility (reward) path  $P_{r_0, r'_0}$  in  $G''$  such that the sum of the prizes collected from the nodes in the path is maximized, while the length of  $P_{r_0, r'_0}$  is no greater than  $L$ , by applying the approximation algorithm due to Bansal *et al.* [1];
- 7: A closed tour  $C'$  for the mobile charger is derived from a closed tour  $C = P_{r_0, r'_0}$ , which is an approximate solution to the problem;
- 8: Each corresponding sensor node  $v$  with non-negative prize in  $C'$  will be fully charged with the amount of energy  $B_v - RE_v$ , and each corresponding edge in  $E_c$  will consume the actual amount of energy needed by the mobile charger.

**Lemma 2:** If the edge weight in  $G' = (V', E'; \pi, w)$  meets the triangle inequality, then the edge weight in  $G'' = (V', E'; \pi, l)$  also meets the triangle inequality.

*Proof:* Following the construction of  $G''$ , we have  $l(e) = \lceil \frac{w(e)}{\delta} \rceil$  for each its edge  $e$ . It is easily verified that the triangle inequality property still holds under the metric  $l(\cdot)$  in  $G''$  if it does hold for metric  $w(\cdot)$  in  $G'$ .  $\square$

Lemma 2 ensures that the approximation algorithms due to Bansal *et al.* [1] is applicable to  $G''$ .

**Theorem 2:** Given a sensor network  $G_s(V_s, E_s)$  with the subset  $V_c (\subseteq V_s)$  of charging sensors and a mobile charger (mobile charger) with the energy capacity  $IE$ , there is an approximation algorithm, Algorithm 2, for the fully charging reward maximization problem, which delivers an approximate solution with the approximation ratio of 4. The time complexity of Algorithm 2 is  $T_{ort}(3|V_c|, 2(|V_c| + |E_c|))$ , where  $T_{ort}(|V|, |E|)$  is the time complexity of the approximation algorithm of Bansal *et al.* [1] for the orienteering problem in a graph with  $|V|$  nodes and  $|E|$  edges.

*Proof:* The proof is contained in the Supplementary Materials.  $\square$

## VI. APPROXIMATION ALGORITHM FOR THE PARTIALLY CHARGING REWARD MAXIMIZATION PROBLEM

In this section we deal with the partially charging reward maximization problem. We assume that each sensor can be charged multiple times per tour due to some energy-critical sensors to be urgently charged to mitigate their dead durations. It must be mentioned that although each sensor  $v \in V_c$  can be charged multiple times during each tour of the mobile charger, the amount of energy charged to it at each time may be different, and the total amount of energy charged to it in the entire tour is no greater than its actual amount of energy demand,  $B_v - RE_v$ .

### A. Algorithm Overview

The idea behind the proposed algorithm is described as follows. Given a sensor  $v$  with its residual energy  $RE_v$ , denote by  $g(e_v)$  the utility gain for charging an amount  $e_v$  of energy to sensor  $v$ , which is defined as follows.

$$g(e_v) = \frac{f(RE_v + e_v) - f(RE_v)}{\delta}, \quad (1)$$

where  $e_v \in [e_{min}, B_v - RE_v]$ , function  $f(x)$  is an increasing submodular function (e.g.,  $f(x) = \log(x+1)$ ) whose utility gain is diminishing with the growth of the value of  $x$ , i.e.,  $f(x+\Delta) - f(x) \geq f(y+\Delta) - f(y)$  if  $0 \leq x \leq y$  and  $\Delta (> 0)$  will be defined later. Notice that  $e_{min}$  is the minimum amount of energy charged to a sensor if the sensor will be charged by the mobile charger in a tour. The rationale behind the adoption of the sub-modular function  $f(\cdot)$  (sometimes it is also referred to as a utility function) is as follows. We model the energy charging to a sensor as a submodular function, which implies that the utility gain margin by charging a sensor with much residual energy is far less than that by charging a sensor with less residual energy, as the latter will be dead if it will not be charged as soon as possible. In case a sensor has run out of its energy, a maximum utility gain  $M$  will be assigned to it.

Recall that the minimum amount of energy charged to a sensor per charging is  $e_{min}$ . The maximum number of chargings to a sensor at each closed tour of the mobile charger thus is no more than  $\lfloor \frac{\max_{v \in V_c} \{B_v\}}{e_{min}} \rfloor$ . Denote by  $K_v = \lfloor \frac{B_v - RE_v}{e_{min}} \rfloor$  the maximum number of possible chargings to sensor  $v$  per tour.  $K_v$  virtual copies  $v_1, v_2, \dots, v_{K_v}$  of sensor  $v$  are created, and each virtual copy  $v_i$  corresponds to an amount  $e_{min}$  of energy charged to sensor  $v$ . The charging utility gain contributed to virtual copy  $v_i$  of sensor  $v$  is  $g_i = \frac{f(RE_v + i \cdot e_{min}) - f(RE_v + (i-1) \cdot e_{min})}{\delta}$ , where  $1 \leq i \leq K_v$ . It can be seen that  $g_1 \geq g_2 \geq \dots \geq g_{K_v}$ , as  $f(\cdot)$  is an increasing submodular function.

The strategy we adopt for the partially charging reward maximization problem in  $G_c(V_c, E_c; l')$  is to reduce the problem to the fully charging reward maximization problem in another graph  $G_1 = (V_1, E_1; \pi_0, l')$ , and a feasible solution to the latter will return a feasible solution to the former. The construction of graph  $G_1 = (V_1, E_1; \pi_0, l')$  from  $G_c(V_c, E_c; l')$  is as follows.  $V_1 = \{v_i \mid v \in V_c, 1 \leq i \leq \lfloor \frac{B_v - RE_v}{e_{min}} \rfloor\}$ , where each virtual copy  $v_i$  of sensor  $v$  represents an amount  $e_{min}$

of energy to charge sensor  $v$ . Let  $(u, v) \in E_c$  be an edge in  $G_c$ . Assume that sensors  $u$  and  $v$  have  $K_u (\geq 1)$  and  $K_v (\geq 1)$  virtual copies in  $V_1$ , which are  $u_1, u_2, \dots, u_{K_u}$  and  $v_1, v_2, \dots, v_{K_v}$  respectively. Then, the set of edges derived from an edge  $(u, v) \in E_c$  in  $G_1$  is  $E_{u,v} = \{(u_i, v_j) \mid (u, v) \in E_c, 1 \leq i \leq K_u, 1 \leq j \leq K_v, u \in V_c, v \in V_c\}$ , and the set of edges derived from nodes  $u$  and  $v$  are  $E_u = \{(u_i, u_j) \mid u \in V_c, 1 \leq i, j \leq K_u, i \neq j\}$  and  $E_v = \{(v_i, v_j) \mid v \in V_c, 1 \leq i, j \leq K_v, i \neq j\}$ , respectively. Thus,  $E_1 = \bigcup_{(u,v) \in E_c} E_{u,v} \cup_{v \in V_c} E_v$ . The length of each edge  $(u_i, v_j) \in E_1$  is equal to the length of edge  $(u, v)$  in  $G_c$ , i.e.,  $l'(u_i, v_j) = l'(u, v)$ , while the lengths of all edges  $(v_i, v_j)$  derived from each node  $v$  are set to zeros, i.e.,  $l'(v_i, v_j) = 0$ .  $G_1(V_1, E_1; \pi_0, l')$  is a node and edge weighted graph with  $\pi_0 : V_1 \mapsto R^+$  and  $l' : E_1 \mapsto R^{\geq 0}$ , and each virtual copy of a sensor will be charged with the amount  $e_{min}$  of energy if the virtual copy is included in a closed tour of the mobile charger.

The auxiliary graph  $G_1(V_1, E_1; \pi_0, l')$  is treated as the original graph  $G_c(V_c, E_c; l')$  in the previous section (Section V), i.e., each virtual copy  $v_i \in V_1$  of every sensor  $v \in V_c$  has the amount  $e_{min}$  of energy to be charged and the prize collected by charging it is  $\pi_0(v_i)$ . Each edge derived from a sensor node  $v$  has a length of zero, and each edge derived from an edge between two different sensor nodes has of identical length as the one of the original edge. Note that the edge weights in  $G_1$  meets the triangle inequality.

Another auxiliary graph  $G' = (V', E'; \pi, w)$  from  $G_1(V_1, E_1; \pi_0, l')$  with  $\pi : V' \mapsto \{1, 2, \dots, n\}$  and  $w : E' \mapsto R^+$  can then be constructed as follows, by applying the reduction technique in Section V. That is, the prize of a virtual copy  $v_i$  of sensor  $v$  is

$$\pi(v_i) = \left\lfloor \frac{f(RE_v + i \cdot e_{min}) - f(RE_v + (i-1) \cdot e_{min})}{\delta} \right\rfloor, \quad \text{for all } i \text{ with } 1 \leq i \leq \left\lfloor \frac{B_v - RE_v}{e_{min}} \right\rfloor, \quad (2)$$

where  $\delta$  is a scaling factor to make the largest prize among the sensors no greater than  $n$ , while the smallest prize among the sensors is no less than 1, which will be defined later in the analysis of the proposed algorithm. There are three corresponding nodes  $v_{i,0}, v_{i,1}$ , and  $v_{i,2}$  in  $G'$  for each virtual node  $v_i$  in  $G_1$ . The only difference is that the prize of each node in  $G'$  is calculated by Eq. (2). As a result,  $G'$  is a graph with a weight on each of its nodes representing the prize collected from the node if the node is charged with an amount of  $e_{min}$  of energy, and a weight on each edge representing the energy consumption of the mobile charger traveling along the edge or the amount of energy for sensor charging if the edge is derived from a node.

Having the auxiliary graph  $G'$ , auxiliary graph  $G'' = (V', E'; \pi, l)$  is then constructed from  $G'$  such that the weight of each edge in  $G''$  is a non-negative integer, by adopting the similar scaling and rounding techniques as we did in Section V. Also, it can be verified that the edge weights of  $G''$  meet the triangle inequality if the edge weights of  $G'$  meet the triangle inequality. The approximation algorithm due to Bansal *et al.* [1] is applied to graph  $G''$ , and an approximate solution  $C$  is then obtained. An approximate solution to the

partially charging reward maximization problem finally can be derived from the solution  $C$  as follows.

Assume that the solution  $C$ , by the approximation algorithm for the fully charging reward maximization problem in  $G_1$ , contains  $k$  virtual copies  $v_{i_1}, v_{i_2}, \dots, v_{i_k}$  of sensor  $v$  with  $1 \leq i_1 \leq i_2, \dots \leq i_k \leq K_v$ , where  $K_v$  is the number of virtual copies of sensor  $v$ . We then form another solution  $C'$  from  $C$ , by replacing the  $k$  virtual copies  $v_{i_1}, v_{i_2}, \dots, v_{i_k}$  of sensor  $v$  with its other  $k$  virtual copies  $v_1, v_2, \dots, v_k$ . It can be seen that  $C'$  is still a feasible solution to the problem, and the length of closed tour  $C'$  is identical to the one of  $C$ , which actually is the total amount of energy consumption of the mobile charger on this closed tour, while the sum of prizes collected from the virtual copies  $v_1, v_2, \dots, v_k$  is no less than that from the  $k$  virtual copies  $v_{i_1}, v_{i_2}, \dots, v_{i_k}$  due to the fact that  $\pi(v_j) \geq \pi(v_{i_j})$ , which will be shown later in Lemma 3, for all  $j$  with  $1 \leq j \leq k \leq K_v$ . The sum of prizes of virtual copies  $v_1, v_2, \dots, v_k$  of sensor  $v$  is  $\sum_{i=1}^k \pi(v_i) = \sum_{i=1}^k \left\lfloor \frac{f(RE_v + i \cdot e_{min}) - f(RE_v + (i-1) \cdot e_{min})}{\delta} \right\rfloor \leq \left\lfloor \frac{f(RE_v + k \cdot e_{min}) - f(RE_v)}{\delta} \right\rfloor$ , which is no greater than the actual prize collected from sensor  $v$  by charging it with the amount  $k \cdot e_{min}$  of energy with  $1 \leq k \leq K_v$ .

Note that following the constructions of the auxiliary graphs  $G'$  and  $G''$ , a given sensor  $v$  may appear in the solution  $C''$  multiple times, each of its appearances at a difference position in  $C''$  corresponds charging a different amount of energy to the sensor, and the total amount of energy charged to sensor  $v$  is no more than  $B_v - RE_v$  per tour.

### B. Approximation Algorithm

The detailed algorithm for the partially charging reward maximization problem is described in Algorithm 3.

In Algorithm 3, it is noted that if energy capacities of different sensors are substantially different, then sensors with large energy capacities may need to be charged many times while sensors with small energy capacities may be charged a few times, depending on the setting of the minimum amount  $e_{min}$  of energy charged per sensor each time. Thus, the number of virtual copies of each sensor and edges in  $G'$  become a function of  $e_{min}$ . The approximate solution may not be achievable within polynomial time. To devise a polynomial time approximation algorithm for the problem and to *fairly charge sensors with different energy capacities*, we assume that each sensor can be charged with a fixed number of charges by the mobile charger per tour. That is, each sensor can be charged no more than  $K$  times per tour, where  $K (\geq 0)$  is a given constant integer. Under this assumption, we can modify the approximation algorithm, Algorithm 3, as follows.

Given a sensor  $v$  with residual energy  $RE_v$ , we assume that the minimum amount of energy charged per charging is  $e_{min}$  and the maximum number of chargings to sensor  $v$  per tour is no more than  $K$ . Then, each time sensor  $v$  can be charged with the amount  $\Delta_v$  of energy with  $\Delta_v = \frac{B_v - RE_v}{K}$ . If  $\Delta_v \leq e_{min}$ , then, the minimum amount of energy charged to  $v$  at each time is  $e_{min}$ , and the number of chargings to  $v$  per tour is no more than  $K' = \left\lfloor \frac{B_v - RE_v}{e_{min}} \right\rfloor \leq \left\lfloor \frac{B_v - RE_v}{\Delta_v} \right\rfloor \leq K$ . Thus, each sensor will be charged no more than  $K$  times per tour.

**Algorithm 3** finding a closed tour for the mobile charger for the partially charging reward maximization problem

**Input:** An undirected metric graph  $G = (V_c, E_c; l')$ , a depot  $r \in V$  and a submodular function, assume that each requested sensor in  $V_c$  is expressed as  $(id_v, RE_v, 0)$ , Let  $\Delta_{max}$  and  $\Delta_{min}$  be the maximum and minimum amounts of charging energy among the sensors, and  $e_{min}$  the minimum amount of energy charged to a sensor,  $\delta > 0$  to be defined in the analysis of this algorithm.

**Output:** A closed tour  $C''$  including the root  $r$  so that the charging reward of all nodes in  $C''$  is maximized, while the total energy consumption of the mobile charger is no more than  $IE$ . Note that a sensor  $v$  can appear multiple times in  $C$  and each of its appearances has a certain amount of energy charged and the total amount of energy charged to it is no more than  $(B_v - RE_v)$  per tour.

- 1: Construct an auxiliary graph  $G_1 = (V_1, E_1)$  from the original graph  $G_c = (V_c, E_c)$ , where for each node  $v \in V$  there are  $K_v = \lfloor \frac{B_v - RE_v}{e_{min}} \rfloor$  virtual copies of sensor  $v$ ,  $v_1, v_2, \dots, v_{K_v}$ . Note that each node in  $G'$  has a weight  $e_{min}$  which is the minimum amount of energy charged to the sensor per time, and each edge has a weight which is the travel distance between two sensors as the endpoints of the edge;
- 2: Calculate the prize of each virtual copy  $v_i$  of sensor  $v$ , which is  $\pi(v_i) \leftarrow \lfloor \frac{f(RE_v + i \cdot e_{min}) - f(RE_v + (i-1) \cdot e_{min})}{\delta} \rfloor$  with  $1 \leq i \leq K_v$ , where  $\delta = \frac{\Delta f_{max}}{n}$  and  $\Delta f_{max} = \max_{v \in V_c} \{f(RE_v + \Delta_v) - f(RE_v)\}$ ;
- 3: Construct another auxiliary graph  $G' = (V', E'; \pi, w)$  from  $G_1$  by applying the novel reduction technique in the previous section. As a result, graph  $G'$  is a node and edge weighted undirected graph, where its edge weight represents the energy consumption by traveling along the link and charging the sensor (an endpoint of the link), and the node weight is the prize collected from the node by charging it with the amount  $e_{min}$  of energy, the prize of a virtual copy  $v_i$  of sensor  $v$  is  $\pi(v_i)$ ;
- 4:  $\delta' \leftarrow \frac{\Delta_{max}}{n^2}$ ;
- 5: Construct the auxiliary graph  $G''(V', E'; \pi, l)$  from  $G'$  by first dividing the weight of each edge  $e$  in  $G'$  by  $K\delta'$ , and then rounding the value up to a nearest integer, i.e.,  $l(e) = \lceil \frac{w(e)}{K\delta'} \rceil$ , where  $K = \max\{K_v \mid v \in V_c\}$ ;
- 6:  $L \leftarrow \lfloor \frac{IE}{K\delta'} \rfloor$ ;
- 7: Find a maximum reward closed tour  $C$  (or a maximum utility path from  $r_{0,0}$  to  $r'_{0,0}$ ) in  $G''$  with the tour length being upper bounded by  $L$ , while the length of the path  $P_{r,r'}$  is no greater than  $L$ , by applying the approximation algorithm for the orienteering problem due to Bansal *et al.* [1];
- 8: A corresponding closed tour  $C'$  is then constructed from  $C$ , by replacing the  $k$  virtual copies of each sensor  $v$  in  $C$  with its the first  $k$  virtual copies, assuming that there are  $k$  virtual copies of sensor  $v$  in  $C$  and  $1 \leq k \leq K_v$  while  $K_v = \lfloor \frac{B_v - RE_v}{e_{min}} \rfloor$ ;
- 9: Construct a closed tour  $C''$  for the partially charging reward maximization problem, by replacing the virtual copies of each sensor in  $C'$  with the sensor itself. If multiple virtual copies of a sensor appear in  $C'$  consecutively, only one copy and the number of such copies are kept. The closed tour  $C''$  then is obtained, which is a sequence of sensors and each appearance of a sensor also contains the amount of energy charged to the sensor.
- 10: **return**  $C''$ .

In the rest of discussion, we assume that  $\Delta_v \geq e_{min}$ ,  $K$  virtual copies  $v_1, v_2, \dots, v_K$  of sensor  $v$  are created, and each of the virtual copies  $v_i$  corresponds to an amount  $\Delta_v$  of energy charged to  $v$ . The prize of a virtual copy  $v_i$  of sensor  $v$  thus is  $\pi(v_i) = \lfloor \frac{f(RE_v + i \cdot \Delta_v) - f(RE_v + (i-1) \cdot \Delta_v)}{\delta} \rfloor$ , where  $1 \leq i \leq K$ .

### C. Algorithm Analysis

We now analyze the performance of the algorithms. To this end, we first show that  $\pi(v_1) \geq \pi(v_2) \geq \dots \geq \pi(v_K)$  for the  $K$  virtual copies of sensor  $v$  by Lemma 3.

**Lemma 3:** For each given sensor  $v \in V_c$  in  $G_c = (V_c, E_c; l')$ , let  $v_1, v_2, \dots, v_K$  be its  $K$  virtual copies, then  $\pi(v_1) \geq \pi(v_2) \geq \dots \geq \pi(v_K)$ .

**Proof:** For any two virtual copies  $v_i$  and  $v_j$  of a sensor  $v$  with  $i < j$ , the prizes collected by charging the same amount  $\Delta_v$  ( $= \frac{B_v - RE_v}{K}$ ) of energy to sensor  $v$  are different. We thus have  $\pi(v_i) \geq \pi(v_j)$  due to

$$\begin{aligned} \frac{\pi(v_j)}{\pi(v_i)} &= \frac{\lfloor \frac{f(RE_v + j \cdot \Delta_v) - f(RE_v + (j-1) \cdot \Delta_v)}{\delta} \rfloor}{\lfloor \frac{f(RE_v + i \cdot \Delta_v) - f(RE_v + (i-1) \cdot \Delta_v)}{\delta} \rfloor} \\ &\leq \frac{\lfloor \frac{f(RE_v + j \cdot \Delta_v) - f(RE_v + (j-1) \cdot \Delta_v)}{\delta} \rfloor}{\lfloor \frac{f(RE_v + j \cdot \Delta_v) - f(RE_v + (j-1) \cdot \Delta_v)}{\delta} \rfloor}, \\ &\quad \text{as } f(x + \Delta) - f(x) \geq f(y + \Delta) - f(y) \text{ if } x < y. \\ &= 1. \end{aligned} \tag{3}$$

□

We now determine the value of  $\delta$  such that the minimum prize  $\pi(v_i)$  of any virtual copy of a sensor  $v$  is no less than 1, while the maximum prize  $\pi(u_j)$  of a virtual copy  $u_j$  of a sensor  $u$  is no greater than  $n$ . Denote by  $\Delta f_{min} = \min_{v \in V_c} \{f(B_v) - f(B_v - \Delta_v)\}$  and  $\Delta f_{max} = \max_{v \in V_c} \{f(RE_v + \Delta_v) - f(RE_v)\}$ , where  $\Delta_v = \frac{B_v - RE_v}{K}$ . Assume that  $\Delta f_{min} \geq \frac{\Delta f_{max}}{n}$ . Then,  $\delta$  is defined as follows.

$$\delta = \frac{\Delta f_{max}}{n}. \tag{4}$$

Its correctness is shown in Lemma 4.

**Lemma 4:** Let  $\delta = \frac{\Delta f_{max}}{n}$ , then  $1 \leq \pi(v_i) \leq n$  for any virtual copy  $v_i$  of a sensor  $v \in V_c$  with  $1 \leq i \leq K$ .

**Proof:** We show the claim by two cases. Case 1: assume that  $u_j$  is a virtual copy of sensor  $u$  in  $G'$  that has the minimum prize, then  $j = K$ , following Lemma 3. We have

$$\begin{aligned} \pi(u_K) &= \lfloor \frac{f(RE_u + K \cdot \Delta_u) - f(RE_u + (K-1) \cdot \Delta_u)}{\delta} \rfloor \\ &= \lfloor n \cdot \frac{f(B_u) - f(B_u - \Delta_u)}{\Delta f_{max}} \rfloor \geq \lfloor n \cdot \frac{\Delta f_{min}}{\Delta f_{max}} \rfloor, \\ &\geq \lfloor n \cdot \frac{n}{\Delta f_{max}} \rfloor, \text{ since } \Delta f_{min} \geq \Delta f_{max}/n \\ &= 1. \end{aligned} \tag{5}$$

Case 2: assume that  $v_i$  is a virtual copy of sensor  $v$  in  $G'$  that has the maximum prize, then  $i = 1$ , following Lemma 3.

We then have

$$\begin{aligned}\pi(v_1) &= \lfloor \frac{f(RE_v + \Delta_v) - f(RE_v)}{\delta} \rfloor \\ &= \lfloor n \cdot \frac{f(RE_v + \Delta_v) - f(RE_v)}{\Delta f_{max}} \rfloor \\ &\leq \lfloor n \cdot \frac{\Delta f_{max}}{\Delta f_{max}} \rfloor, \quad \text{as } v_1 \text{ has the maximum prize} \\ &= n.\end{aligned}\tag{6}$$

Thus, for any virtual copy  $v_j \in V'$  derived from any sensor  $v$ , the range of its prize  $\pi(v_j)$  is between 1 and  $n$ .  $\square$

We now have the following theorem.

**Theorem 3:** Given a sensor network  $G_s(V_s, E_s)$ , a subset  $V_c (\subseteq V_s)$  of sensors to be charged by a mobile charger with the energy capacity  $IE$ , there is an approximation algorithm, Algorithm 3, for the partially charging reward maximization problem in  $G_c = (V_c, E_c; l')$ , assuming that each sensor can be charged no more than  $K$  times and  $K \geq 1$  is a given constant. Algorithm 3 delivers a solution with approximation ratio of 4. The time complexity of Algorithm 3 is  $T_{ort}(3K|V_c|, (K^2|V_c| + 4K|E_c| + 3K|V_c|)/2)$ , where  $T_{ori}(|V|, |E|)$  is the time complexity of the approximation algorithm of Bansal *et al.* [1] for the orienteering problem in a graph with  $|V|$  nodes and  $|E|$  edges.

*Proof:* Following the constructions of auxiliary graphs  $G' = (V', E'; \pi, w)$  and  $G'' = (V', E'; \pi, l)$ , which are derived from another auxiliary graph  $G_1 = (V_1, E_1; \pi_0, l')$  where  $G_1$  is derived from the graph  $G_c$ , each virtual copy  $u_j$  of a sensor  $u$  in  $G_1$  has three copies  $u_{j,0}, u_{j,1}$ , and  $u_{j,2}$  in  $G'$  and  $G''$  with  $1 \leq j \leq K$ . Following Algorithm 3, an approximate solution  $C$  delivered at Step 7 by the proposed approximation algorithm, Algorithm 1, for the charging utility maximization problem in  $G''$ , contains  $k$  ( $1 \leq k \leq K$ ) virtual copies  $v_{i_1}, v_{i_2}, \dots, v_{i_k}$  of sensor  $v$  with  $1 \leq i_1 \leq i_2, \dots \leq i_k \leq K$ . Another approximate solution  $C'$  can then formed by replacing the  $k$  virtual sensors  $v_{i_1}, v_{i_2}, \dots, v_{i_k}$  of each sensor  $v$  in  $C$  with its the first  $k$  virtual sensors  $v_1, v_2, \dots, v_k$ . Clearly,  $C'$  still is a feasible solution to the problem, and the tour length of both  $C$  and  $C'$  are identical. The sum of prizes of virtual copies  $v_1, v_2, \dots, v_k$  of sensor  $v$  is no less than that of its other  $k$  virtual copies  $v_{i_1}, v_{i_2}, \dots, v_{i_k}$  as  $\pi(v_1) \geq \pi(v_2) \geq \dots \geq \pi(v_k)$  by Lemma 3. Also, the sum of prizes collected from the virtual copies  $v_1, v_2, \dots, v_k$  of sensor  $v$  is no greater than the actual prize collected from sensor  $v$  by charging it with an amount  $k \cdot \Delta_v$  of energy.

We now analyze the actual amount of energy of the mobile charger used for charging the sensors and traversing the edges derived from the closed tour  $C$ . Let  $n_e(C)$  be the number of edges in  $C$  with  $p$  virtual sensor nodes (the mobile charger can be treated as a sensor). Clearly,  $n_e(C) = 3p$  following the similar discussion we did in the proof body of Theorem 2. Let  $v$  be a virtual sensor in  $C$ , then there are three nodes  $v_0, v_1, v_2$  in  $G'$  and  $G''$ . Assume that the amount of energy that virtual sensor  $v$  will be charged is  $A$ . Following the construction of  $G'$ , there are two edges  $(v_1, v_0)$  and  $(v_2, v_0)$  in  $G'$  that are related to energy charging to virtual sensor  $v$ , and each of them is assigned a weight is  $A/2$ , i.e.,  $w(v_1, v_0) = A/2$  and  $w(v_2, v_0) = A/2$ . These edge weights are then converted into

integers in  $G''$ , i.e.,  $l(v_1, v_0) = \lceil \frac{A/2}{K\delta'} \rceil$  and  $l(v_2, v_0) = \lceil \frac{A/2}{K\delta'} \rceil$  where  $\delta' = \frac{\Delta_{max}}{n^2}$ . Thus, the amount of energy assigned to virtual sensor  $v$  (via these two edges) may be larger than its actual demand  $A$ . However, it can be seen that the difference between the actual demand and the amount of extra energy assigned to sensor  $v$  is no greater than  $2\delta'$ . For each edge  $(u, v) \in E_c$  derived from two virtual sensors in  $C$ , its energy consumption was rounding up, the amount of energy assigned for the mobile charger passing through the edge may also be larger than its actual need, and this extra amount of energy however is no more than  $\delta'$ . The total amount of extra energy assigned to the virtual sensors and traveling edges in  $C$  is no more than  $3(p-1)\delta'$  due to the fact that the mobile charger node will not be charged by itself. Thus, there is at most the amount of extra energy  $3K(n-1)\delta'$  assigned to the sensors and travelling edges in any  $C$  as there are at most  $n$  sensor nodes to be charged in the network and each sensor node has at most  $K$  virtual sensor copies in  $G'$  and  $G''$ . In other words, the amount of unused energy of the mobile charger per tour is no more than  $3K(n-1)\delta'$ .

Following Step 6 of Algorithm 3, let  $IE'$  be the actual amount of energy  $IE'$  ( $\leq IE$ ) used by the mobile charger for charging the sensors and travelling edges that are derived from the closed tour  $C$ . It then can be seen that  $IE' \geq (IE - 3(p-1)\delta') \geq (IE - 3K(n-1)\delta') = IE - \frac{3K(n-1)\Delta_{max}}{Kn^2} > IE - \frac{3\Delta_{max}}{n}$ , where  $\Delta_{max}$  is the maximum amount of energy to charge a sensor. When the number of requested charging sensors  $n$  ( $= |V_c|$ ) in the network is quite large, the term  $\frac{3\Delta_{max}}{n}$  approaches zero, i.e., there is almost no any energy left when the mobile charger returns its depot.

Let  $\Pi(v)$  be the actual prize collected from sensor  $v$  by charging it an amount  $k \cdot \Delta_v$  of energy per tour of the mobile charger, then

$$\begin{aligned}\Pi(v) &= \lfloor \frac{f(RE_v + k \cdot \Delta_v) - f(RE_v)}{\delta} \rfloor \\ &= \lfloor \sum_{i=1}^k \frac{f(RE_v + i \cdot \Delta_v) - f(RE_v + (i-1) \cdot \Delta_v)}{\delta} \rfloor \\ &\geq \sum_{i=1}^k \lfloor \frac{f(RE_v + i \cdot \Delta_v) - f(RE_v + (i-1) \cdot \Delta_v)}{\delta} \rfloor \\ &= \sum_{i=1}^k \pi(v_i).\end{aligned}\tag{7}$$

A closed tour  $C''$  finally can be derived from  $C'$  by replacing each virtual copy of a sensor with the sensor and the amount of energy charged in the tour, which is an approximate solution to the partially charging reward maximization problem in  $G_c$ . Thus, the total prize collected from all charged sensors in  $C''$  is

$$\begin{aligned}\sum_{v \in C''} \Pi(v) &\geq \sum_{v_i \in C'} \pi(v_i), \quad v_i \text{ is a virtual copy of sensor } v \in V_c \\ &\geq \sum_{v_{j_i} \in C} \pi(v_{j_i}), \quad v_{j_i} \text{ is a virtual copy of sensor } v \\ &\quad \text{and } v_{j_i} \text{ was replaced by } v_i \text{ in } C' \\ &\geq \frac{OPT}{4}, \quad \text{by Theorem 2,}\end{aligned}\tag{8}$$

where  $OPT$  is the cost of the optimal solution.

The rest is to analyze the time complexity of Algorithm 3. The construction of auxiliary graph  $G_1(V_1, E_1)$  takes  $O(|V_1| + |E_1|)$  time, as  $|V_1| = K|V_c|$ ,  $|E_1| = K(K - 1)|V|/2 + 2K|E_c|$ , and  $K$  is constant. The construction of the auxiliary graph  $G'$  takes  $O(|V_c| + |E_c|)$  time, as it contains  $|V'| = 3K|V_c|$  nodes and  $|E'| = K(K - 1)|V_c|/2 + 2K|V_c| + 2K|E_c| = (K^2|V_c| + 4K|E_c| + 3K|V_c|)/2$  edges. The time complexity of Algorithm 3 thus is  $T_{ort}(3K|V_c|, (K^2|V_c| + 4K|E_c| + 3K|V_c|)/2)$ .  $\square$

## VII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed algorithms by experimental simulations. We also study the impact of important parameters on their performance.

### A. Simulation Environment

We consider a WSN consisting of from 50 to 200 sensors randomly deployed in a  $1,000 \times 1,000 m^2$  square area. The base station and the depot of the mobile charger are co-located at the center of the monitoring area. The battery capacity of each sensor is set at  $10.8 kJ$  [17]. The residual energy of each sensor is randomly generated in the range of  $(0, 10.8] kJ$ . A sensor will send a recharging request to the base station once its energy level is below a given threshold, which is 20% of its energy capacity. The energy capacity  $IE$  of the mobile charger is set to  $300 kJ$ , and it consumes  $600 J$  per meter when travelling [16]. The default value of  $K$  in Algorithm 3 is 3. The submodular function used in Algorithm 3 is  $\log(x+1)$ . Unless otherwise specified, these parameters will be adopted in default settings. Each value in all figures is the average of the results by applying each mentioned algorithm 50 times on different network topologies with the same network size.

We evaluate the performance of the proposed algorithms against that of the following three state-of-the-art heuristics.

The first one finds a minimum spanning tree (MST) in  $G_c$ , and then derives a cycle from the MST using the triangle inequality property. If the total energy consumption on the traveling of the mobile charger and charging the sensors in the cycle is no greater than the energy capacity  $IE$  of the mobile charger, the solution is a feasible solution. Otherwise, the heuristic removes a node  $v$  from the cycle such that the ratio of the reward to its distance in the cycle is minimized, and a shorter cycle then is formed. This procedure continues until the total energy consumption in the resulting cycle is no more than  $IE$ .

The second one charges sensors greedily. That is, the next sensor to be charged is the sensor with the least residual energy, and the mobile charger ends its tour if all its energy runs out.

The third one first reduces the original problem to a Capacitated Minimum Spanning Tree (CMST) problem, and then extends the Esau-Williams algorithm [3] to jointly consider sensor charging energy and travelling energy [20].

For simplicity, we refer to the first heuristic for the fully and partially charging reward maximization problems as algorithms  $Heu\_FULL$  and  $Heu\_PART$ , the second heuristic as

algorithms  $LEF\_FULL$  and  $LEF\_PART$ , and the third heuristic as algorithms  $CMST\_FULL$  and  $CMST\_PART$ , respectively. We refer to algorithms 2 and 3 in this paper as algorithms  $App\_FULL$  and  $App\_PART$ , respectively.

### B. Performance of Different Algorithms

We first evaluate the two proposed algorithms against the three mentioned heuristic algorithms, by varying network sizes from 50 to 200. Fig 3 shows the total reward, the total amount of energy charged to sensors and spent on the travelling of the mobile charger, and the running times of all mentioned algorithms. It can be seen from Figures 3(a), (b), (c), and (d) that the total reward by algorithm  $App\_FULL$  is around 25%, 35%, and 45% more than those by algorithms  $CMST\_FULL$ ,  $Heu\_FULL$ , and  $LEF\_FULL$  respectively when the network size is 100, while the total energy for sensor charging is roughly 6%, 8%, and 10% more than those by algorithms  $CMST\_FULL$ ,  $Heu\_FULL$ , and  $LEF\_FULL$ , respectively. It is noticed that the better solution delivered by algorithm  $App\_FULL$  is at the expense of more running time, compared with the other algorithms. Notice that the running times of different algorithms are obtained in a desktop with limited computing power, which however can be much improved if a power-full server is deployed at the base station. In addition, it can also be seen from Fig. 3 (c) that more energy will be spent on travelling in the solutions delivered by each of the algorithms with the growth of network size, because sensors to be charged in a large network are very likely distributed in a wider region far from the depot of the mobile charger. Since the energy capacity  $IE$  of the mobile charger is fixed, the amount of energy used for sensor charging will decrease with the growth of network size, as shown in Fig. 3(b). Furthermore, the total reward collected in the solutions delivered by all mentioned algorithms increase, since there are more sensors to be charged and there are much more opportunities to select the sensors with larger rewards to be charged. Similar performance for algorithms  $App\_PART$ ,  $CMST\_PART$ ,  $Heu\_PART$ , and  $LEF\_PART$  can also be observed from Figures 3(e), (f), (g), and (h), respectively. In addition, from Fig. 3(b) and Fig. 3(f) It can be seen that the energy charged to sensors by algorithm  $App\_FULL$  is slightly less than that by algorithm  $App\_PART$ , the rationale behind is that unlike fully charging, partial charging can avoid charging the next sensor with less residual energy (i.e.,  $RE_v$ ) that is far away from the currently being charged sensor, thereby saving energy that is spent on travelling (as shown by the comparisons of Fig. 3(c) and Fig. 3(g)).

### C. Impact of Parameters

We then study the impact of different parameters on the performance of different algorithms.

We start by investigating the impact of the energy capacity  $IE$  of the mobile charger on the performance of different algorithms, by varying  $IE$  from  $200 kJ$  to  $350 kJ$ . Figures 4(a), (b), (c), and (d) depict the total reward, the total amounts of energy on the travelling and sensor charging, and the running times of algorithms  $App\_FULL$ ,  $CMST\_FULL$ ,  $Heu\_FULL$ , and  $LEF\_FULL$ , respectively.

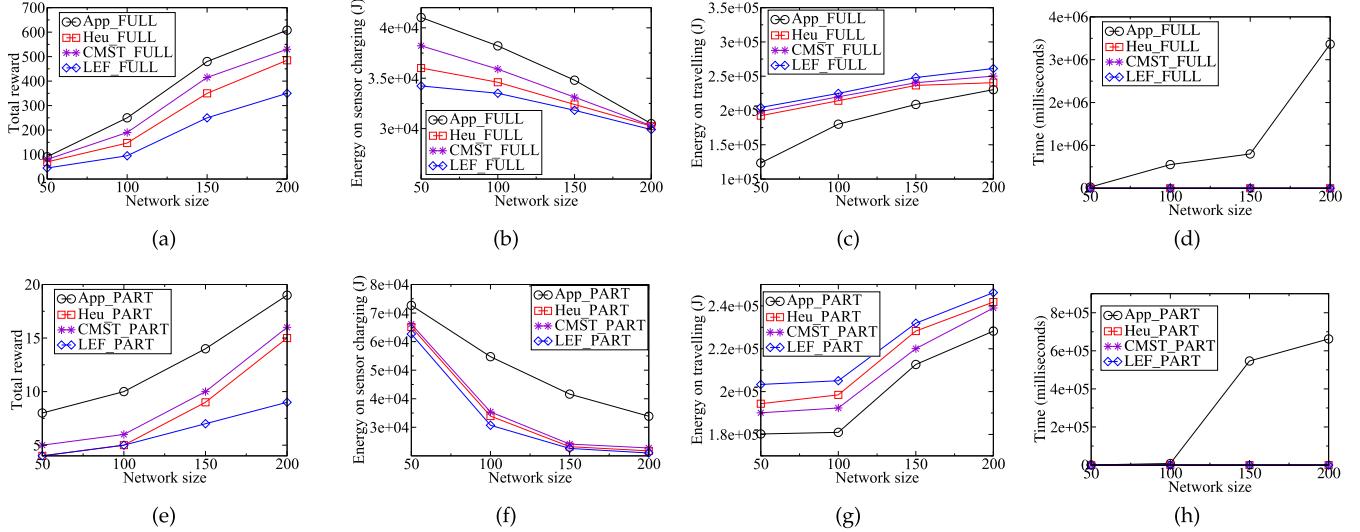


Fig. 3. The performance of different algorithms by varying network sizes. (a) The total energy on sensor charging. (b) The total energy on travelling. (c) The total reward. (d) The running times of algorithms. (e) The total energy on sensor charging. (f) The total energy on travelling. (g) The total reward. (h) The running times of algorithms.

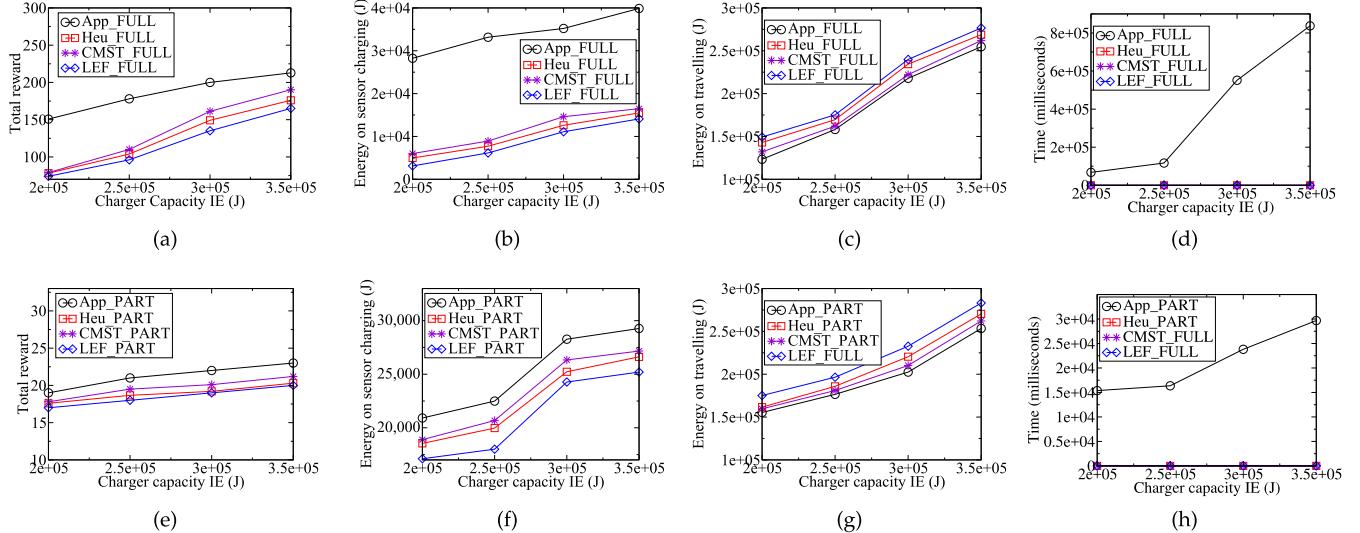


Fig. 4. Impacts of the energy capacity  $IE$  of the mobile charger on the performance of different algorithms. (a) The total energy on sensor charging. (b) The total energy on travelling. (c) The total reward. (d) The running times of algorithms. (e) The total energy on sensor charging. (f) The total energy on travelling. (g) The total reward. (h) The running times of algorithms.

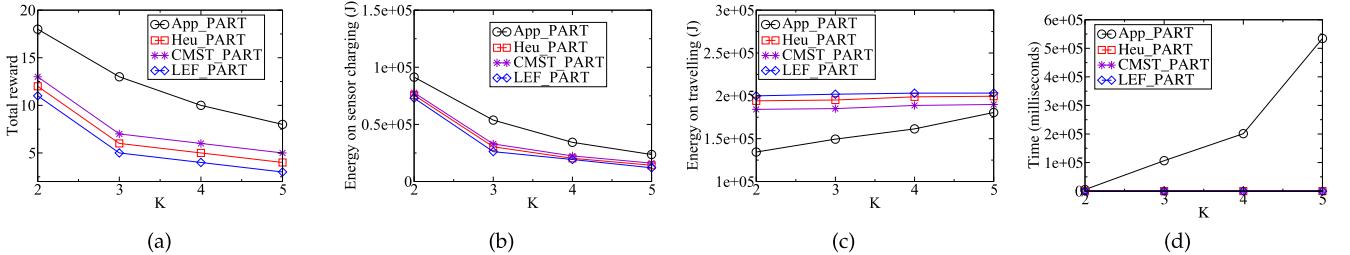


Fig. 5. The impact of  $K$  on the performance of algorithms  $App\_PART$ ,  $Heu\_PART$ ,  $CMST\_PART$ , and  $LEF\_PART$ . (a) The total reward. (b) The total energy on sensor charging. (c) The total energy on travelling. (d) Running times

It can be seen from Figures 4(a) and 4(c) that the total reward collected by algorithm  $App\_FULL$  is around 10%, 19%, and 23% more than that by algorithms  $CMST\_FULL$ ,  $Heu\_FULL$ , and  $LEF\_FULL$  respectively when  $IE = 350,000$ , while the amount of energy spent on traveling of the mobile charger is also larger than that by other algorithms  $CMST\_FULL$ ,  $Heu\_FULL$ , and  $LEF\_FULL$ . Furthermore, it can be seen that the reward collected by all

algorithms grow with the increase of the value of  $IE$ , as a larger  $IE$  enables the mobile charger to charge more sensors. The running time of algorithm  $App\_FULL$  increases with the growth of the value of  $IE$ . Similarly, from Figures 4(d), (e), and (f), it can be observed that the total award collected by algorithm  $App\_PART$  is much more than that by either of algorithms  $Heu\_PART$ ,  $CMST\_FULL$ , and  $LEF\_FULL$ .

We then study the impact of the maximum number  $K$  of charges to a sensor per tour on the performance of different algorithms, which is shown in Fig. 5. We can see from this figure that with the increase of  $K$ , the reward and energy charged to sensors by algorithms App\_PART and Heu\_PART slightly decrease. The rationale behind is that with a larger  $K$ , algorithm App\_PART will spend more energy in travelling, since the mobile charger can be scheduled to charge other sensors in different locations after its partial charging to a sensor, which can be seen in Fig. 5(c). The running time of algorithm App\_PART grows with the increase of  $K$ , as a larger  $K$  implies that more virtual sensor nodes are derived from each sensor, thereby increasing the network size of  $G_1$ .

### VIII. CONCLUSIONS

In this paper we studied the use of a mobile charger to charge energy to sensors wirelessly in a rechargeable sensor network with the aim to maximize the total reward collected from the charged sensors, subject to the energy capacity of the mobile charger. We first formulated this mobile charging scheduling problem as fully and partially charging reward maximization problems under the assumptions of full and partial energy charging to each sensor per tour, respectively. We then showed that both problems are NP-hard, and devised constant approximation algorithms for them. We finally evaluated the performance of the proposed algorithms through experimental simulations. Simulation results demonstrate that the proposed algorithms are promising, and outperform state-of-the-arts heuristics.

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